Unit 5 Linear Equations and Inequalities

Watch the following instructional video. In your handout:
i) Copy down the given notes and examples
ii) Complete the assigned questions https://youtu.be/gjTfh3x2Dgc

5.1 Review of Graphing and Linear Equations

A linear equation is a graph of a straight line

Two ways to write down the equation of a straight line are:

- **Standard Form**: $Ax + By - C = 0$ where *A* is a whole number $>$ zero; and *B* & *C* are integers.
- **Slope Intercept Form**: $y = mx + b$ where *m*= slope and *b* = *y*-intercept

We can graph these equations by plotting at least three points on the grid. Then we draw a straight line through the points.

Graphing Standard Form – Method 1

Step 1: To find y - *intercept*, set $x = 0$

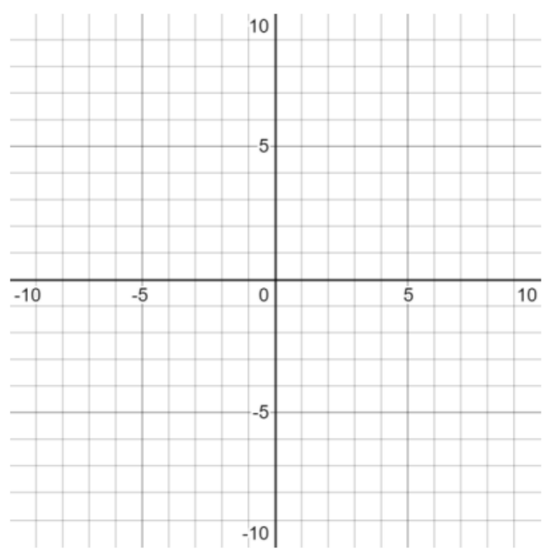
To find x - *intercept*, set $y = 0$

Step 2: Then pick any value of x and solve for y

Step 3: plot the three points from the first two steps, and draw a straight line through them

Note: when $x = 0$ we have the $y - intercept$ and when $y = 0$ we have the $x - intercept$

Example 1: $Graph\ 3x + 2y = 12$



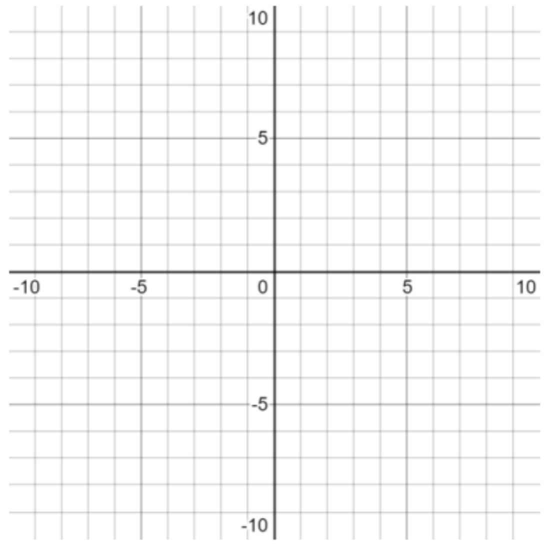
Graphing Standard Form – Method 2

Step 1: Use algebra to change the equation into $y = mx + b$ form (*slope – intercept* form)

Step 2: Plot the $y - intercept$

Step 3: Map out the slope (use slope as $\frac{rise}{run}$)

Example 1: $Graph\ 3x + 2y = 12$



Summary

To graph a linear equation:

Step 1: Find at least three ordered pairs (points) that are solutions to the linear equations

Step 2: plot the corresponding points and connect them with a straight line

OR

Convert to slope intercept form and map out slope from the $y - intercept$

5.2 Solving Systems of Linear Equations Algebraically

- Two or more equations viewed together are called a system of equations
- The ordered pair(s) that the equations have in common is the **solution**.
- The solution to the equations must ***satisfy*** all the equations.

Example 1: Is (2, -2) a solution to the system of equations:

$2x - y = 6$ and $-x + 3y = -8$?

Answer:

Example 2: Is (-1, 3) a solution to the system of equations:

$2x + y = 1$ and $-x - 3y = -9$?

Answer:

To solve a system of linear equations we can use two methods. These are the ***method of elimination or method of substitution.***

Method of Elimination

Step 1: Write the equation of the system in the form $Ax + By = C$

Step 2: multiply the terms of one or both of the equations by constant so that the coefficients of x and y are the same for

Step 3: add or subtract the equations to eliminate one of the variables. Solve the remaining equation.

Step 4: substitute the value you solved for in Step 3 into either of the original equations, and solve for t the other variable

Step 5: verify that your solution satisfies both equations.

Example 1:

Solve the following system of equations:

$2x - 3y = 2$ and $x + 2y = 8$

Example 2:

Solve the following system of equations:

$4x - 3y = 5$ and $3x - 2y = 8$

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If the graph of the lines given by a system of equations are ***parallel,*** then that system cannot be solved, and we say that it has ***no solution***.

Example 3:

Solve the following system of equations:

$3x - 2y = 1$ and $-6x + 4y = 3$

If the graph given by a system of linear equations ***overlap*** (they are the same line), then that system has ***infinitely many solutions***.

Example 4:

Solve the following system of equations:

$2x + 5y = 2$ and $6x + 15y = 6$

Method of substitution

Step 1: Solve one equation for one of its variables in terms of the other

Step 2: Substitute the equation from step one into the other equation and solve it

Step 3: Take the value solved for in Step 2, and substitute the value into any equation containing both variables

Step 4: Verify your solution.

Example 1:

Solve the following system of equations:

$2x + 3y = 1$ and $3x - y = 7$

Example 2:

Solve the following system of equations:

$4x - y = 2$ and $x - 3y = -5$

Solving Problems Involving Systems of Linear Equations

Step 1: Read the question carefully. Assign variables to your unknowns (assign variables which make sense) for example use t for time, c for cost etc.

Step 2: Express all equations in terms of your two variables.

Step 3: Use elimination or substitution to solve for the unknowns.

Example 1

Adult tickets for the school play are $12 and children's tickets are $8. If a theatre holds 300 seats and the sold-out performance brings in $3280, how many children and adults attended the play?

Example 2

A small airplane makes a 2400 kilometer trip in 7 1/2 hours, and makes the return trip in six hours. If the plane travels at a constant speed, and the wind blows at a constant rate, find the airplanes airspeed and the speed of the wind. ($speed = \frac{distance}{time}$)

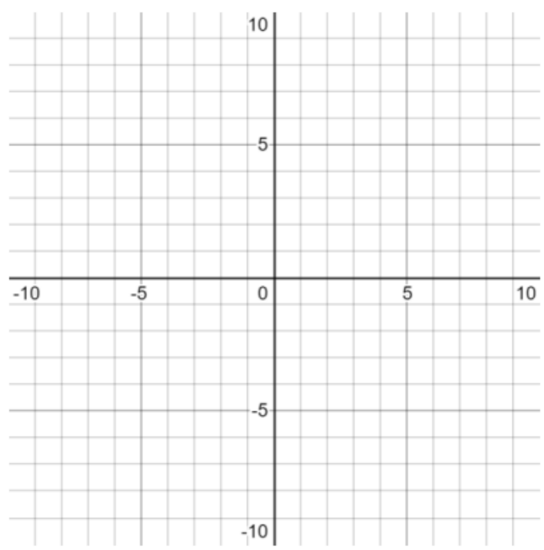
Example 3:

A chemist has two acid solutions in stock, one that is a 50% solution and the other an 80% solution. How much of each solution should be mixed to obtain 100 milliliters of a 68% solution?

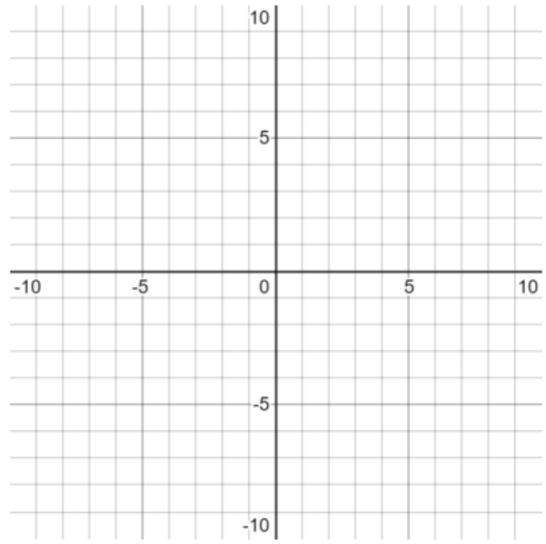
PRACTICE PROBLEMS

1. Graph the following equations:

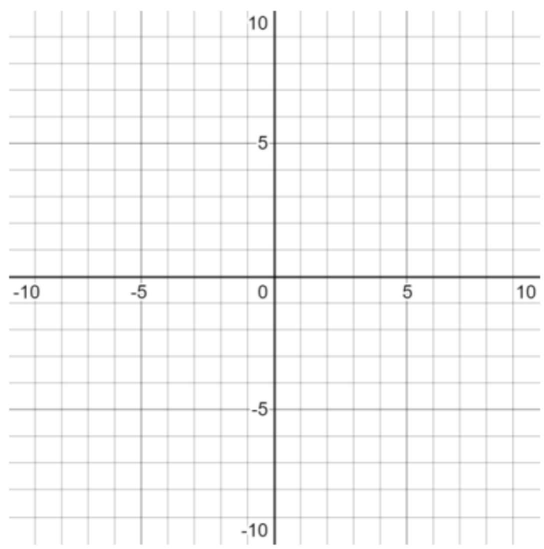
a) $2x + 3y = 6$



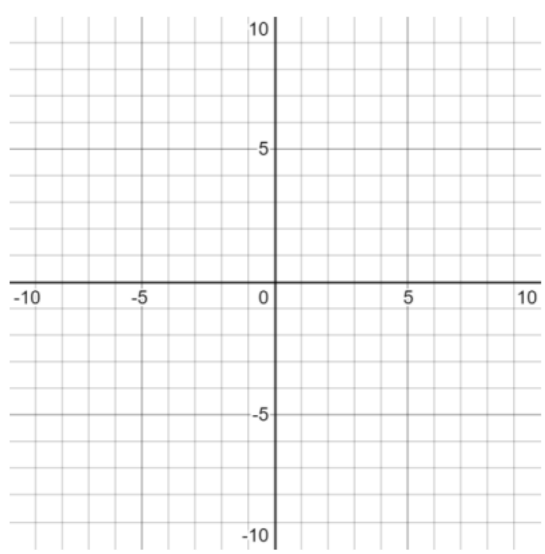
b) $2x + y = -4$



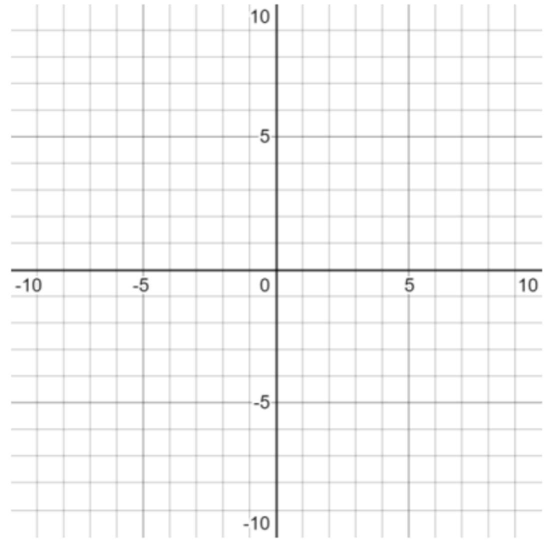
c) $2x + \frac{1}{2}y = 2$



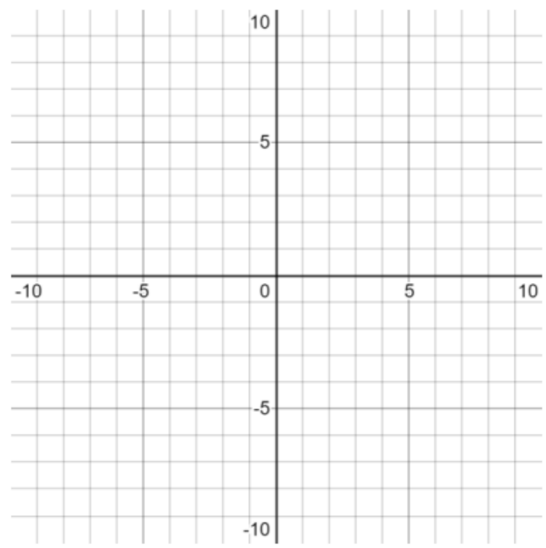
d) $3x + 2y = 5$



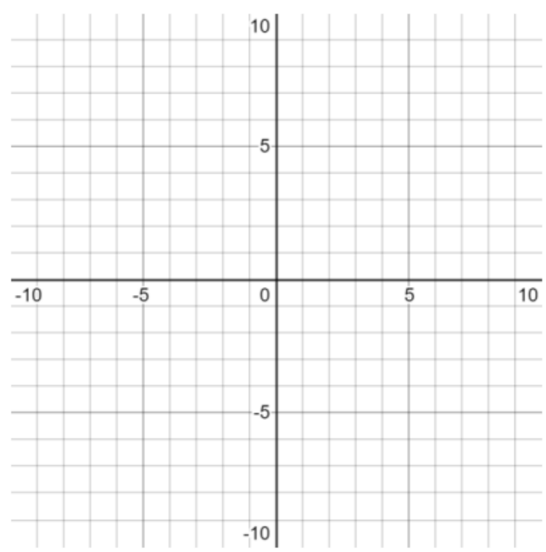
e) $\frac{2}{3}x - 0.4y = 2$



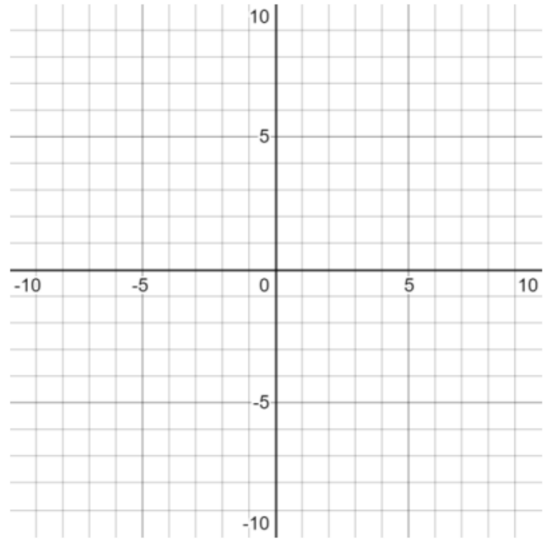
f) $y = -2x - 1$



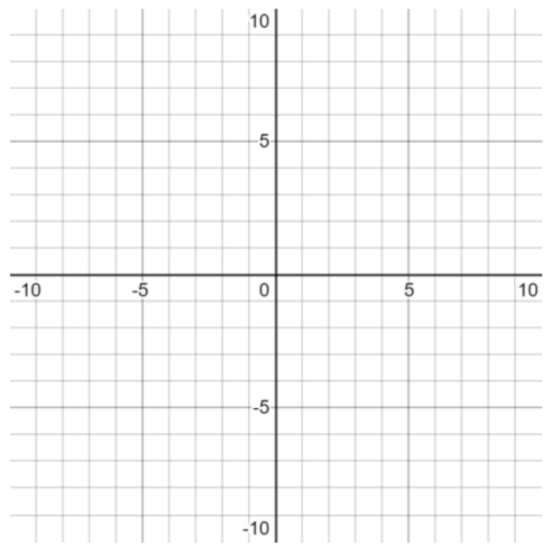
g) $y = \frac{3}{4}x - 1$



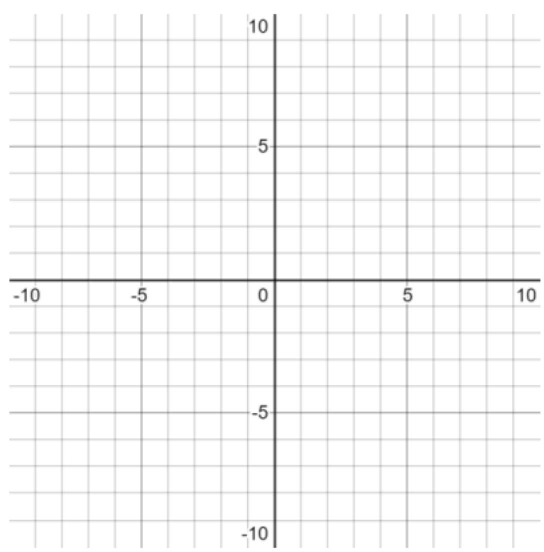
h) $x = 2(y - 1) + 1$



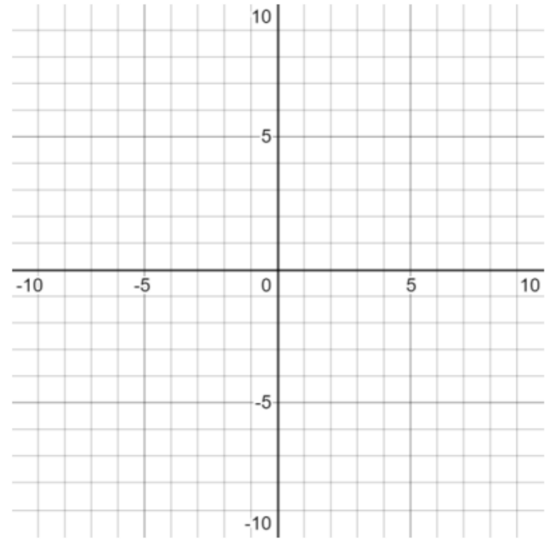
i) $2(x - y) + 6 = 0$



j) $x = 3$



k) $y = -3$



2. Solve the following systems of equations algebraically (use elimination **or** substitution)

a) $3x + 5y = 17$

$4x - y = -8$

b) $4x + 3y = 1$

$3x + 2y = 2$

c) $5x - 3y = \frac{21}{2}$

$2x + 5y = -2$

d) $3x - 2y = 6$

$-6x + 4y = -6$

e) $y = 3x + 4$

$2x - 3y = 2$

f) $y = -2x$

$x + 4y = 21$

g) $3x - 2y = 6$

$-6x + 4y = -12$

h) $\frac{x}{3} + \frac{y}{4} = 1$

$\frac{x}{2} - \frac{y}{8} = \frac{7}{2}$

i) $6x - y = 0$

$8x - 3y = 25$

j) $2s + t = -3$

$3s + 2t = -4$

k) $y = \frac{1}{3}x + 2$

$2x - 6y = -12$

l) $\frac{x}{3} - \frac{y}{4} = \frac{1}{12}$

$\frac{x}{6} - \frac{3y}{2} = -\frac{7}{8}$

3. Solve the following:

a) Jean has $50,000 to invest. He invests some in the stock market which earns 8%, and some in bonds that earned 6% on his investments. If the total interest earned was to $3500, how much did Jean invest in stocks and how much in bonds?

b) Stephanie has 80 coins, consisting of dimes and quarters. If the total value of her coins is $15.20, how many dimes does she have

c) A plane travels 2835 kilometers in seven hours with a tailwind, but only 1827 kilometers with a headwind in the same time. Find the speed of the plane, and the speed of the wind.

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d) Tickets for the girl's volleyball game are $2 each or three for $5. The team sold 300 tickets and the total amount of money taken in was $528. How many people bought only a single ticket?

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5.3 Linear Inequalities

- The solution of a linear inequality is a section of the **coordinate plane** that takes up half of it.
- We graph the inequality like we would a regular linear equation
- Then using inequality sign we determine what side of the line to shade

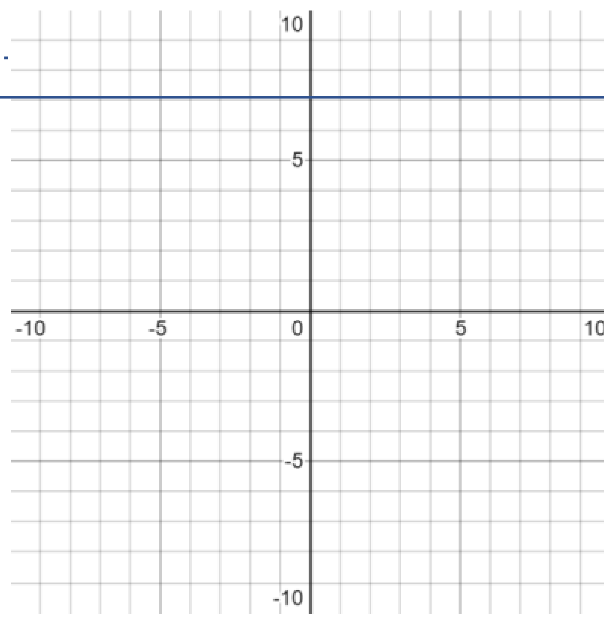
Graphing (solving) a Linear Inequality

Step 1: Graph the linear inequality equation. Use a solid line for $\leq$ or $\geq$, and a dashed line for $<$ or $>$.

Step 2: The graph (from step 1) divides the coordinate plane into two regions. We need to shade one of the regions ***(the solution region)*** based on the inequality.

To choose the region, pick a test point **not on the line** and substitute the point into the equation. If the inequality stays true, shade the region that includes that point. If not true, shade the other region.

Example 1: Graph $2x - 3y \leq 6$



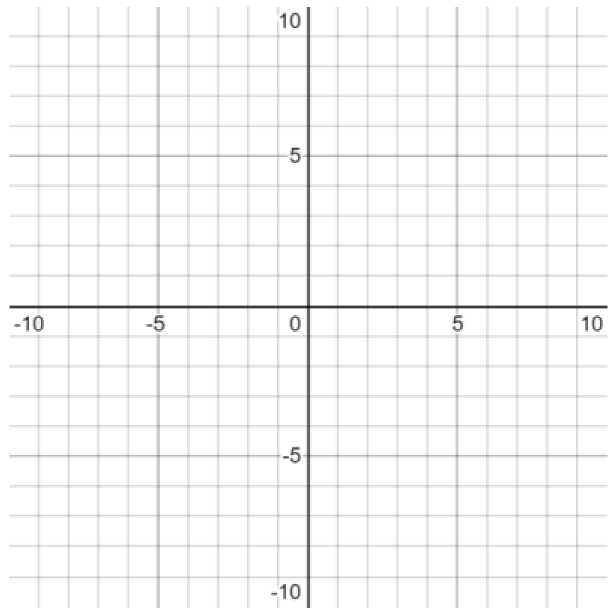
Graphing (solving) Systems of Linear Inequalities

- Steps to follow here are the same as one linear inequality
- The only difference is that the solution must be the intersection of all the inequality equations
- This is the region where all points satisfy all the equations in the system.

Example 1:

Solve: $2x - y > 3$

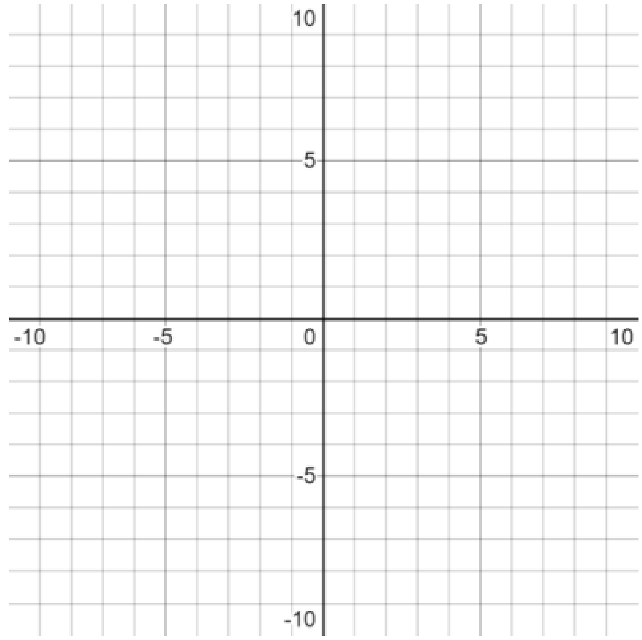
$x + y \geq 3$



Example 2:

Solve: $3x + 2y \leq 6$

$x - 2y > 4$



Example 3:

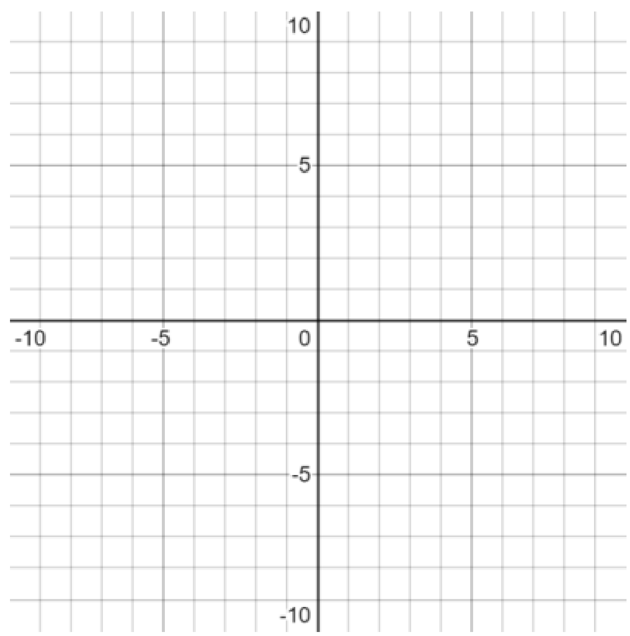
Solve the system of linear inequalities:

$x \geq -3$

$y \geq -4$

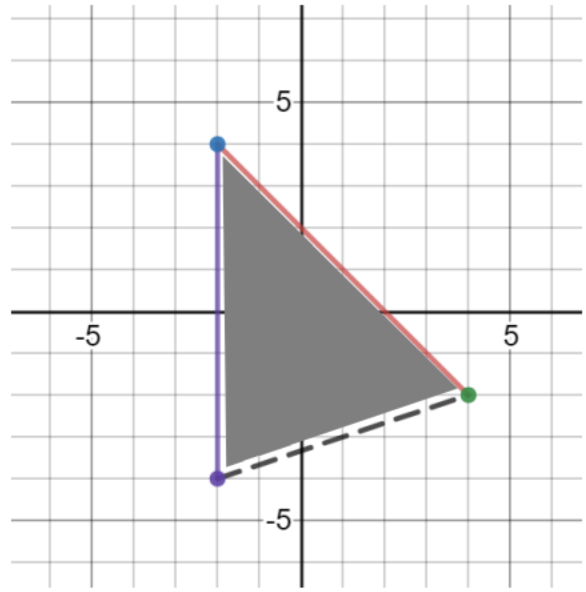
$y > 2x - 4$

$x + 6y \leq 15$



Example 4:

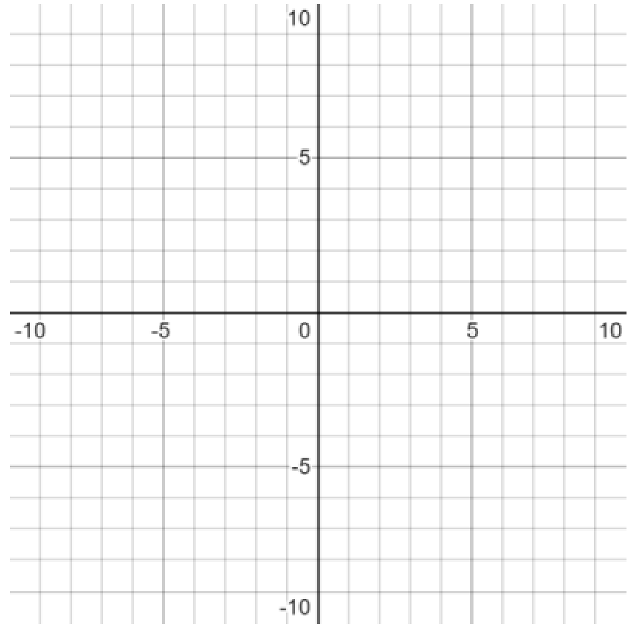
Write a system of linear inequalities that has the given graph



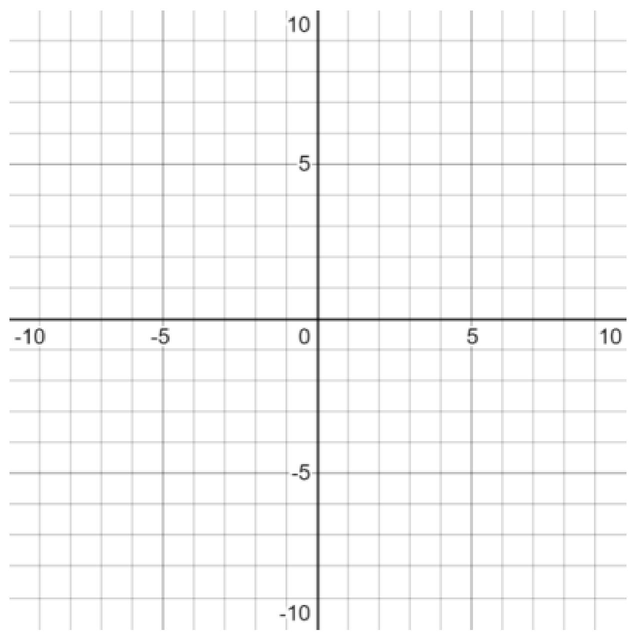
PRACTICE PROBLEMS

1. Graph the following inequalities

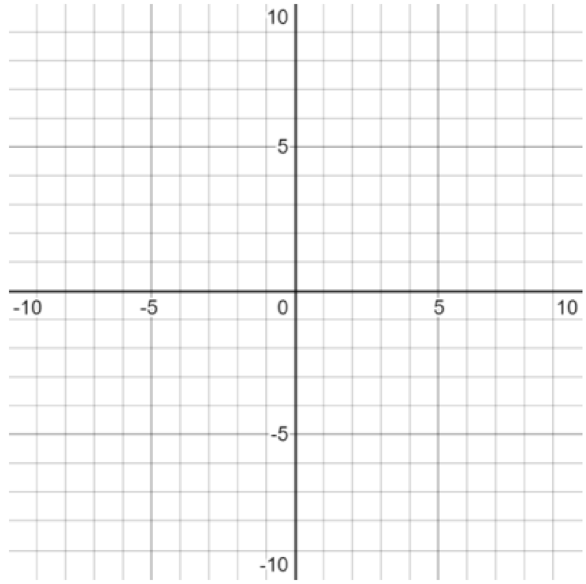
a) $3x + y \geq 6$



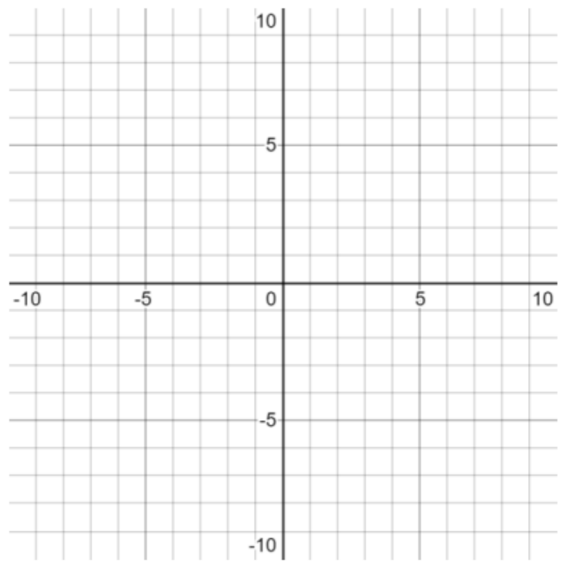
b) $2x - y < 4$



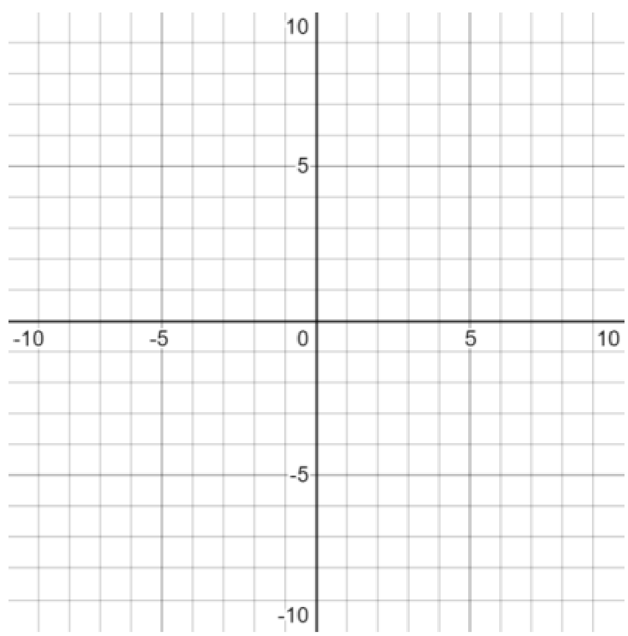
c) $\frac{1}{3}x + \frac{2}{3}y \geq 2$



d) $y \geq \frac{1}{2}x + 3$

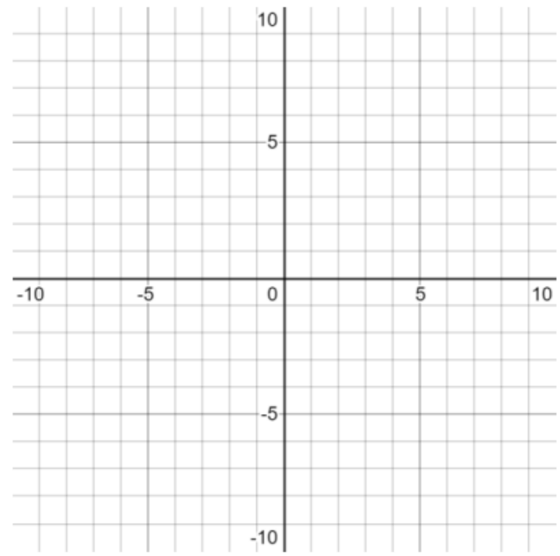


e) $y < -\frac{4}{3}x + 2$



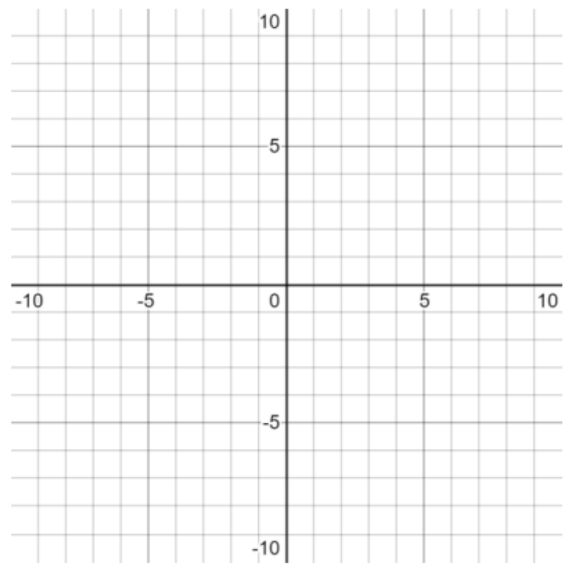
f) $y \geq x$

$2y < -x + 2$



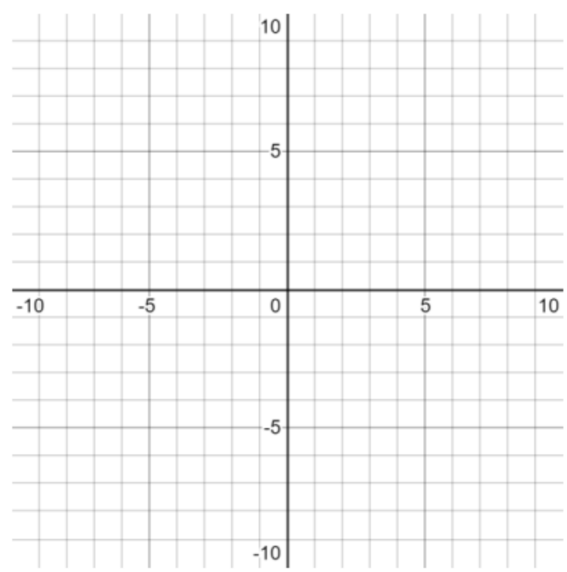
g) $x + 2y > 4$

$3x - 2y \leq 6$



h) $x + y \leq 2$

$x + y \geq -2$

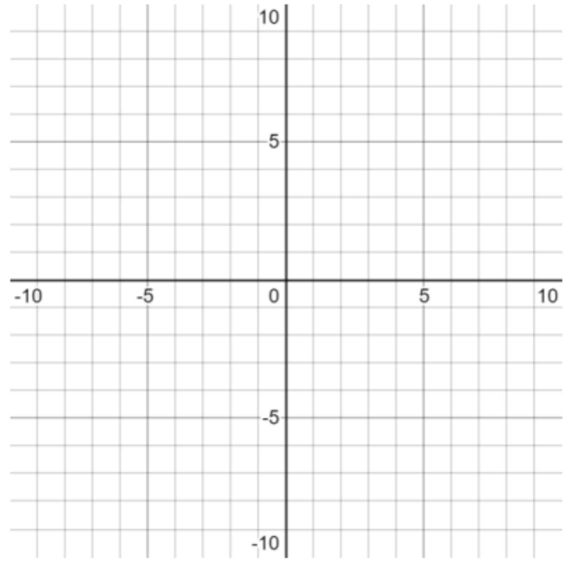


i) $4x + 5y < 20$

$2x - y \leq 4$

$x \geq 0$

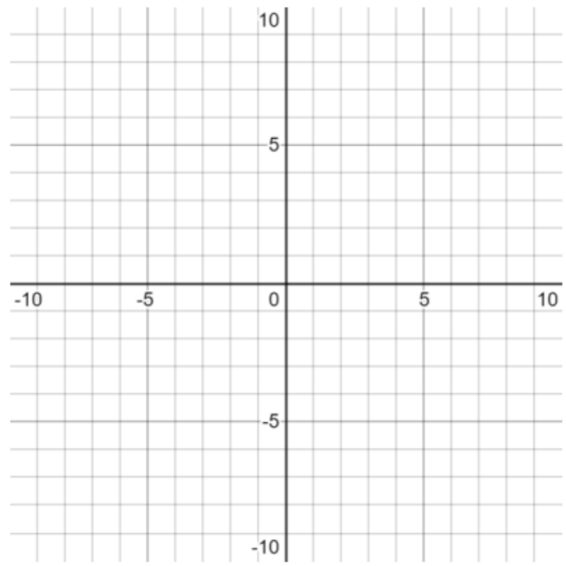
$y \geq 0$



j) $x - y \leq 1$

$x - y \geq -3$

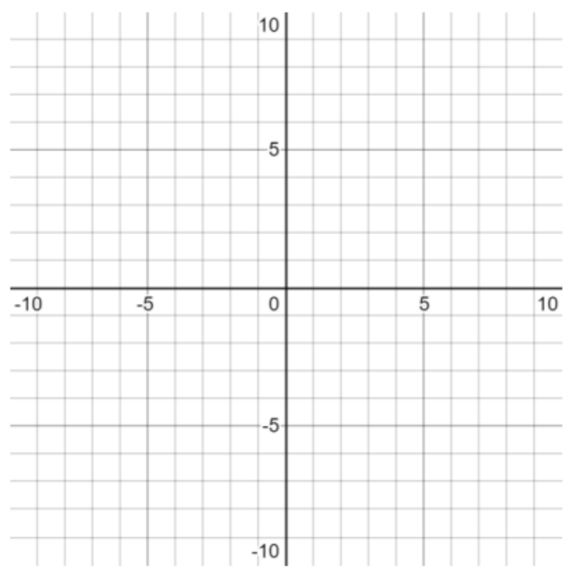
$-1 \leq x \leq 3$



k) $x - y \leq 2$

$x + 2y \leq 4$

$x \leq -1$

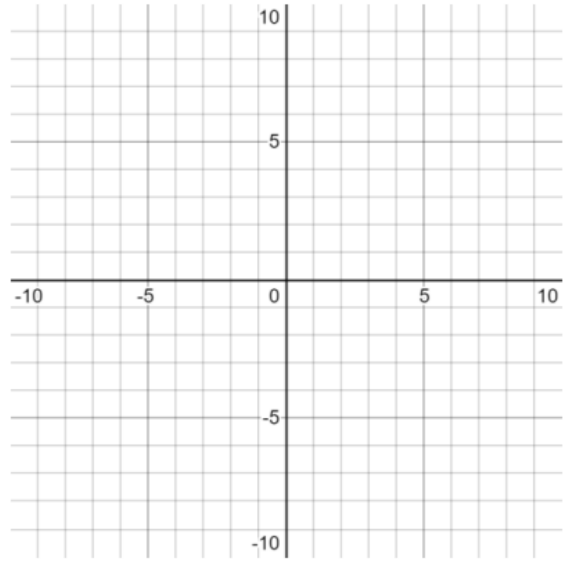


l) $x + y \leq 4$

$2x - y \geq 2$

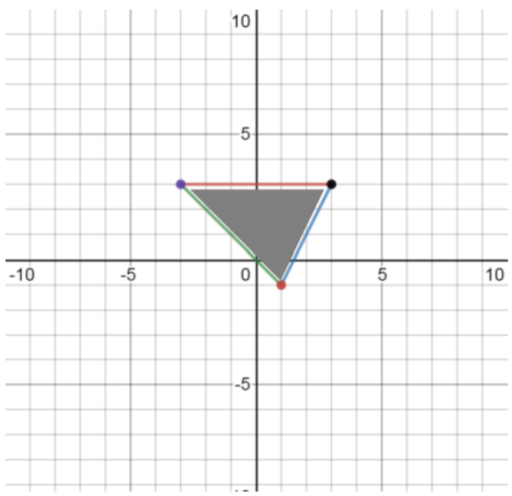
$x \geq 0$

$y \geq 0$

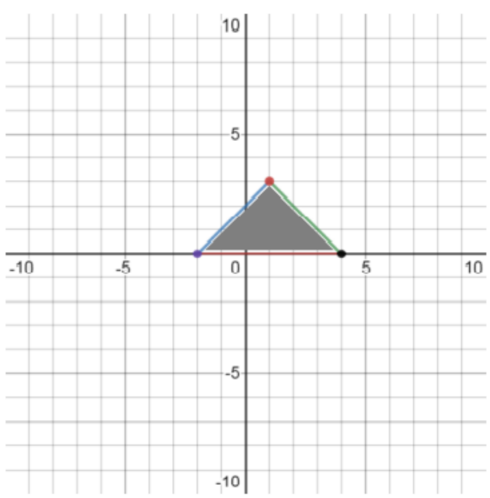


Write a system of inequalities for the following graphs

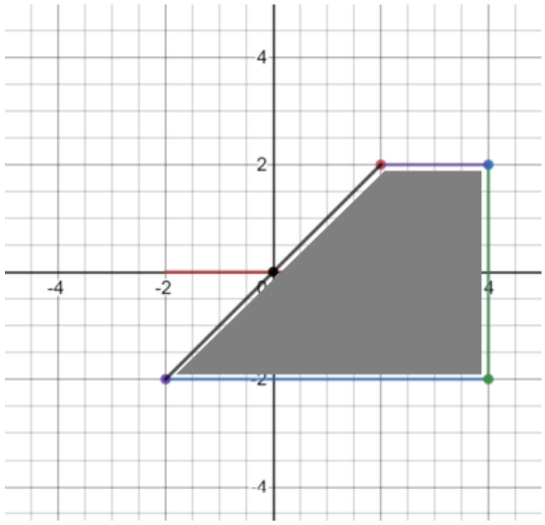
a)



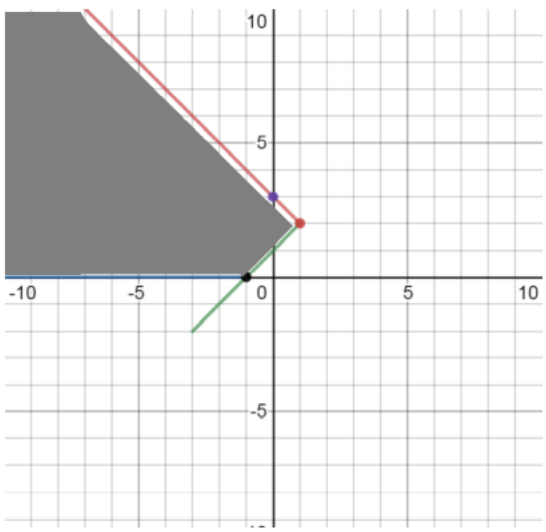
b)



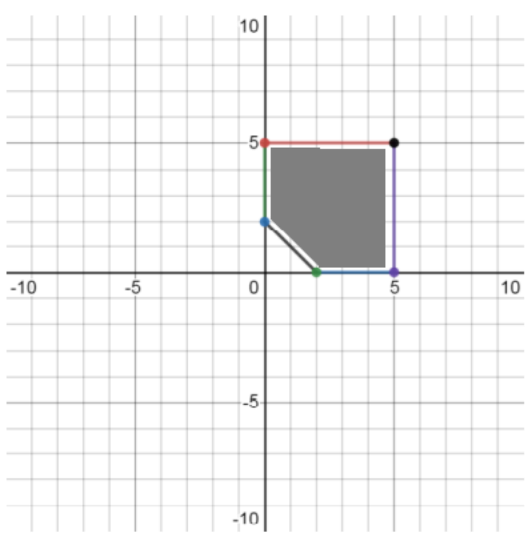
c)



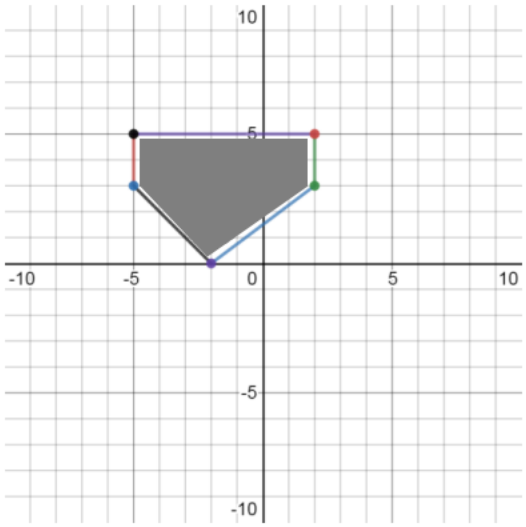
d)



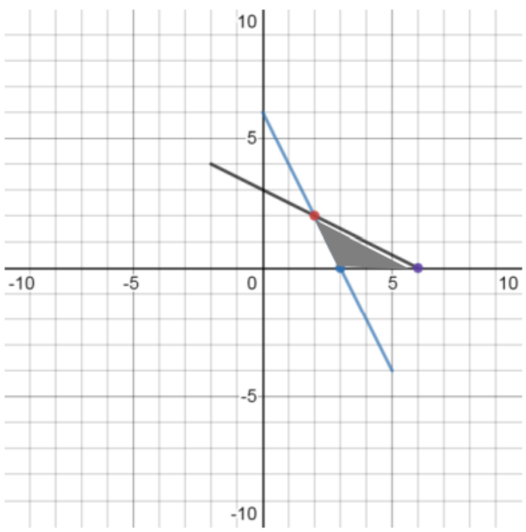
e)



f)



g)



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5.4 Linear Programming (or Linear Optimization)

Optimization is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model.

- Linear inequalities can be used to solve optimization problems
- We call this linear programming

We will deal with two-variable linear programming models. They contain two parts:

1. An ***objective function*** tells us the quantity we want to maximize/minimize.

2. The system of control constraints consists of linear inequalities whose solutions is called the ***feasible solution*** with an area called the ***feasible region***.

The optimal solution is at the **vertices** of the feasible regions.

We find the solution by testing the objective function at each vertex.

Steps to follow for a linear programming problem

Step 1: Sketch the region determined by the system of constraints

Step 2: Find the vertices of the region

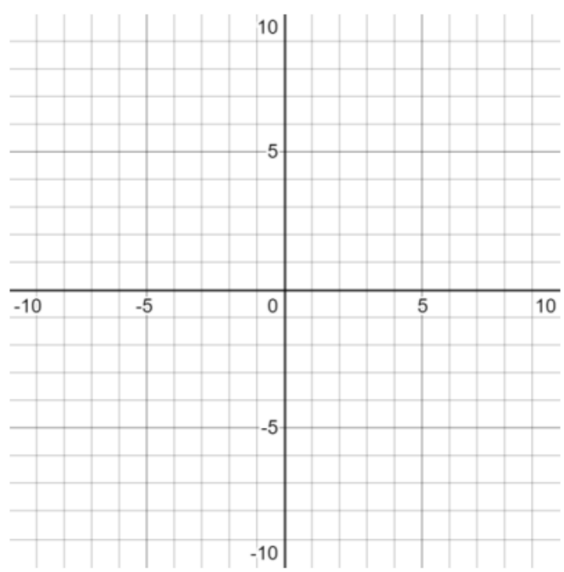
Step 3: Calculate the value of the objective function C at each vertex of the region

Step 4: Find the maximum or minimum values of C

Example 1:

Find the maximum values of the objective function given by $C = 7x + 3y$; subject to the following constraints:

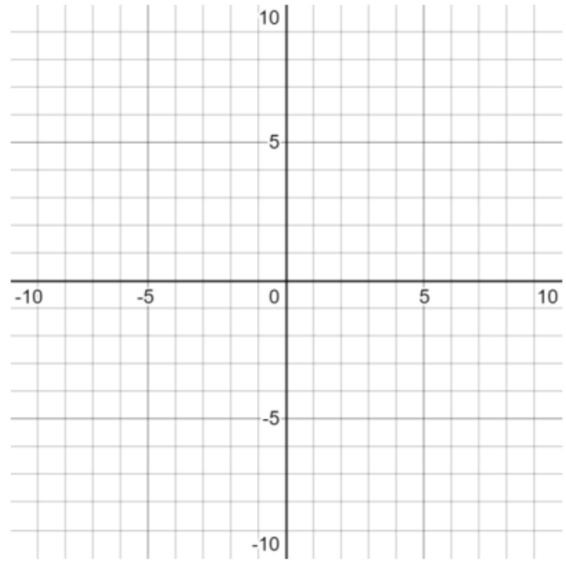
$x - y \geq -4$, $2x + y \leq 10$, $x \geq 0$, $y \geq 0$



Example 2:

Find the maximum and minimum values of the objective function given by $C = 3x + 2y$; subject to the following constraints:

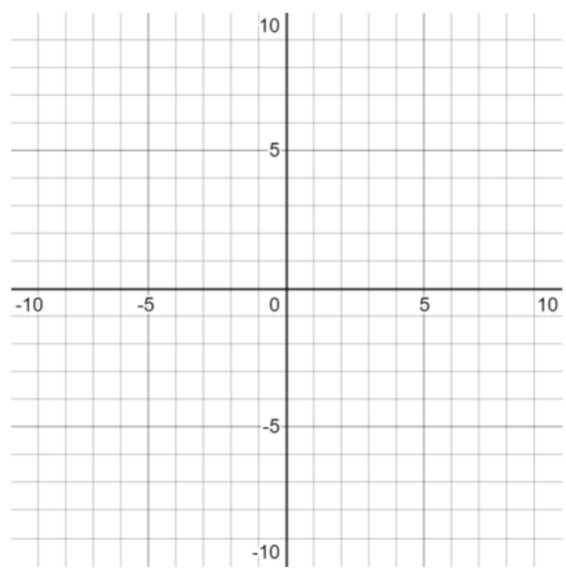
$3x + y \leq 15$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$



Example 3:

Find the maximum and minimum values of the objective function given by $C = 3x + 4y$; subject to the following constraints:

$2x + y \geq 8$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$



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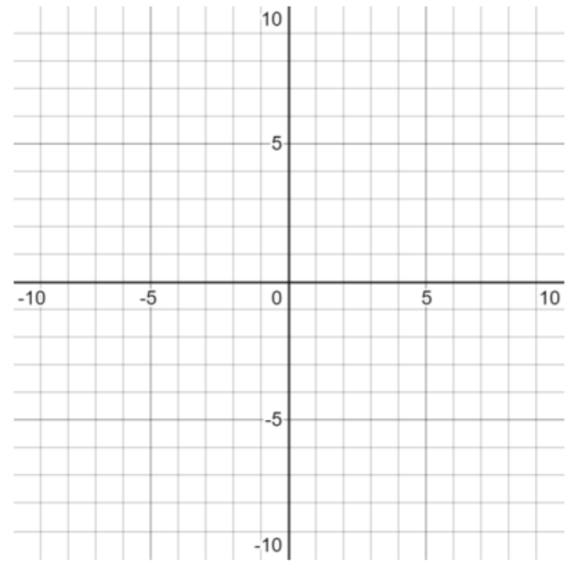
Example 4:

Find the maximum and minimum values of the objective function given by $C = 2x + 6y$; subject to the following constraints:

$2x + 3y \geq 12$, $x + 3y \geq 9$, $x \geq 0$, $y \geq 0$



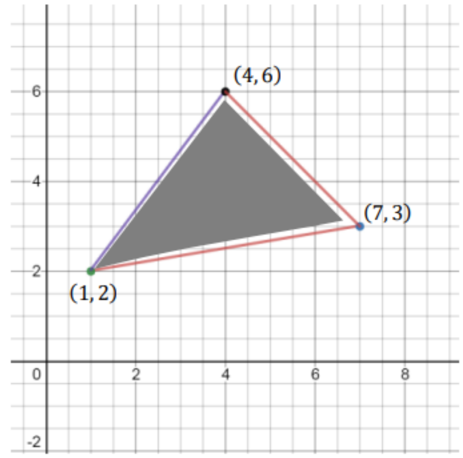
Summary of Linear Programming

1. If the linear programming problem has an optimal solution, either ***the maximum or minimum*** of the objective function, then it ***occurs at a vertex of the feasible region***.
2. ***If the feasible region is closed and bounded*** open brackets (examples 1, 2, and 3), then the objective function has ***both a maximum and a minimum***
3. When the ***feasible region is not closed*** (example 4), the objective function may have a ***maximum only, minimum only, or neither.***

PRACTICE PROBLEMS

1. Find the maximum and minimum values of the given region given the objective function.

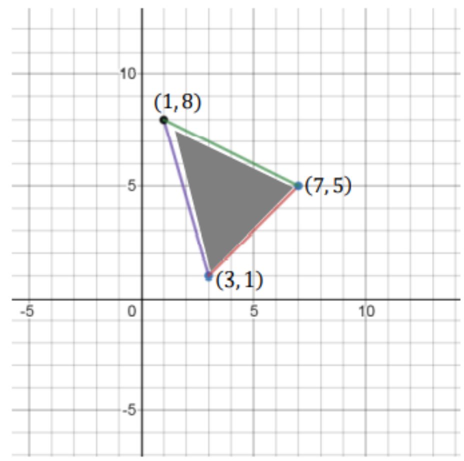
a) $C = 2x - 3y$



Max =

Min =

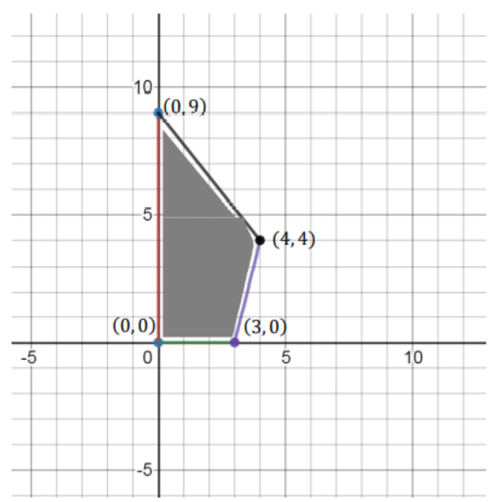
b) $C = x + 3y$



Max =

Min =

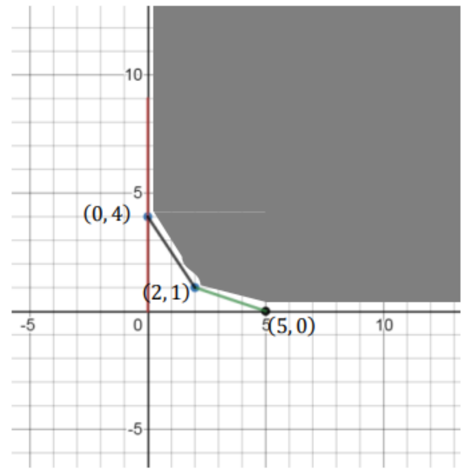
c) $C = 3x - 2y$



Max =

Min =

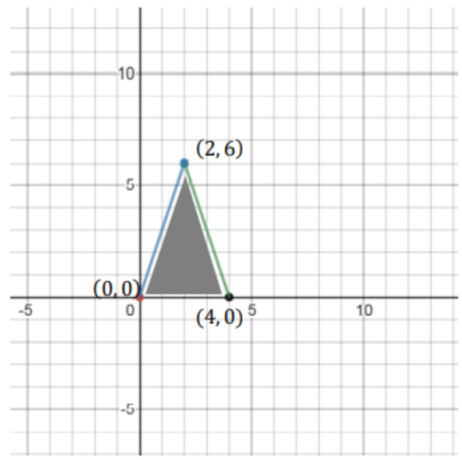
d) $C = 2x + 4y$



Max =

Min =

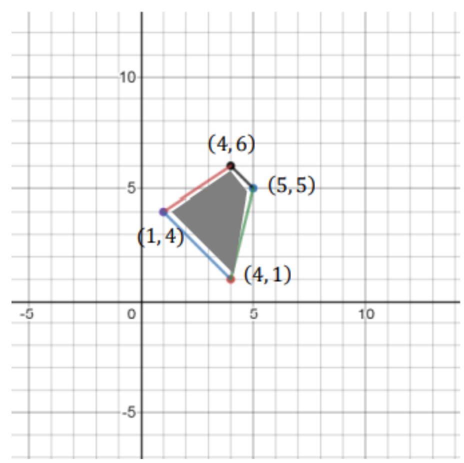
e) $C = 3x + y$



Max =

Min =

f) $C = 3x + 3y$



Max =

Min =

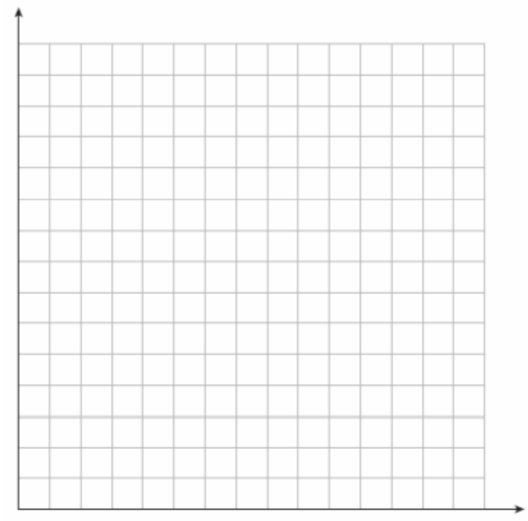
2. $Maximize\ C = 6x + 4y$

$Subject\ to : x + 2y \leq 10$

$3x + y \leq 15$

$x \geq 0$

$y \geq 0$



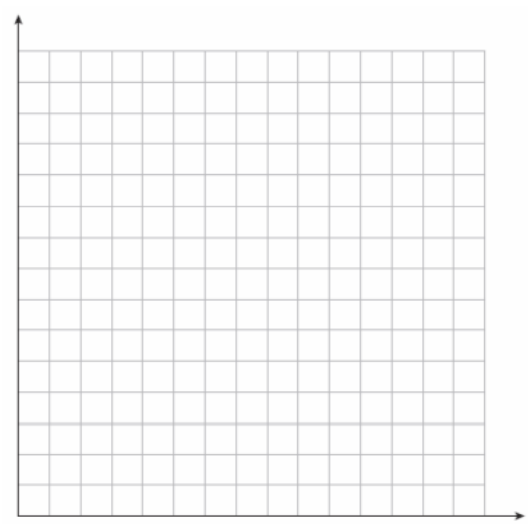
3. $Maximize\ C = 8x + 10y$

$Subject\ to : 2x + y \leq 12$

$x + 3y \leq 21$

$x \geq 0$

$y \geq 0$



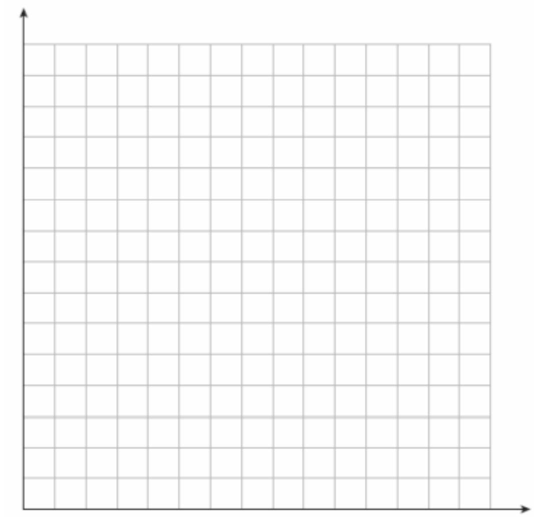
4. $Maximize\ C = 6x + 8y$

$Subject\ to: 2x + y \geq 8$

$x + 2y \leq 10$

$x \geq 0$

$y \geq 0$



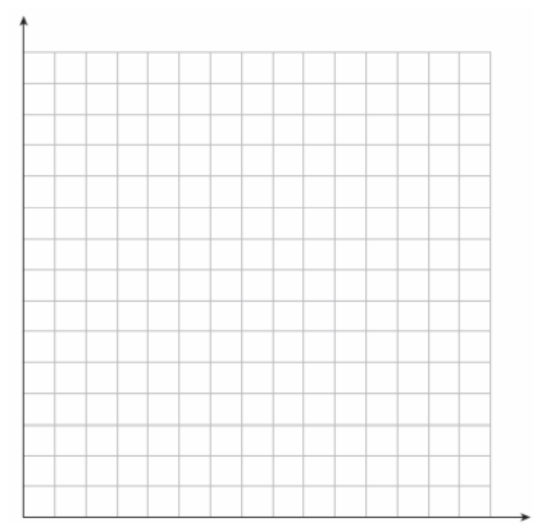
4. $Maximize\ C = 6x + 3y$

$Subject\ to: 4x + 3y \geq 24$

$4x + y \leq 16$

$x \geq 0$

$y \geq 0$

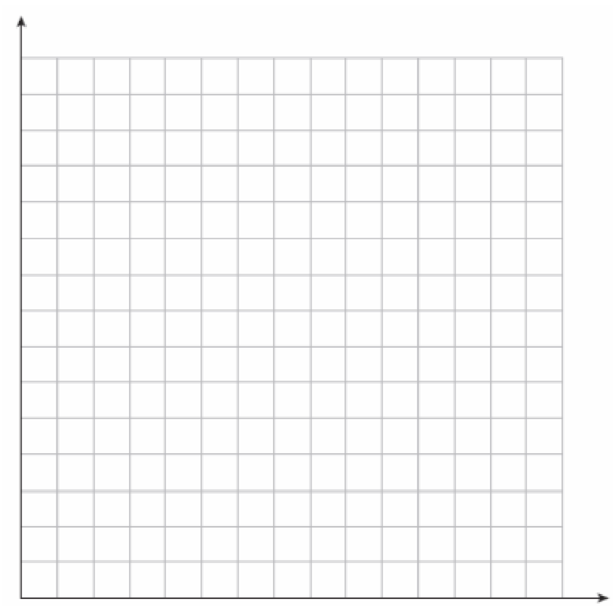


5.5 Applications of Linear Programming

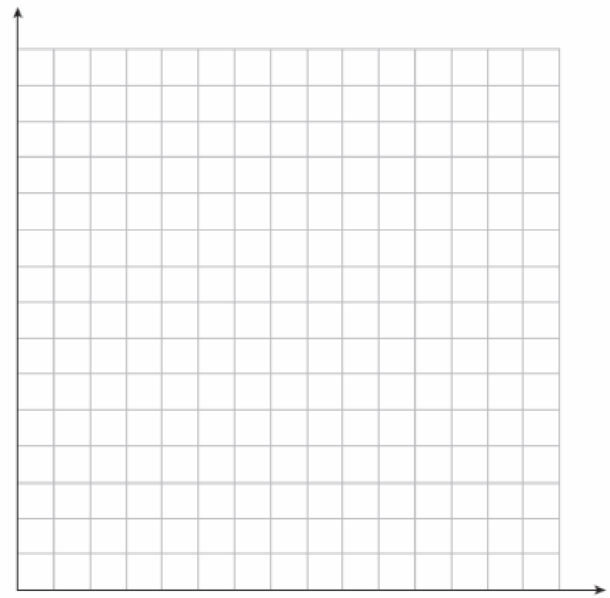
- Optimization problems can be used in the business world
- When companies are developing business plans, they generally want to make sure they have detailed plans to optimize success
- We can use the techniques we just learned to solve optimization problems

Example 1:

A parkade can fit at most 100 vehicles (cars and trucks) on its lot. A car takes up 100 ft^2 and a truck 200 ft^2. There is 12000 ft^2 of space available. The parkade charges $20 per car an $35 per truck to park each week. How many of each vehicle will bring the maximum revenue?



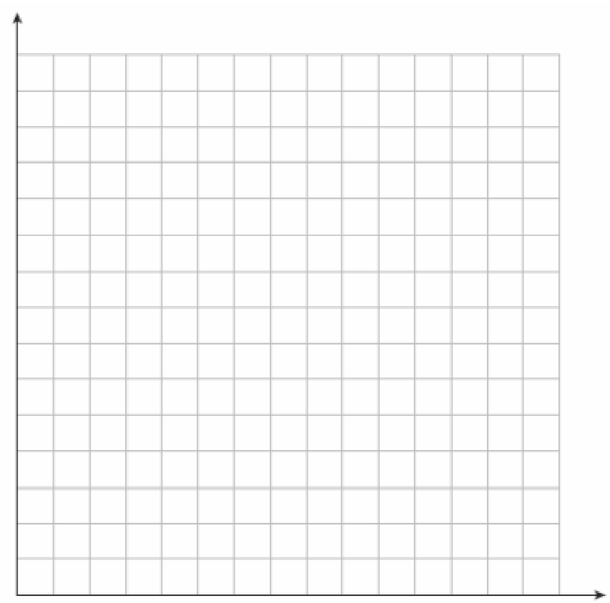
Example 2:



A Company makes custom scooters and bicycles. The limited work area restricts the number of vehicles that can be built in one week.

- no more than 10 scooters can be built
- no more than 8 bicycles can be built
- no more than a total of 12 vehicles can be built.

A $150 profit is made from a scooter and $100 profit is made from a bicycle. What is the company's maximum weekly profit?



Example 3:

Maple Ridge runs two recycling centres. **Centre *X*** costs $40 to run for an hour. In a typical hour 14 kilograms of glass and 10 kilograms of aluminum dropped off at centre *X*.

Centre *Y* costs $50 per hour to run, with 10 kilograms of glass and 20 kilograms of aluminum dropped off per hour.

The city has a contract to deliver at least 140 kilograms of glass and 190 kilograms of aluminum each day to a manufacturer.

How many hours per day should the city open each centre to minimize its cost and still meet the manufacturers needs?

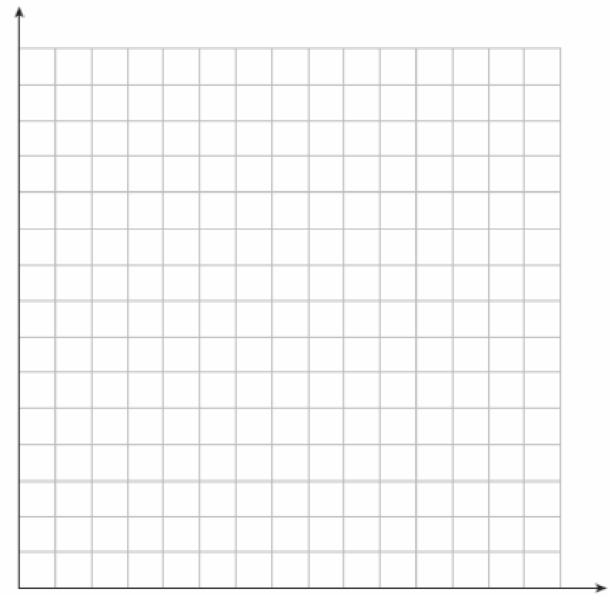
PRACTICE PROBLEMS

1. A butcher shop makes hamburger patties and sausages. Hamburger patties sell for \$2 and sausages sell for \$1.50. The butcher noticed that they always sell at least twice as many sausages as hamburger patties, but never more than 100 hamburger patties and 300 sausages.

   Let *H* represent the number of hamburger patties sold.

   Let *S* represent the number of sausages sold.

   Determine the number of hamburger patties and sausages the butcher shop should sell to maximize profit.

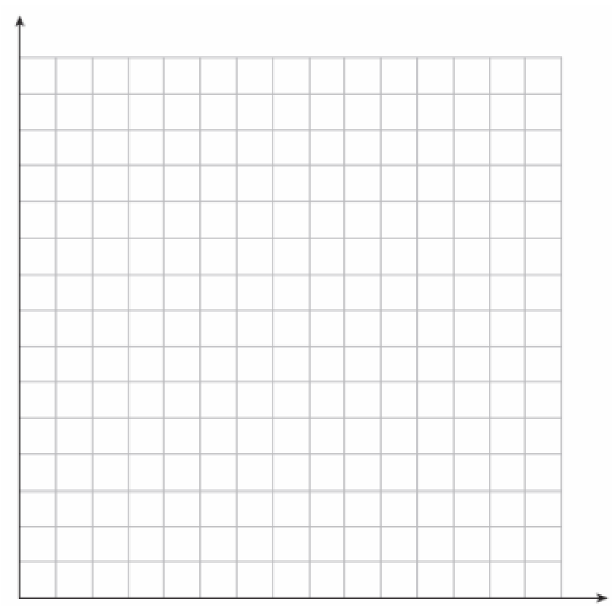


2. A publisher makes romance and adventure novels. Romance novels sell for \$9 and adventure novels for \$7.50. The publishers notice that each month they sell at least three times as many adventure novels as romance novels, but never more than 1500 novels a month.

Let r represent the number of romance novels sold.

Let a represent the number of adventure novels sold.

Determine the number of romance novels and adventure novels the publisher should sell to maximize profit.

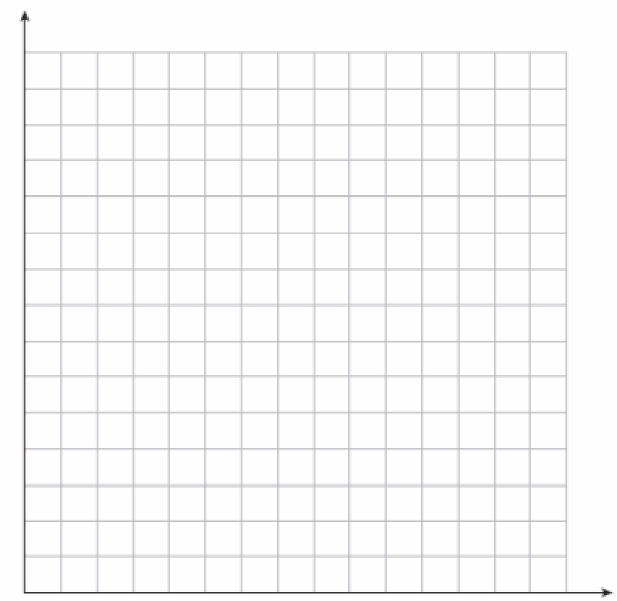


3. April notices the number of people and dogs in a dog park. There are more than twice as many people as dogs. There are at least 10 dogs. There are no more than 50 people and dogs in total.

   Let d represent the number of dogs in the park.

   Let p represent the number of people in the park.

   Determine the maximum and minimum number of legs at the dog park.



4. A student council is ordering signs for the winter dance. Signs can be made in letter-size or poster-size. No more than 30 of each size is wanted. No more than 50 signs are needed altogether.

   Letter-size costs $8.75 each, and poster-size signs cost $14.50 each.

   Let l represent the number of letter-size signs.

   Let p represent the number of poster-size signs.

   What combination of the two sizes of signs would result in the lowest cost to the council?