***IMPORTANT NOTE*: The videos for this unit of Math 11F are still being produced, therefore are not yet available. They should be completed soon.**

UNIT 6 LINEAR INEQUALITIES

REVIEW

MATHEMATICAL STATEMENTS

- An ***equation*** states that two expressions have the same value.
- An ***inequality*** compares the values of two expressions.
- A ***compound statement*** combines two statements using "and" or "or".
- Solving a mathematical statement means to find all the values that "satisfy" the statement, to find all the values for which the statement is true.
- The ***solution set*** can be used to describe the solution of a statement.
- A ***graph***, either on a ***number line*** or on a ***coordinate plane*** can be used to illustrate the solution set.

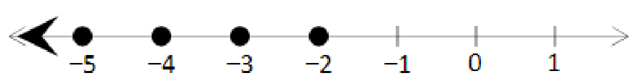
Example 1

Write the solution set and then graph the solution on a number line.

a) $3 - 2x > 5$, where $x \in \mathbb{I}$

- Solve like an equation.
- Multiply/divide by a negative, reverse inequality.

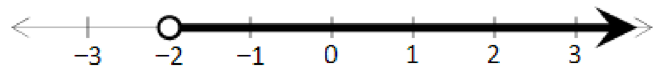
$$3 - 2x > 5$$
$$-2x > 2$$
$$\frac{-2x}{-2} < \frac{2}{-2}$$
$$x < -1$$



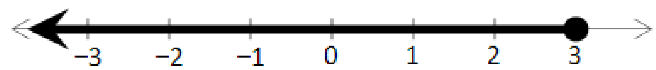
Example 2

$x > -2$ and $x \leq 3$, where $x \in \mathbb{R}$

- $x > -2$

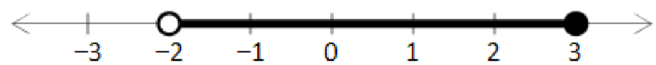


- $x \leq 3$



- The elements of the solution set are the values of x that are real numbers "greater than −2" <u>and</u> "less than or equal to 3".

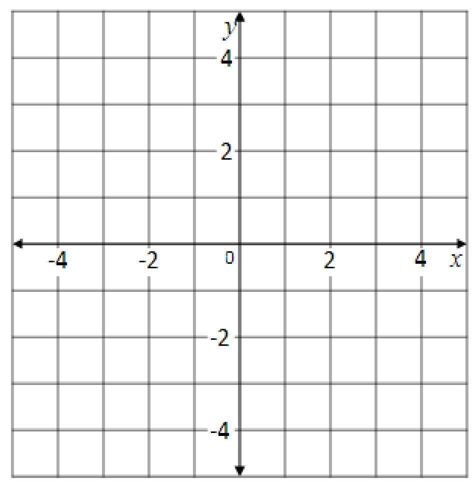
$-2 < x \leq 3, x \in \mathbb{R}$



THE COORDINATE PLANE

On the coordinate plane shown, label:

- the ***origin***
- the ***x-axis*** and the ***y-axis***
- ***Quadrant I, Quadrant II, Quadrant III, Quadrant IV***



An ***order pair***, (x,y) describes the location of a point on the coordinate plane.

RELATIONS

- A ***relation*** describes the relationship between two quantities. x and y are usually used to represent the two quantities, but other variables can be used.

example: The price of a cup of coffee is $3. The relationship between the cost and the number of cups purchased can be described:

- in ***words***: The cost of the coffee in dollars is three times the number of cups purchased.

- with an ***equation***: $C = 3n$, $n \in \mathbb{I}$, $n \geq 0$, where *C* represents the dollar cost of the coffee and *n* represents the number of cups of coffee purchased.

  Any variable introduced to be used in the equation <u>must</u> be defined.

n is the ***independent variable*** of this relation. It is the quantity for which the values are selected; we decide how many cups to buy.

C is the ***dependent variable*** of this relation. It is the quantity for which the values are calculated; it depends on the value of *n*.

- as a ***set of ordered pairs***: $\{ (0,0) , (1,3) , (2,6) , (3,9) , (4,12) , \ldots \}$

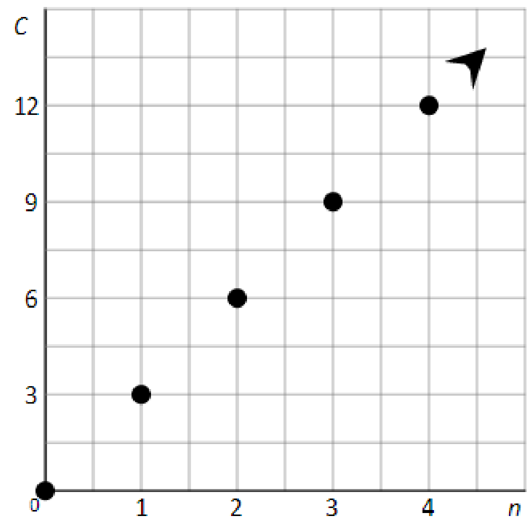
Every ordered pair *satisfies the equation*, so they are all solutions of the equation.

- with a ***graph***:

  The horizontal axis is for the independent variable; the vertical axis is for the dependent variable.

  Each point of the graph represents one ordered pair from the relation.

  These points are ***discrete*** because the points are separate and not connected; the graph would be ***continuous*** if all the points are connected.



The relation's equation describes the coordinates of the graph's points.

LINEAR RELATIONS

Equations of all linear relations can be written in the form $Ax + By = C$.

The ***x-intercept*** is the point on the linear graph that is also on the x-axis.

- The x-intercept will be point $(a,0)$ or just the number a.

The ***y-intercept*** is the point on the linear graph that is also on the y-axis.

- The y-intercept will be the point $(0,b)$ or just the number b.

Example 3

Calculate the x-intercept of $2x - 3y = 6$ without graphing.

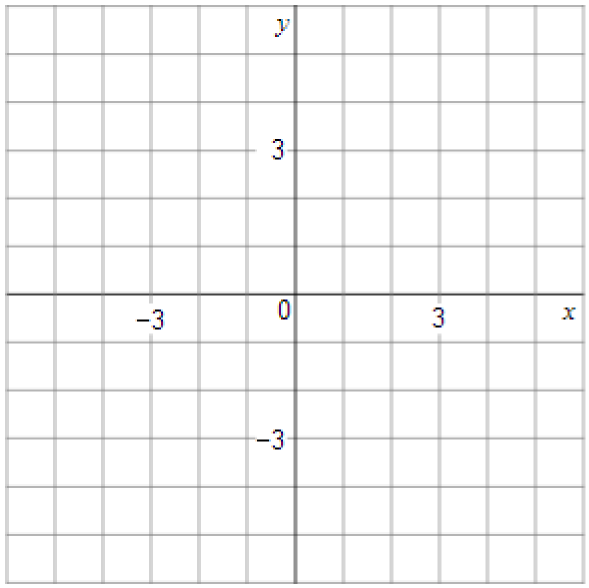
- The y-coordinate will be 0. $\quad 2x - 3(0) = 6$
- Solve to find the x-coordinate when y is 0. $\quad 2x = 6$
- Solve to find the x-coordinate when y is 0. $\quad x = 3$

Answer: The x-intercept is (3,0) or just 3

Linear graphs can be drawn using two points; intercepts are easiest.

Example 4

Graph $3x-2y=12$ using intercepts



The ***slope-intercept form*** of a linear relation's equation is $\boldsymbol{y = mx + b}$, where:

- the ***slope***; $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- b is the y-intercept of the function.

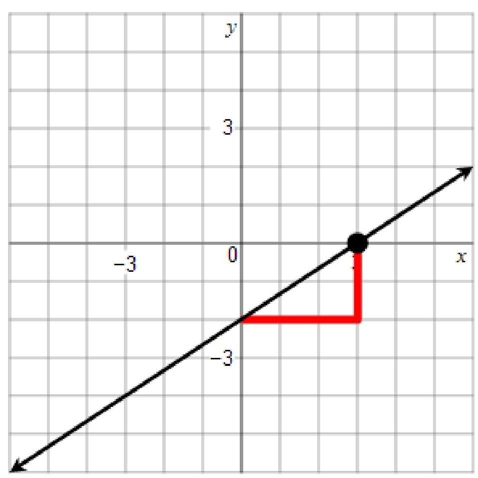
Example 5

Determine the slope and y-intercept of the straight line graph described by $2x - 3y = 6$ and then graph the relation.

- Write the equation in slope-intercept form, $y = mx + b$

$$2x - 3y = 6$$
$$-3y = -2x + 6$$
$$\frac{-3y}{-3} = \frac{-2x + 6}{-3}$$
$$y = \frac{2}{3}x - 2$$

slope, $m = \frac{2}{3}$ and y-intercept, $b = -2$



Example 6

Describe the graph of $x = -4$ and the graph of $y = 5$.

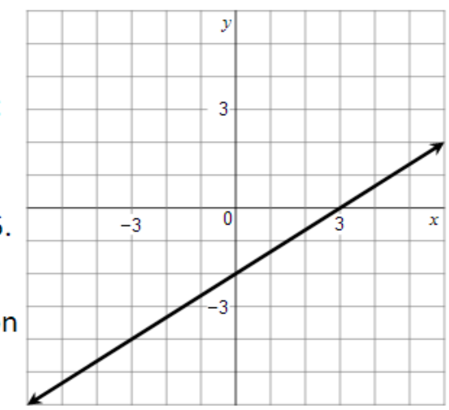
- The graph of $x = -4$ is a vertical line; points with x-coordinate -4.
- The graph of $y = 5$ is a horizontal line; points with y-coordinate 5.

LINEAR INEQUALITIES

A ***linear inequality in two variables*** is a mathematical statement like a linear equation in two variables with the equal symbol replaced with an inequality symbol, >, <, ≥, ≤.

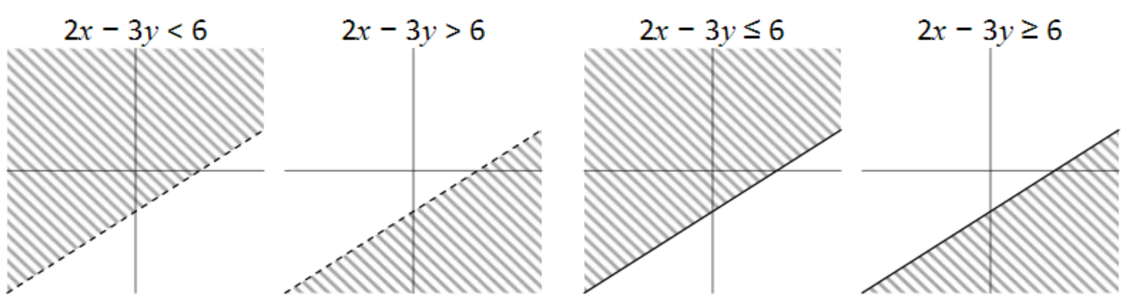
The solution is the set of all ordered pairs that satisfy the statement.

Consider the graph of the equation $2x - 3y = 6$



- All the points on the line have coordinates that satisfy the equation; $2x - 3y$ will equal 6.
- For all the other points, $2x - 3y$ will not equal 6.
  - (−2,2) is on one side of the line. For this point, $2x - 3y$ is less than 6. All the points on this side of the line satisfy the inequality $2x - 3y < 6$.
  - (3,−3) on one side of the line. For this point, $2x - 3y$ is greater than 6. All the points on this side of the line satisfy the inequality $2x - 3y > 6$.

The graph of an inequality in two variables must represent all (and only) the points with coordinates that satisfy the statement.



dashed line	solid line
points on it do not satisfy the statement	points on it also satisfy the statement

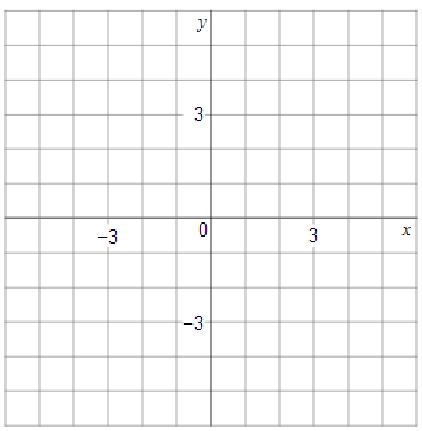
Graphing a Linear Inequality in Two Variables

- Draw the ***boundary line***:
    - dashed or solid? dashed for > or < ; solid for ≥ or ≤
    - where? graph of the corresponding equation
- Determine the ***half-plane*** containing the solution:
    - select a test point on one side of the boundary - if it satisfies the inequality, the solution is on this side; if not, the solution is on the other side
    - if the domain and range are real numbers, shade that half-plane; if the domain and range are integers, "***stipple***" the half-plane with discrete points

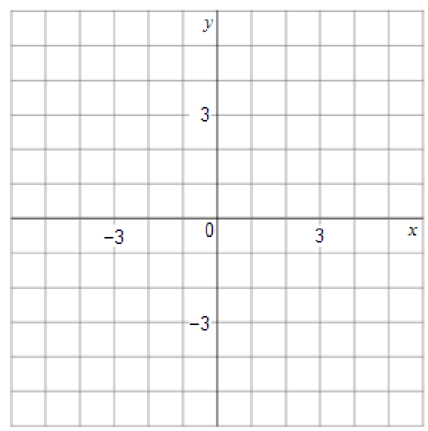
<u>Examples:</u>

Graph the following inequalities:

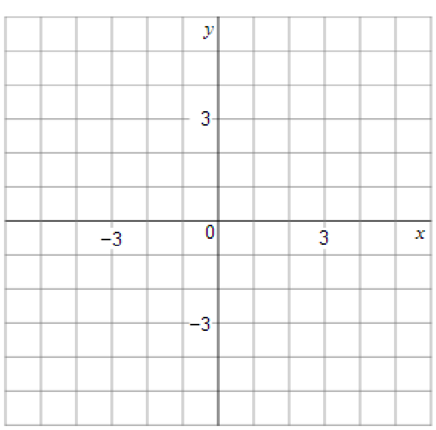
a) $\{(x,y) \mid 3x + 4y \geq -12 , x \in \mathbb{R}, y \in \mathbb{R}\}$



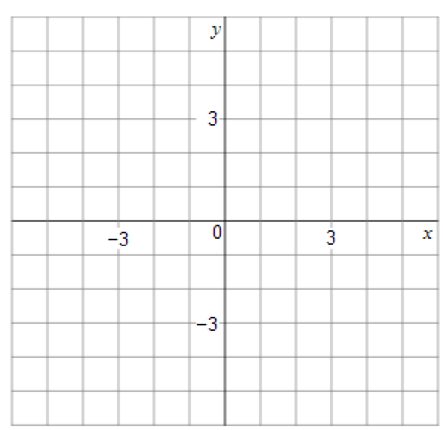
b) $\{(x,y) \mid y < -\frac{5}{2}x + 1 , x \in \mathbb{I}, y \in \mathbb{I}\}$



c) $\{(x,y) \mid x > -4 , x \in \mathbb{R}\}$



d) $\{(x,y) \mid y \leq 0 , y \in \mathbb{I}\}$



A ***system of inequalities*** is a combination of two or more inequalities.

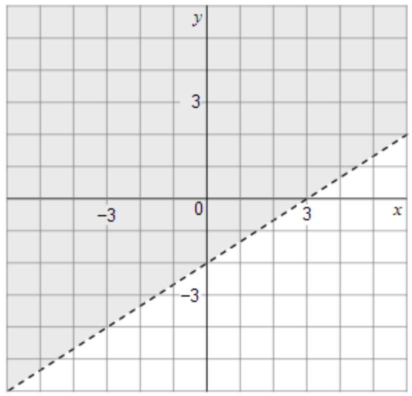
The solution of a system is the set of all ordered pairs that satisfy all the inequalities.

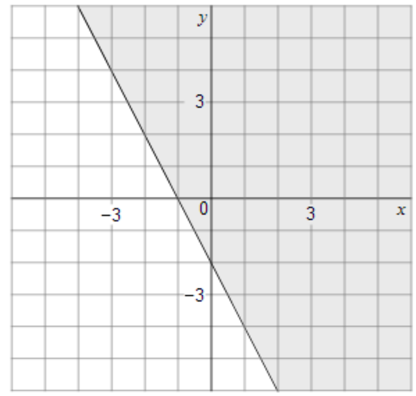
Consider the graph of the inequalities

$2x - 3y < 6$

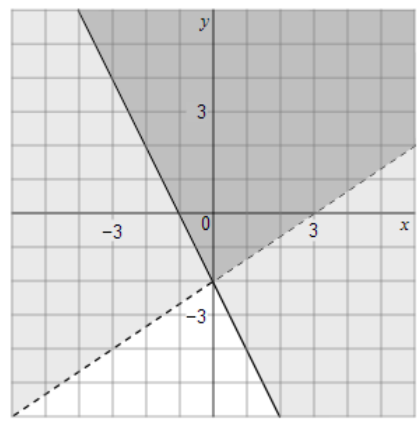
and

$y \geq -2x - 2$





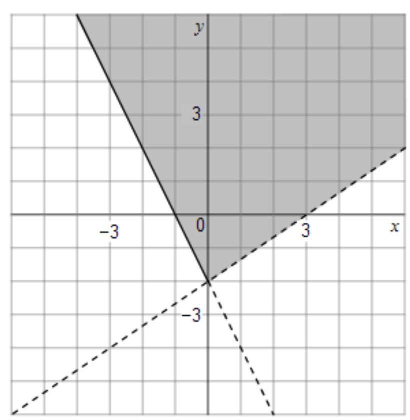
The solution of the system of linear inequalities $2x - 3y < 6$ and $y \geq -2x - 2$ are the points that satisfy both statements; that would be the intersection of the graphs for both inequalities.



Note: Some of the points on the boundary of the second inequality are in the solution set. Those that are in the shaded region of the first inequality are; those outside that shaded region are not.

Solving Systems of Inequalities

- Draw each inequality; the graph of the solution is the region where all the inequalities overlap.



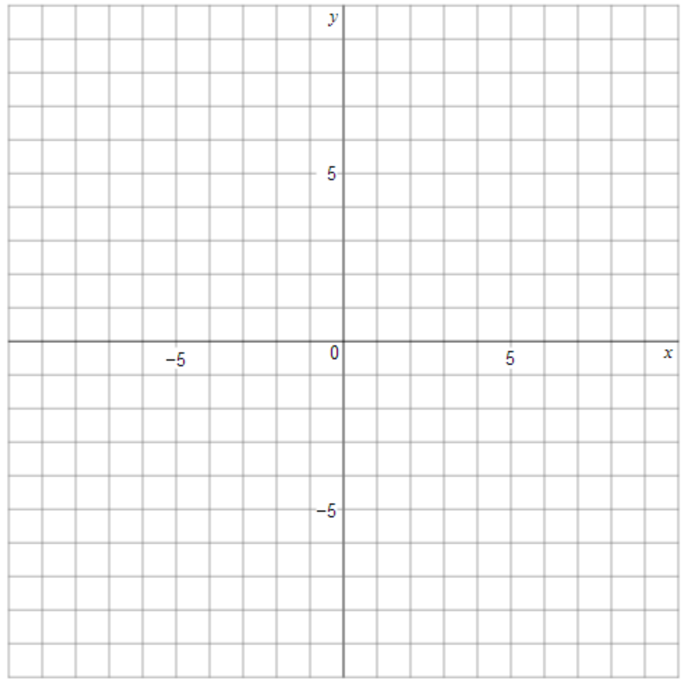
Example

Graph the following systems. Describe the region of the solution set.

a) For $x \in \mathbb{R}$ and $y \in \mathbb{R}$,

$\{ (x,y) \mid x + y < -3 \}$

$\{ (x,y) \mid x - 2y < -6 \}$



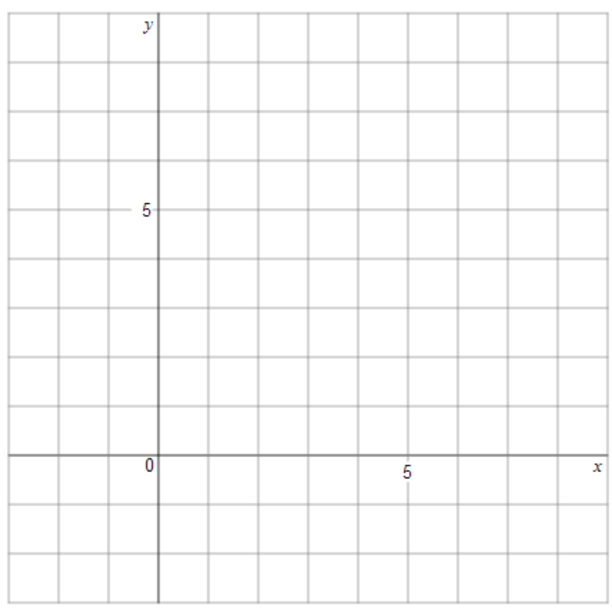
b) For $x \in \mathbb{I}$ and $y \in \mathbb{I}$,

$\{ (x,y) \mid y \geq 2x - 2 \}$

$\{ (x,y) \mid y \leq -x + 7 \}$

$\{ (x,y) \mid x \geq 0 \}$

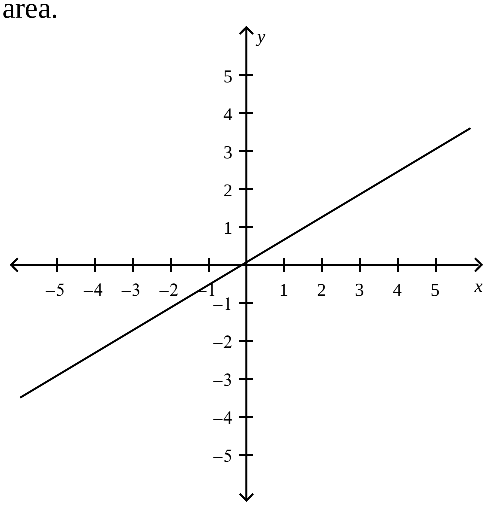
$\{ (x,y) \mid y \geq 0 \}$



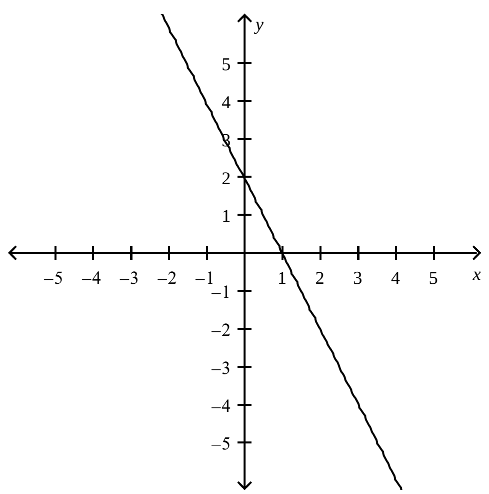
PRACTICE PROBLEMS

1. Is the point (0, 0) in the solution set for the linear inequality $10y - 12x > 5$?

2. Which side of the boundary line is the solution set for the linear inequality $5y + 3x \leq 0$? Shade the solution area.



3. Which side of the boundary line is the solution set for the linear inequality $4y + 8x \leq 2$? Shade the solution area.

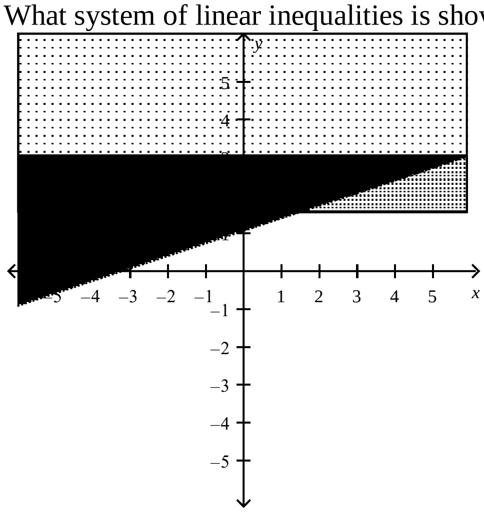


4. Graph the solution set for the linear inequality $x + y \geq 1$.

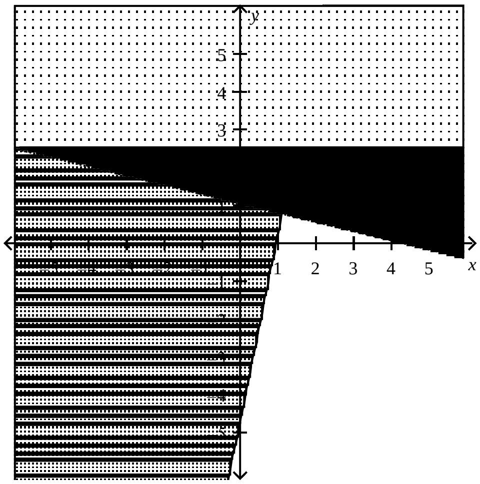
5. Graph the solution set for the linear inequality $3y - 6x < -1$.

6. Graph the solution set for the linear inequality $5y - 2x \leq 15$.

7. What system of linear inequalities is shown here?



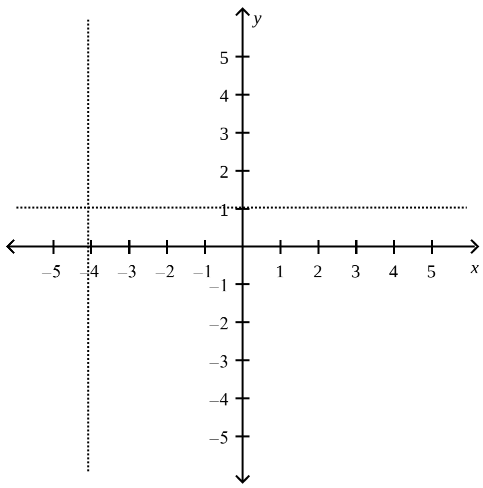
8. What system of linear inequalities is shown here?



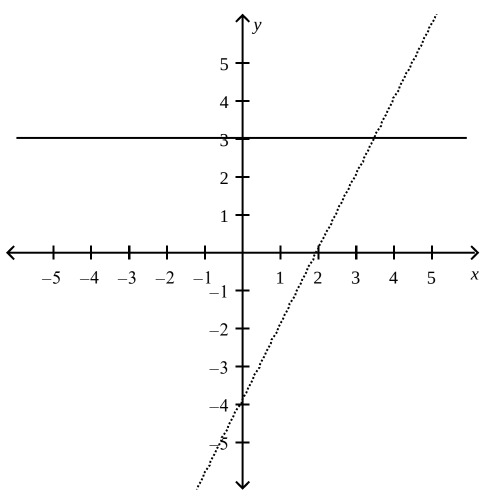
9. Graph the system of linear inequalities:
$\{(x, y) \mid x + y \leq 2, x > -3, x \in R, y \in R\}$

10. Graph the system of linear inequalities:
$\{(x, y) \mid x + y \leq 2, x > -3, x \in W, y \in W\}$

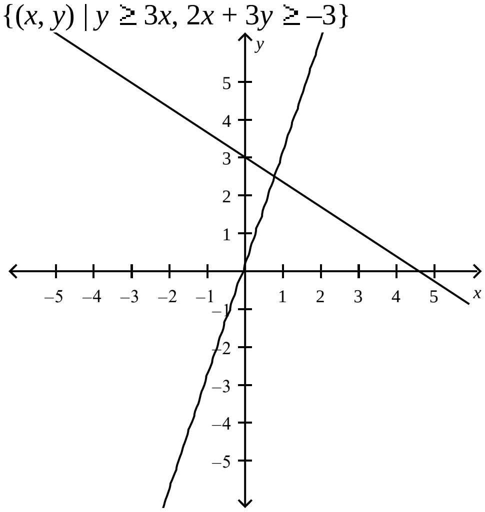
11. Complete the graph of the solution set for the following system of inequalities.
$\{(x, y) \mid y < 1, x > -4\}$



12. Complete the graph of the solution set for the following system of inequalities.
$\{(x, y) \mid y \geq 3, y > 2x - 4\}$



13. Complete the graph of the solution set for the following system of inequalities.
$\{(x, y) \mid y \geq 3x,\ 2x + 3y \geq -3\}$



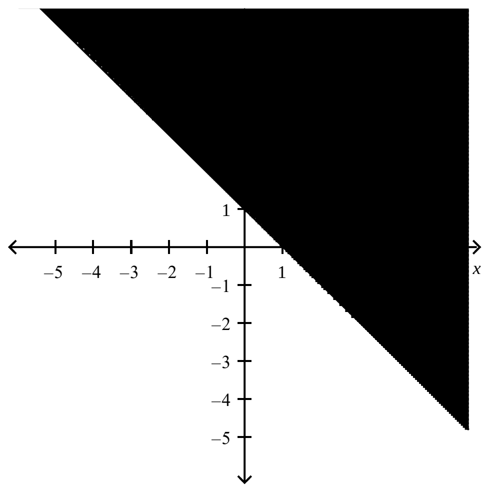
14. Graph the solution set for the following system of inequalities.
$\{(x, y) \mid x \leq 4,\ y > x - 1,\ x \in \mathrm{R},\ y \in \mathrm{R}\}$

15. Graph the solution set for the following system of inequalities.
$\{(x, y) \mid x + 2y \leq 2,\ y + 2 > x,\ x \in \mathrm{R},\ y \in \mathrm{R}\}$

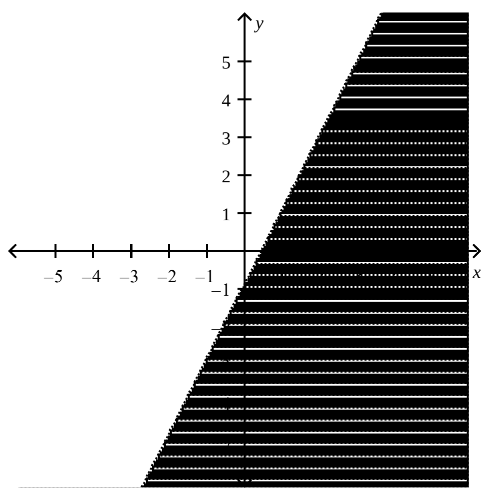
ANSWERS

1. no   2. below the line   3.below the line or to the left of the line

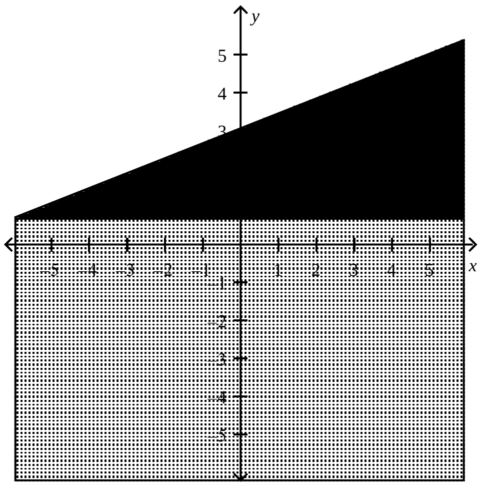
4.



5.



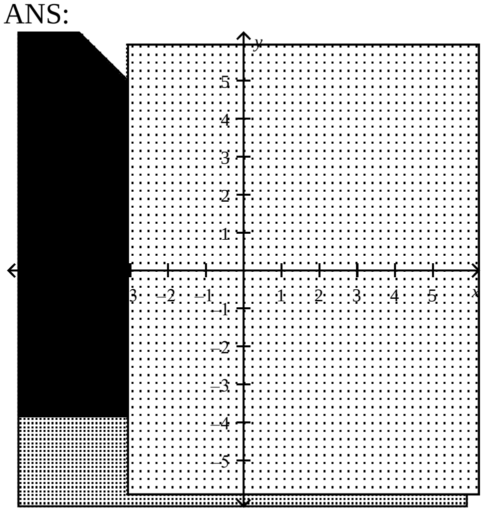
6.



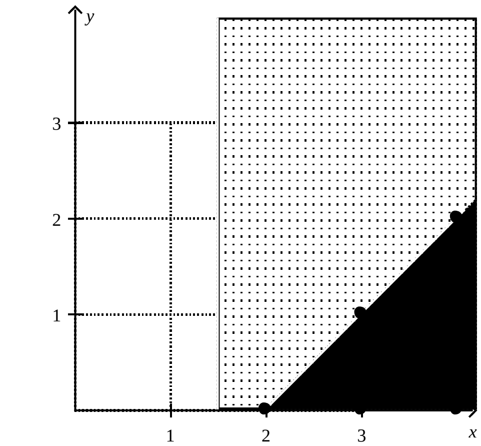
7. ANS:
{(*x*, *y*) | $y \geq 1.5$, $3y - x > 3$, $x \in$ R, $y \in$ R}

8. ANS:
{(*x*, *y*) | $x + 4y > 4$, $y \geq 5x - 5$, $x \in$ R, $y \in$ R}

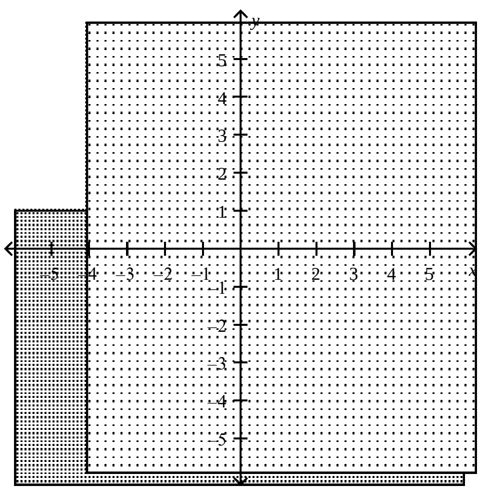
9. ANS:



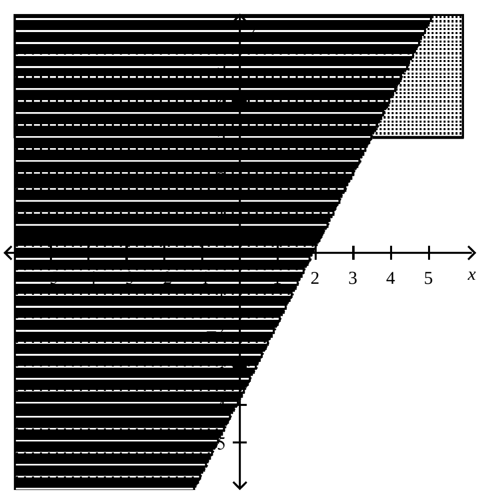
10



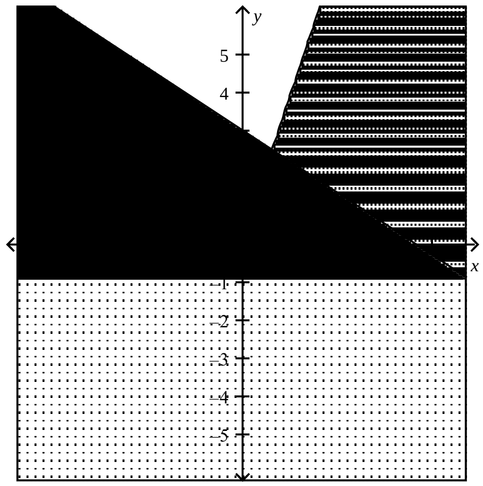
11.



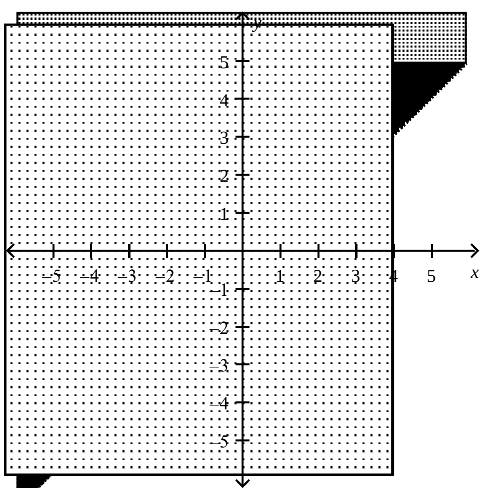
12.



13.



14.



15.