INVESTING MONEY

Watch the following instructional video. In your handout:
i) Copy down the given notes and examples
ii) Complete the assigned questions https://youtu.be/fuVcMYshLg4

The goal of investing money is to accumulate interest. There are two types of interest that we will look at. These are **Simple Interest** and **Compound Interest.**

SIMPLE INTEREST

Here are some terms that we need to be familiar with.

Principal (P): The original amount of money invested (or taken out as a loan)
Interest Rate (r): The percentage of interest that you will be paid (or pay if you have a loan)
Interest (I): The amount of money earned in an investment (or paid in a loan)
Term (t): The total time that you have invested (or borrowed) the money for.
Future Value: The total value of the investment at the end of the term. It's calculated by adding **P** and **I**

Simple interest is calculated using the formula:

$$I = P \times r \times t$$

where, **r** is expressed as a decimal and **t** in years

Example 1:

Marty invested in a $2500 guaranteed investment certificate (GIC) at 2.5% simple interest, paid annually, with a term of 10 years.

How much interest will accumulate over the term of Marty's investment?

What is the future value of his investment at maturity (at the end of the term)?

Unless otherwise stated, an interest rate is assumed to be annual (sometimes referred to as per annum). However, even though interest rates are annual, interest can be paid out at different intervals, such as annually, semi-annually, quarterly, monthly, weekly, and daily.

Example 2:

Sunni invested $15 000 in a savings account. She earned a simple interest rate of 8% paid semi-annually on her investment. She intends to hold the investment for 2.5 years, when she will withdraw all the money to buy a car.

a) Determine the value of the investment at each half year until she withdraws the money

After 0.5 years		After 1.5 years		After 2.5 years	
After 1 year		After 2 years			

b) What is the future value of her investment at maturity.

Note:
For simple interest the frequency of interest payments **does not affect the value of your investment at the end of the term.** However, you would not have to wait until the end of the year for interest to be added to your account.

Example 3:
Ingrid invested $5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday. Ingrid figures that she will need $8000 for the holiday.
a) How long will Ingrid need to invest the money for?

b) What is Ingrids **rate of return**?
(**Rate of Return**: Interest earned divided by amount of money invested. Usually expressed as a percentage)

Example 4:

Grant invested $20 000 in a simple interest Canada Savings Bond (CSB) that paid interest annually.
If the future value of the CSB is $29 375 at the end of 5 years, what is the interest rate?

Grant cashed in the bond after 4.5 years because a house that he had been admring came up for sale and he needed a down payment. How much money did he have for the down payment?

Practice

1. Determine the future value of a simple interest investment with a 4-year term on a principal of $400 at 1.9%.

2. Determine the future value of a simple interest investment where 3% interest paid quarterly for 3 years on $700.

3. Principal of $80 is invested at 2.3% simple interest, paid annually, for 4 years. What is the rate of return?

4. Patrick invested $4000 for 9 years. At the investment's maturity, its value was $5476. What was the annual simple interest rate?

5. Rosa invested $600 at 3.9% simple interest. At the investment's maturity, its value was $1302. How long was the money invested?

COMPOUND INTEREST

In **Simple Interest**, only your principal (the amount you first invested) is used to determine amount of interest earned. **Compound Interest** is paid on both the principal and the accumulated interest.

Comparing Simple and Compound Interest

Example 1:

Both Evan and Rina received a $1000 prize in a story writing contest.

Evan bought a $1000 simple interest GIC with his prize money. It has a 3-year term and earns 3.6% interest paid annually.

Rina bought a $1000 compound interest GIC with her winnings. It also has a 3-year term and earns 3.6% interest paid annually.

Compare Evan and Rina's investments.

Evan

Term (Year)	Value at Start of Year ($)	Interest Rate	Interest Earned ($)	Value at End of Year ($)

Rina

Term (Year)	Value at Start of Year ($)	Interest Rate	Interest Earned ($)	Value at End of Year ($)

Compound Interest Formula:

$$\mathbf{A} = \mathbf{P}\left(1+\frac{r}{n}\right)^{\mathbf{nt}}$$

where A is the accumulated amount of money, P is the principal (amount invested), t is the term (number of years money is invested for) , r is the interest rate (as a decimal) and n is the number of compounding periods per year (n= 1 if compounding period is yearly or annually)

Example 2:

Both Robert and Allan received a $2000 prize in a math contest. Robert bought a $2000 simple interest GIC with his money. It has a 5-year term and earns 2.6% paid annually. Allan bought a $2000 compound interest GIC. It also has a 5-year term and earns 2.6% paid annually. Calculate the future value of both investments at maturity.

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Compounding Frequencies

As with simple interest, compound interest can be paid yearly or at other pre-determined intervals. The frequency of interest payments will change the value of ***n*** in the compound interest formula. The table below shows some commonly used interest payment frequencies.

Compounding Frequency	Value of ***n***
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52

Determine the Future Value of an Investment with Semi-annual Compounding

Example 2

Max has invested a $23,000 in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. What is the future value of the investment after 10 years?

Practice

Determine the future value and the total interest earned for $1,400 invested for 15 years at 8.6% compounded semi-annually.

Determine the Future Value of an Investment with Quarterly Compounding

Example 3

Determine the future value and the total interest earned for $2,300 invested for 6 years at 7.5% compounded quarterly.

Practice

Determine the future value and the total interest earned for $520 invested for 8 years at 4.5% compounded quarterly.

The compound interest earned (***I***) on an investment at the end of any compounding period is the difference between the value of the investment at that time (***A***) and the original principal (***P***): I = A — P

Calculate Interest Earned

Example 4

Determine the total interest earned if $6,500 is invested in an account paying 6% compounded monthly for five years.

Practice

Determine the total interest earned if $520 is invested in an account paying 4.5% compounded monthly for 8 years.

The more frequent the compounding and the longer the term, the greater the impact of the compounding on the principal and the greater the future value will be.

Compare Interest on Investments with Different Compounding Periods

Example 5 Hanna wants to invest $3000 so that she can renovate her living room in about 3 years; she has the following investment options (semi-annual/ monthly/ weekly/ daily) at 4.8%:

Principal (**P**)	$3000 semi-annual	$3000 monthly	$3000 weekly	$3000 daily
Interest Rate (***r***)				
Compounding Periods				
Calculation				
Accumulated (***A***)				

Compound Interest and Future Value (Part 2)

Use TVM Solver on TI-83 Graphing Calculator (or a TVM app)

Step 1. Use TVM Solver

→ Press APPS

→ Select #1 Finance

→ Select #1 TVM Solver Step 2. Input all givens

N =
I% =
PV =

PMT =
FV = P/Y
= C/Y=

PMT: END BEGIN

Step 3. Determine the unknown

→ Go to the unknown

→ Press ALPHA

→ Press Enter

For Single-Payment Investment

N = # of Years
I% = Interest Rate
PV = Present Value (Use negative value)
PMT=Monthly Payment (Use zero if not making regular payments)
FV = Future Value
P/Y = # of Payments Per Year (Use 1)
C/Y = # of Compounding Periods Per Year
PMT: END BEGIN=Payment at the end /beginning

http://www.fncalculator.com/financialcalculator?type=tvmAdvancedCalculator

Determine Future Value Using TVM App

Example 1: Jennifer invested $4,300 in a 10-year Canada Savings Bond (CSB) that will earn 3.8% compounded annually. What is the future value of Jennifer's investment after 10 years?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Example 2 Max has invested a $23,000 in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. What is the future value of the investment after 10 years?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Example 3 Determine the future value and the total interest earned for $2,300 invested for 6 years at 7.5% compounded quarterly.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Practice: Determine the future value and the total interest earned if $520 is invested in an account paying 4.5% compounded monthly for 8 years.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Doubling time is the time it takes for an investment to double in value. A simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years.

The Rule of 72 is most accurate when the interest is compounded annually.

Estimate Doubling Time for Investments

Example 4 Chris invests $5000 by purchasing Canada Savings Bonds, which earns 9%, compounded semi-annually. Estimate and determine the doubling time.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Practice

Use the Rule of 72 to estimate the doubling time and then determine the doubling time.

Present Value	Interest Rate (%)	Compounding and Payment Frequency	Term (years)
200	4.8	monthly	
1750	5.6	semi-annually	
50	8.4	quarterly	
5500	6.5	semi-annually	

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

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Compound Interest Present Value

The present value of an investment that earns compound interest can be determined using a TVM calculator or the formula

$$\mathbf{A} = \mathbf{P}\left(1+\frac{r}{n}\right)^{\mathbf{nt}}$$

Determine the Present Value of Investments Earning Compound Interest

Example 1: Ginny is 18 years old. She has inherited some money from her parents. Ginny wants to invest some of the money so that she can buy a home when she turns 30. She estimates that she will need about $170,000 to buy a home.

a) How much does she have to invest now, at 6.5% compounded annually?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) What is the ratio of future value to present value? (NOTE: NOT the rate of return)

c) How would the ratio change if the interest rate decreased to 6% but was compounded semi-annually?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Determine the Present Value of Investments that is Compounded Quarterly

Example 2: Lana and Matt are computer scientists. They researched the costs to set up a software company. They estimate that $40 000 will be enough. They plan to set up the company in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.

a) How much money should they have invested?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) How much interest will they earn over the term of their investment?

Practice:

Mike invested in a 10-year GIC that has matured. Mike's investment is currently worth $13,009 and has been earning 9.6%, compounded monthly. How much money did he invest?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Example 3: Determine an Unknown Interest Rate and Unknown Term

Laura has invested $15,500 in a Registered Education Savings Plan (RESP). She wants her investment to grow to at least $50,000 by the time her newborn enters university, in 18 years.

a) What interest rate, compounded annually, will result in a future value of $50,000? Round your answer to two decimal places.

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y=

b) Suppose that Laura wants her $15,500 to grow to at least $60 000 at the interest rate from part a). How long will this take?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y=

Practice: An investment of $400 grew to $625 in 10 years. What was the annual interest rate if the interest was compounded monthly?

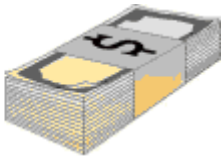
N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y=

Practice: An investment of $250 grew to $1000 at 6% interest, compounded semi-annually. How long did it take for the investment to grow?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y=

Investments Involving Regular Payments

Regular-Payment VS Single Payment



Some investments, such as CSBs, lock in money for specified periods of time, thus limiting access to the money, but offer higher interest rates. Other investments, such as savings accounts, are accessible at any time but offer lower interest rates.

For Regular-Payment Investment

N = Number of Payments (# of years × P/Y)

I% = Interest Rate

PV = Present Value (Use zero)

PMT = Monthly Payment (Use negative value)

FV = Future Value

P/Y = Number of Payments Per Year

C/Y = Number of Compounding Periods Per Year

PMT: END BEGIN = Payment at the end

Determine the Future Value of an Investment Involving Regular Deposits

Example 1 Dora deposits $500 into her savings account at the end of every 6 months. The account earns 3.8%, compounded semi-annually.

a) How much money will be in the account at the end of 5 years?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) How much of this money will be earned interest?

Practice: If Dora's deposits were only $400 every 6 months instead of $500 every 6 months, and that the interest rate on her account remains 3.8%, compounded semi-annually.

a) At the end of 5 years, how much less would the future value of the account be?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) How much interest would Dora earn?

Determine the Interest Rate of a Regular Payment Investment

Example 2

Jeremiah deposits $750 into an investment account at the end of every 3 months (regular deposits). Interest is compounded quarterly, the term is 3 years, and the future value is $10 059.07. What annual rate of interest does Jeremiah's investment earn?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Practice: Determine the interest rate each situation below, assuming the term remains 3 years, and the future value remains $10 059.07.

a) Jeremiah made payments of $800 every 3 months.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) Jeremiah made payments of $1500 every 6 months, and interest was compounded semi-annually.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Determine the Regular Payment Amount of an Investment

Example 3:

Celia wants to have $300,000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded annually.

a) What regular payments must Celia make at the end of each year to meet her goal of $300,000?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) How much interest will she earn over the 20 years?

Practice: Determine Celia's payment amount in each situation.

a) The payment frequency is every 6 months for 20 years (assume compounding is also every 6 months.

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y=

b) The payment frequency is every month for 20 years (assume compounding is also every month).

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y=

Regular-Payment VS Single Payment Investment

The future value of a single deposit has a greater future value than a series of regular payments of the same total amount. Small deposits over a long term can have a greater future value than large deposits over a short term because there is more time for compound interest to be earned.

Determine the Term of a Regular Payment Investment

Example 4: Lois makes regular $1,000 payments into an investment account at the end of every 6 months. Her investment earns 3.5%, compounded semi- annually. How many years will it take for her investment to grow more than $18,000?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Example 5. How long will it take for $20,000 payments every 3 months to be $1,080,978.04 if the interest rate is 4.75%, compounded quarterly?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Practice: How long will it take for $1000 payments every 6 months to grow to more than $10 000 if the interest rate is 7.5%, compounded semi-annually?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

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Solving Investment Portfolio Problems

An investment portfolio can be built from different types of investments, such as single payment investments (for example, CSBs and GICs) and investments involving regular payments. Some of these investments, such as CSBs, lock in money for specified periods of time, thus limiting access to the money, but offer higher interest rates. Other investments, such as savings accounts, are accessible at any time but offer lower interest rates.

Investments that involve greater principal amounts invested or greater regular payment amounts when contracted tend to offer higher interest rates.

Determine the Future Value and Doubling Time of an Investment Portfolio

Example 1: Phyllis started to build an investment portfolio for her retirement. She purchased a $500 Canada Savings Bond (CSB) at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually. Three years ago, she also purchased a $4000 GIC that earned 6%, compounded monthly.

a) What was the value of Phyllis's portfolio 10 years after she started to invest?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) Phyllis found a savings account that earned 4.9%, compounded semi- annually. She redeemed her portfolio and invested all the money in the savings account. Estimate (rule of 72) and calculate how long will it take her to double her money?

Practice: Andy is 17 years old and in Grade 12. When he was born, his parents deposited $100 each month into a savings account, earning an average annual interest rate of 3%, compounded monthly. On his 7th birthday, his grandparents bought him a 10-year $5000 GIC that earned 4%, compounded annually. He plans to start a 4-year history degree next year. He plans to redeem both investments now and combine them into one investment account that earns 4.2%, compounded quarterly, for one year until he starts school.

a) How much will Andy's parents' investment be worth when he redeems it?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) How much will his grandparents' investment be worth when he redeems it?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

c) How much will Andy's new investment account be worth when he starts school?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Compare the Rates of Return of Two Investment Portfolios

Example 2: Jason and Malik are each hoping to buy a house in 10 years. They want their money to grow so they can make a substantial down payment. Calculate Jason and Malik's rate of return.

Jason's Portfolio:

A 10-year $2000 GIC that earns 4.2%, compounded semi-annually

A savings account that earns 1.8% compounded weekly, where he saves $55 every week for 10 years

A 5-year $4000 bond that earns 3.9%, compounded quarterly, which he will reinvest in another 5-year bond at an interest rate of 4.1% compounding quarterly.

Malik's Portfolio:

A $5600 tax-free savings account (TFSA) that earns 2.2%, compounded monthly

The purchase, at the end of each year, of a 10-year $500 CSB that earns 3.6%, compounded annually

A savings account that earns 1.6%, compounded monthly, where she saves $200 every month for 10 years.

a) Determine the rate of return for Jason's investment portfolio.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) Determine the rate of return for Malik's investment portfolio.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Rate of return is a useful measure for comparing investment portfolios.

Practice

1. Tenzin has $9000 to invest for 2 years. Which investment option will earn her more interest? How much more interest?

A. 5% simple interest, paid quarterly

B. 7% compound interest, paid annually

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

2. Freda has $14 000 to invest for 10 years. Which investment option will earn her more interest? How much more interest?

A. 2.5% simple interest, paid daily

B. 1.25% compound interest, paid annually

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

3. Determine the future value and the total interest earned for the investment.

Principal (*P*) ($)	**Compound Interest Rate per Annum (%)**	**Compounding Frequency**	**Term**
16 000	5.4	monthly	4.5 years

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

4. Determine the future value and the total interest earned for the investment.

Principal (*P*) ($)	**Compound Interest Rate per Annum (%)**	**Compounding Frequency**	**Term**
9000	2.25	semi-annually	3 years

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

5. Use the Rule of 72 to estimate the investment's doubling time and then determine the actual doubling time.

Principal (*P*) ($)	**Compound Interest Rate per Annum (%)**	**Compounding Frequency**	**Term**
5000	4.5	monthly	?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Analyzing Loans (Part 1)

The large majority of commercial loans are compound interest loans, although simple interest loans are also available. The cost of a loan is the interest charged over the term of the loan. It can be determined by **Cost = Total Payment – Principal**.

A loan can involve regular payments (monthly or bi-weekly) over the term of the loan or a single payment (lump-sum) at the end of the term.

NOTE: Borrow only what you can afford.

Solve for the Future Value of a Loan with a Single Loan Payment

Example 1: Tina's employer lent her $10,000 at a fixed interest rate of 6%, compounded annually. The loan is to be repaid in a lump-sum payment at the end of 5 years.

a) How much will Tina need to pay her employer on the maturity date? What is the cost of her loan?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) Suppose the interest was compounded monthly instead. How much will Tina need to pay her employer on the maturity date? What is the cost of her loan?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

For the lump-sum payment option, N = Number of Years, PMT = 0, P/Y = 1.
Loans can be paid off at any-time.

Practice: A school ordered $1020 in books. Suppose that the bookstore offered the school a loan at 4%, compounded monthly, for 1.5 years. How much would the school need to repay the loan in a lump-sum payment on the maturity date?

What is the cost of the loan?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Example 2: Stan borrowed $1500 at 7.2%, compounded monthly, to buy a riding lawn mower for his summer business. He arranged to pay off the loan in 4 months, with a single payment. What amount did Stan need to pay on the maturity date? What is the amount of total interest Stan paid?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Practice: If Stan chooses to pay off the loan in 8 months with a lump-sum payment, what amount did Stand need to pay on the maturity date? What is the cost of the loan?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Solve for the Present Value and Interest of a Loan with a Single Payment

Example 3: Anna wants to get a home improvement loan for renovation. Her bank charges 3.6%, compounded quarterly. She has a GIC that will mature in 5 years. When her GIC reaches maturity, Anna wants to use all the money to repay the loan with one lump-sum payment of no more than $20,000.

a) How much can she borrow now? b) How much interest will she pay?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Practice: Matt needs a loan that he will not have to pay back for 18 months. The interest rate for the loan is 4.9%, compounded quarterly. On the maturity date, Matt wants to make a lump-sum payment of no more than $12,000.

a) What is the most that Matt can borrow?

b) How much interest will Matt pay on his loan?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

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Analyzing Loans (Part 2)

For the regular payment (monthly, be-weekly, etc.) option, **N** = Total Number of Regular Payments (Years × P/Y), **FV** = Remaining Amount to Pay Off, **P/Y** = Number of Regular Payments per Year

Solve for the Term and Total Interest of a Loan with Regular Payments

Example 1: Lex borrowed $12 000 at 5% interest, compounded monthly, to fund his research into a viable kryptonite weapon. Lex negotiated regular loan payments of $350 at the end of each month until the loan is paid off.

a) In which month will Lex have at least half of the loan paid off?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) How long will it take Lex to pay off the loan?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Total Payment = (Number of Payments) x (Amount of Each Payment)

c) How much interest will Lex have paid by the time he has paid off the loan?

Practice: Amber paid $1025 for her prom dress. She used her mother's credit card, which charges 18.9% compounded daily. Amber plans to make $50 payments each month.

a) When will Amber have paid half the cost of her dress?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) How long will it take Amber to repay the total amount?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

c) How much interest will Amber pay?

Mortgage Payments

Solve for the Payment and Interest of a Loan with Regular Payments

Example 1: John is negotiating with his bank for a mortgage on a house. He needs to make a 10% down payment on the purchase price of $225,000. Then the bank will offer a mortgage of the remaining, at 3.75%, compounded semi- annually, with a term of 20 years and with monthly mortgage payments.

a) What is the amount of the down payment?

b) What is the amount John needs for his mortgage?

c) What will the monthly mortgage payment be? What will the cost (interest paid) of the mortgage be?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

d) How much will he pay altogether?

Unlike loans, mortgage cannot be paid off at any-time unless you pay the penalty. There will be a chosen duration that the mortgage will be paid back in, which is called the amortization. It is usually 25, 30 or 35 years, depending on your regular payment amount. The interest rate is less likely to stay the same throughout the entire amortization. You will need to re-negotiate the rate with the bank once every few years and that's called the term. It is usually 3, 4, 5 or 6 years, depending on the economy and rate fluctuation.

Practice: Justin and Jen bought a house for $300 000. They made a 20% down payment and negotiated a mortgage at 3.82% per annum compounded semi-annually. The mortgage is amortized over 25years.

a) Determine the amount of the down payment.

b) Determine the amount to be financed.

c) Determine the monthly mortgage payment.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

d) What would their house really cost if the interest rate remained the same throughout the amortization period?

Example 3. Ryan borrowed $5 000 at 7.25%, compounded quarterly, for 2 years.

He decided to make regular monthly payments over the 2 years.

a) What is the cost of the loan?

b) How much did he pay altogether?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Relate Payment and Compounding Frequency to Interest Period

Example 4: Bill has been offered the following two loan options for borrowing $8000.
Which option is better?

Option A: He can borrow at 4.06% interest, compounded annually, and pay off the loan in annual payments of $1 800.05 at the end of each year.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Option B: He can borrow at 4.06% interest, compounded weekly, and pay off the loan in weekly payments of $34.62 at the end of each week.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Practice

1. Anya wants to renovate her house. To pay for the renovation, she took out a loan of $30 000 with an interest rate of 2.9%, compounded semi-annually. The loan must be repaid in 15 months with monthly payments. How much interest will Anya pay?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

2. Carlos was approved for a mortgage to finance his new house that he purchased for $325 000. He made a down payment that was 20% of the purchase price. The mortgage is compounded semi-annually at an interest rate of 4.2%. Carlos will repay the mortgage in 25 years with regular monthly payments. How much interest will he have to pay?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

3. Kristina took out a bank loan for $60 000 that must be repaid with regular monthly payments of $1100. The bank charges her an interest rate of 3.0%, compounded monthly. How much interest will Kristina pay?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

4. Dante wants to buy a truck that costs $35 000 and he has two different options to finance the purchase.
Option A: Finance the purchase through the dealership by making regular weekly payments for 4 years at an interest rate of 5.0%, compounded daily.
Option B: Finance the purchase with a bank loan by making regular monthly payments for 4 years at an interest rate of 5.0%, compounded daily.
What is the total paid for the cheaper option?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

Buy, Lease, or Rent?

Lease is a contract for purchasing the use of property, such as a building or vehicle, from the leaser for a specified period. Equity is the difference between the value of an item and the amount still owing on it; it can be thought of as the portion owned. Asset is an item or a portion of an item owned; also known as property.

Solve a Problem that Involves Leasing, Buying or Renting a Vehicle

Example 1: Mandy needs a vehicle for work, on average, 12 days each month.
She has three options:

She could lease a vehicle, which requires a down payment of $4000 and lease payments of $380 per month. She would need insurance at $1220 each year (which could be paid monthly) and would have to pay for repairs and some maintenance, which would average $50 each month. For the 4-year lease she is looking at, she would have no equity in the vehicle at the end of the term.

She could buy a vehicle for $32 800 and finance it for 4-year term at 4.5% interest, compounded monthly. She would have the same insurance, repairs, and maintenance costs that she would have with leasing. However, the equity of the vehicle would be considered an asset.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

She could rent at $49.99 per day, with un-limited kilometers.

Which option is the most economical choice?

Solve a Problem that Involves Leasing or Buying a Water Heater

Example 2: The water heater in Tom's home stopped working, so he needs a new one. Tom works for minimum wage. After paying his monthly expenses, he has $35 saving left every month. He has an unused credit card that charges 18.7%, compounded daily. He has two options:

a) He could buy a water heater for $712.99, plus an installation fee of $250, using his credit card. He could afford to pay no more than $35 each month. What are costs of buying?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

b) Tom could lease from his utility company for $17.25 per month. This would include parts and service. What are the costs with leasing?

c) To lease or to buy, which option is better?

When deciding whether to rent, buy (with or without financing), or lease, each situation is unique. Since each situation is unique, it is impossible to generalize about whether renting, leasing, or buying is best. A cost and benefit analysis should take everything into account. Costs include initial costs and fees, short- term costs, long-term costs, disposable income, the cost of financing, depreciation and appreciation, penalties for breaking contracts, and equity. Benefits include convenience, commitments, flexibility, and personal needs or wants, such as how often you want to buy a new car.

Appreciation is an increase in the value of an asset over time. Depreciation is a decrease in the value of an asset over time.

Solve a Problem that Involves Leasing or Buying Office Space

Example 3: Lance started his own business 2 years ago. His business has grown quickly, and his office is no longer big enough. He is considering these two options:

a) He could sign a 3-year lease on office space, with monthly rental payments of $2,000. What are the costs of leasing over 15 years?

b) He could purchase a property for $285,000 and renovate so it could be used as an office. A 5% down payment would be required, and he would take out a 15-year mortgage at 5%, compounded semi-annually, with monthly payments. Assume the property has appreciated by 34.6% over the 15 years. What are the costs of buying over 15years?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

c) Which option is better if Lance decides to retire and sell his business in 15 years?

Chapter Review

1. Yu needs a car. He can lease a car for 3 years for $300 per month and a down payment of $4100. He can purchase a new car for $28 000, which would be financed with a bank loan at an interest rate of 5.2%, compounded monthly, and a down payment of $3700. He would pay off this loan with regular monthly payments. He can also rent a car at $75 per day. What is the total cost of leasing the car?

2. Nigel is purchasing a house for $225 000 that appreciates at a rate of about 3% per year. He will finance this purchase with a 20-year mortgage at an interest rate of 4.5%, compounded semi-annually, with monthly payments, where he is required to make a 15% down payment. How much does he pay monthly?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

3. Jace needs special equipment for his job as a landscaper. He has two options. He can buy the equipment which costs $9600. Jace will finance this purchase through the vendor by making regular monthly payments over 4 years at an interest rate of 6.2%, compounded monthly. At the end of the 4 years, the equipment will be worthless. He can also lease the equipment at a cost of $180 per month. Both options require a down payment of $750. What is the total cost of the cheaper option?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

4. Arrange the investments in the order of least to greatest return and give the return.

A. $15 000 invested for 8 years at a simple interest rate of 4.7%
B. $20 000 invested for 6 years at a compound interest rate of 5%
C. $10 000 invested for 15 years at a compound interest rate of 3%

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

5. Trahn wants to buy a new sound system but he has only $1000, half the amount he needs. When can Trahn buy the sound system if he invests his money at 3.7%, compounded quarterly, to the nearest year?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

6. Consider these rates:
   - Bank A offers 5.4%, compounded annually.
   - Bank B offers 5.35%, compounded quarterly.
   - Bank C offers 5.2%, compounded monthly.

   Rank the rates from greatest to least return on an investment of $8000 for a term of 5 years.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

7. Reggie plans to retire in 30 years, when he is 65. He estimates that he will need $360 000 to supplement his company pension. How much money should he invest per quarter, at 5% compounded quarterly, to meet his goal?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

8. Paige took out a $12 000 loan from the bank to pay for equipment for her business. The bank offered her an interest rate of 7.0%, compounded semi-annually. The loan is to be repaid in 4 years. What amount did Paige need to pay back?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

9. Vladimir is buying a house that costs $375 000. He has negotiated a mortgage with the bank that requires a down payment of 12% of the cost of the house. He will pay off the mortgage with regular monthly payments over 25 years at an interest rate of 2.8%, compounded semi-annually. How much will each monthly payment be?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

10. Vernon wants to buy a car that costs $24 000 and he has two different options to finance the purchase.

    Option A: Finance the purchase through the dealership by making regular quarterly payments for 9 years at an interest rate of 2.5%, compounded daily.

    Option B: Finance the purchase with a bank loan by making regular monthly payments for 9 years at an interest rate of 2.5%, compounded daily.

    What is the total cost of the cheaper option?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

11. Jose needs a truck for his job. He can lease a truck for 4 years for $500 per month and a down payment of $5600. He can purchase a new truck for $41 000 which would be financed with a bank loan at an interest rate of 4.4%, compounded monthly, and a down payment of $6100. He would pay off this loan with regular monthly payments over 4 years. He can also rent a truck at $85 per day. What is the total cost of buying the truck?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END

12. Emily is purchasing a house for $185 000 that appreciates at a rate of about 1.5% per year. She will finance this purchase with a 15-year mortgage at an interest rate of 3.9%, compounded semi-annually, with monthly payments, where she is required to make a 10% down payment. How much are her regular monthly payments?

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
							END