
• **Watch the following screencast video before beginning this next section:**
https://youtu.be/gkwsz-Xrd4s

UNIT 6 LINEAR INEQUALITIES

REVIEW

MATHEMATICAL STATEMENTS

- An ***equation*** states that two expressions have the same value.
- An ***inequality*** compares the values of two expressions.
- A ***compound statement*** combines two statements using "and" or "or".
- Solving a mathematical statement means to find all the values that "satisfy" the statement, to find all the values for which the statement is true.
- The ***solution set*** can be used to describe the solution of a statement.
- A ***graph***, either on a ***number line*** or on a ***coordinate plane*** can be used to illustrate the solution set.

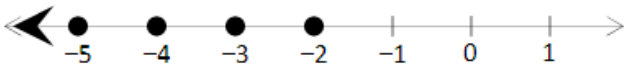
Example 1

Write the solution set and then graph the solution on a number line.

a) $3 - 2x > 5$, where $x \in \mathbb{I}$

- Solve like an equation.
- Multiply/divide by a negative, reverse inequality.

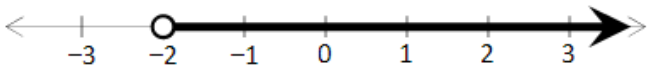
$$3 - 2x > 5$$
$$-2x > 2$$
$$\frac{-2x}{-2} < \frac{2}{-2}$$
$$x < -1$$



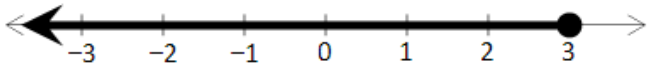
Example 2

$x > -2$ and $x \leq 3$, where $x \in \mathbb{R}$

- $x > -2$

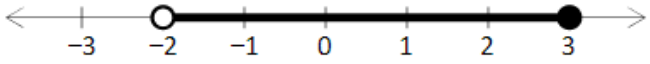


- $x \leq 3$



- The elements of the solution set are the values of x that are real numbers "greater than −2" and "less than or equal to 3".

$-2 < x \leq 3, x \in \mathbb{R}$

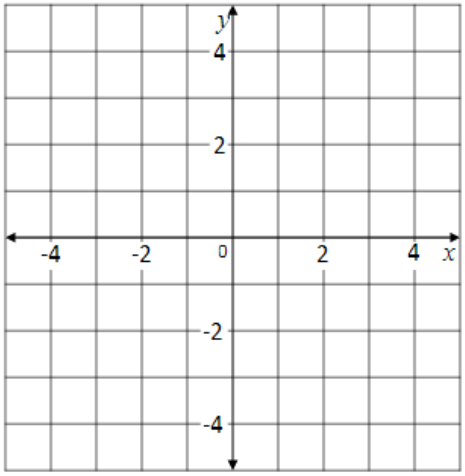


THE COORDINATE PLANE

On the coordinate plane shown, label:

- the ***origin***
- the ***x-axis*** and the ***y-axis***
- ***Quadrant I***, ***Quadrant II***, ***Quadrant III***, ***Quadrant IV***

An ***order pair***, (x,y) describes the location of a point on the coordinate plane.



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RELATIONS

- A ***relation*** describes the relationship between two quantities. x and y are usually used to represent the two quantities, but other variables can be used.

example: The price of a cup of coffee is \$3. The relationship between the cost and the number of cups purchased can be described:

- in ***words***: The cost of the coffee in dollars is three times the number of cups purchased.

- with an ***equation***: $C = 3n$, $n \in \mathbb{I}$, $n \geq 0$, where C represents the dollar cost of the coffee and n represents the number of cups of coffee purchased.

  Any variable introduced to be used in the equation must be defined.

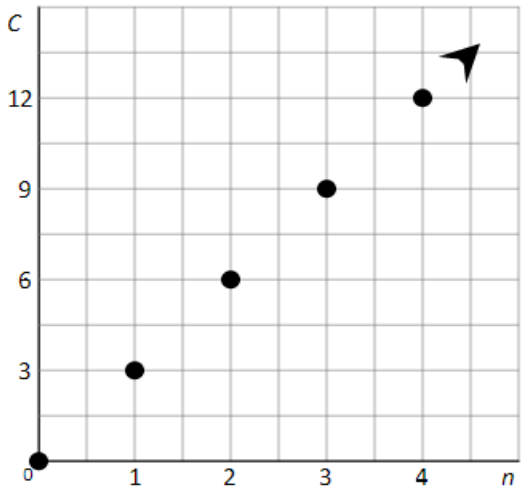
n is the ***independent variable*** of this relation. It is the quantity for which the values are selected; we decide how many cups to buy.

C is the ***dependent variable*** of this relation. It is the quantity for which the values are calculated; it depends on the value of n.

- as a ***set of ordered pairs***: $\{ (0,0) , (1,3) , (2,6) , (3,9) , (4,12) , \ldots \}$

Every ordered pair *satisfies the equation*, so they are all solutions of the equation.

- with a ***graph***:



The horizontal axis is for the independent variable; the vertical axis is for the dependent variable.

Each point of the graph represents one ordered pair from the relation.

These points are ***discrete*** because the points are separate and not connected; the graph would be ***continuous*** if all the points are connected.

The relation's equation describes the coordinates of the graph's points.

LINEAR RELATIONS

Equations of all linear relations can be written in the form $Ax + By = C$.

The ***x-intercept*** is the point on the linear graph that is also on the x-axis.

- The x-intercept will be point $(a,0)$ or just the number a.

The ***y-intercept*** is the point on the linear graph that is also on the y-axis.

- The y-intercept will be the point $(0,b)$ or just the number b.

Example 3

Calculate the x-intercept of $2x - 3y = 6$ without graphing.

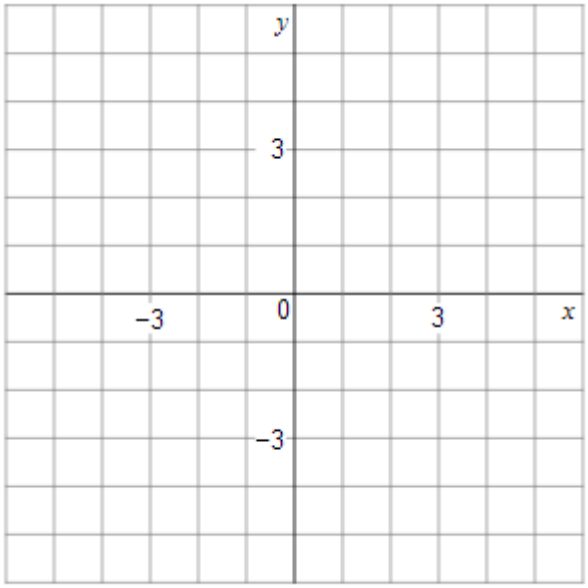
- The y-coordinate will be 0. $2x - 3(0) = 6$
- Solve to find the x-coordinate when y is 0. $2x = 6$
- Solve to find the x-coordinate when y is 0. $x = 3$

Answer: The x-intercept is (3,0) or just 3

Linear graphs can be drawn using two points; intercepts are easiest.

Example 4

Graph $3x-2y=12$ using intercepts



The ***slope-intercept form*** of a linear relation's equation is $y = mx + b$, where:

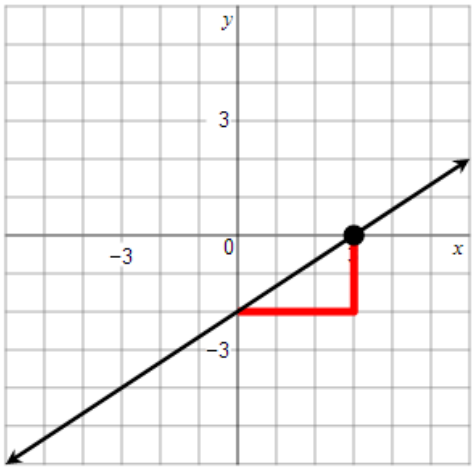
- the ***slope***; $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- b is the y-intercept of the function.

Example 5

Determine the slope and y-intercept of the straight line graph described by $2x - 3y = 6$ and then graph the relation.

- Write the equation in slope-intercept form, $y = mx + b$

$$2x - 3y = 6$$
$$-3y = -2x + 6$$
$$\frac{-3y}{-3} = \frac{-2x + 6}{-3}$$
$$y = \frac{2}{3}x - 2$$



slope, $m = \frac{2}{3}$ and y-intercept, $b = -2$

Example 6

Describe the graph of $x = -4$ and the graph of $y = 5$.

- The graph of $x = -4$ is a vertical line; points with x-coordinate -4.
- The graph of $y = 5$ is a horizontal line; points with y-coordinate 5.

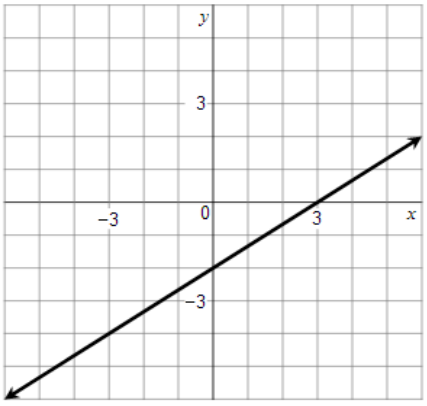
- Watch the following screencast video before beginning this next section:
https://youtu.be/OcUowsztJV4

LINEAR INEQUALITIES

A ***linear inequality in two variables*** is a mathematical statement like a linear equation in two variables with the equal symbol replaced with an inequality symbol, >, <, ≥, ≤.

The solution is the set of all ordered pairs that satisfy the statement.

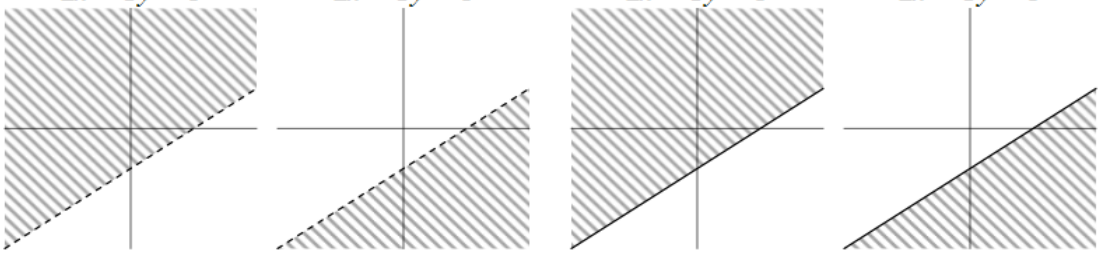
Consider the graph of the equation $2x - 3y = 6$



- All the points <u>on</u> the line have coordinates that satisfy the <u>equation</u>; $2x - 3y$ will equal 6.
- For all the other points, $2x - 3y$ will <u>not</u> equal 6.
  - (−2,2) is on one side of the line. For this point, $2x - 3y$ is less than 6. All the points on this side of the line satisfy the inequality $2x - 3y < 6$.
  - (3,−3) on one side of the line. For this point, $2x - 3y$ is greater than 6. All the points on this side of the line satisfy the inequality $2x - 3y > 6$.

The graph of an inequality in two variables must represent all (and only) the points with coordinates that satisfy the statement.

$2x - 3y < 6$ $2x - 3y > 6$ $2x - 3y \leq 6$ $2x - 3y \geq 6$



dashed line
points on it do not satisfy the statement

solid line
points on it also satisfy the statement

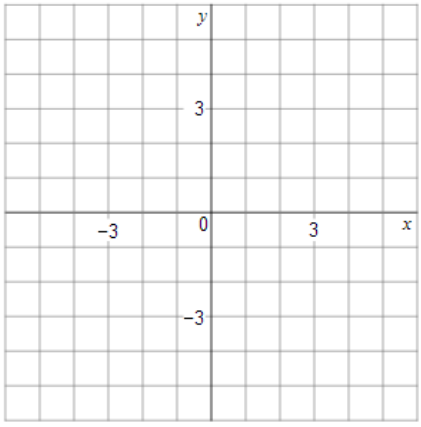
Graphing a Linear Inequality in Two Variables

- Draw the ***boundary line***:
  - dashed or solid? dashed for > or < ; solid for ≥ or ≤
  - where? graph of the corresponding equation
- Determine the ***half-plane*** containing the solution:
  - select a test point on one side of the boundary - if it satisfies the inequality, the solution is on this side; if not, the solution is on the other side
  - if the domain and range are real numbers, shade that half-plane; if the domain and range are integers, "***stipple***" the half-plane with discrete points

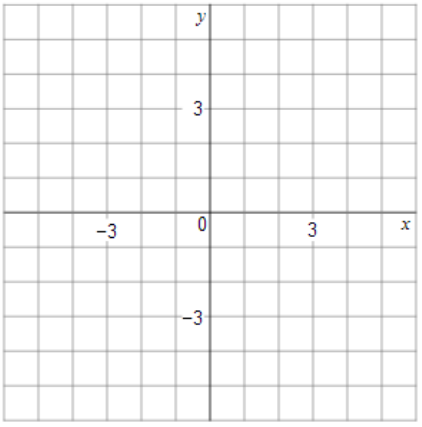
Examples:

Graph the following inequalities:

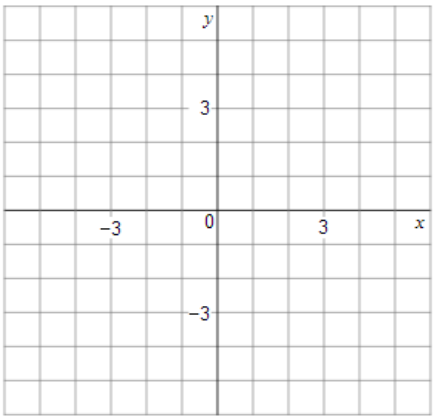
a) $\{(x,y) \mid 3x + 4y \geq -12\ ,\ x \in \mathbb{R},\ y \in \mathbb{R}\}$



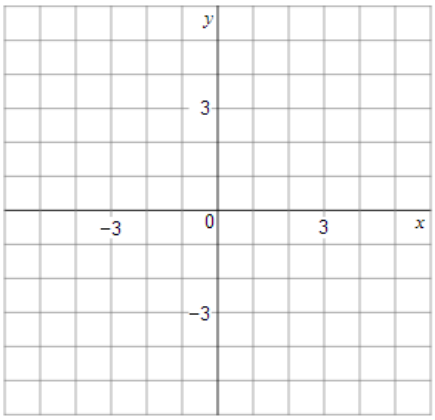
b) $\{(x,y) \mid y < -\frac{5}{2}x + 1\ ,\ x \in \mathbb{I},\ y \in \mathbb{I}\}$



c) $\{(x,y) \mid x > -4, x \in \mathbb{R}\}$



d) $\{(x,y) \mid y \leq 0, y \in \mathbb{I}\}$



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https://youtu.be/00HcfRdb3kI

A ***system of inequalities*** is a combination of two or more inequalities.

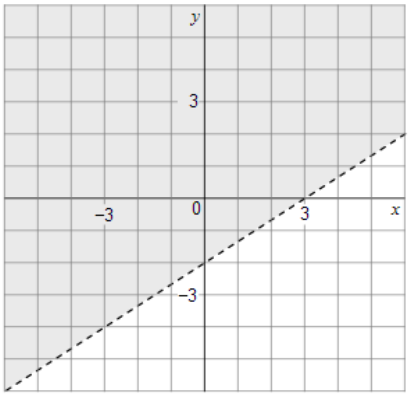
The solution of a system is the set of all ordered pairs that satisfy <u>all</u> the inequalities.

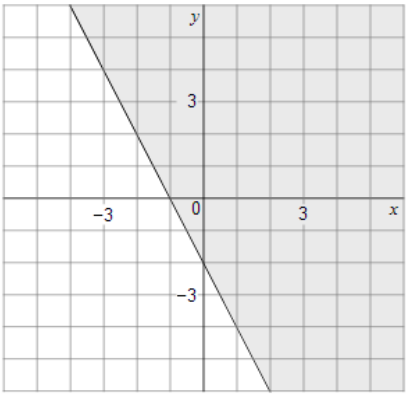
Consider the graph of the inequalities

$2x - 3y < 6$

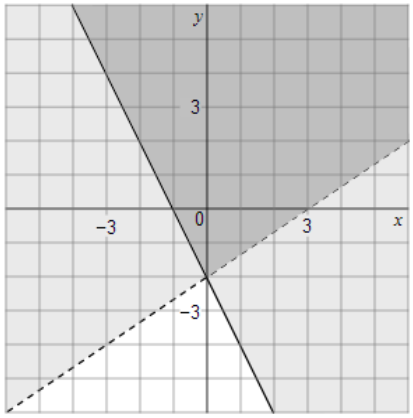
and

$y \geq -2x - 2$





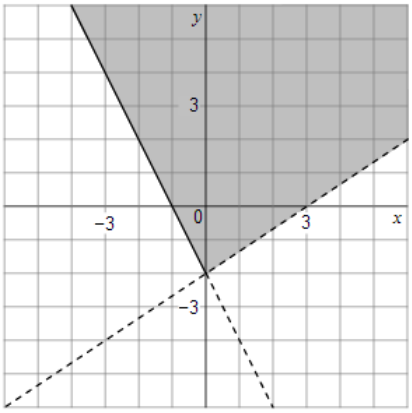
The solution of the system of linear inequalities $2x - 3y < 6$ and $y \geq -2x - 2$ are the points that satisfy both statements; that would be the intersection of the graphs for both inequalities.



Note: Some of the points on the boundary of the second inequality are in the solution set. Those that are in the shaded region of the first inequality are; those outside that shaded region are not.

Solving Systems of Inequalities

- Draw each inequality; the graph of the solution is the region where all the inequalities overlap.



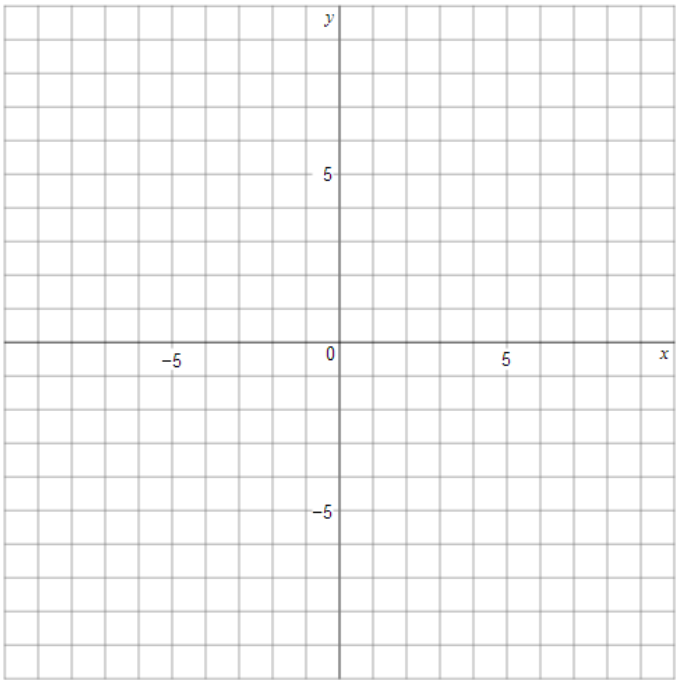
Example

Graph the following systems. Describe the region of the solution set.

a) For $x \in \mathbb{R}$ and $y \in \mathbb{R}$,

$\{(x,y) \mid x + y < -3\}$

$\{(x,y) \mid x - 2y < -6\}$



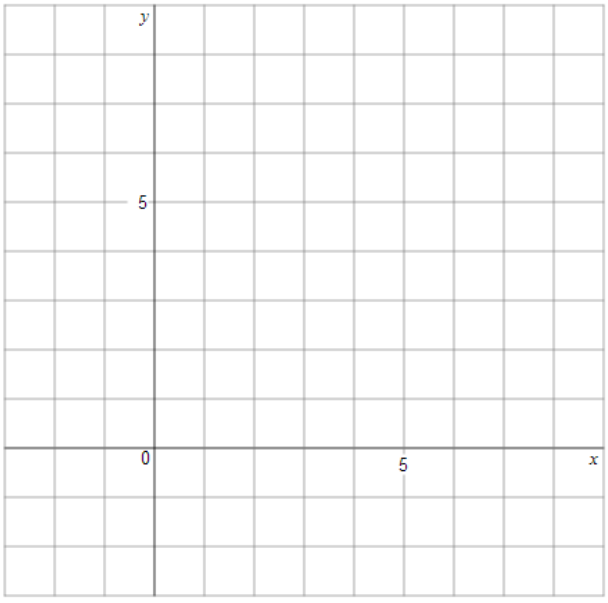
b) For $x \in \mathbb{I}$ and $y \in \mathbb{I}$,

$\{(x,y) \mid y \geq 2x - 2\}$

$\{(x,y) \mid y \leq -x + 7\}$

$\{(x,y) \mid x \geq 0\}$

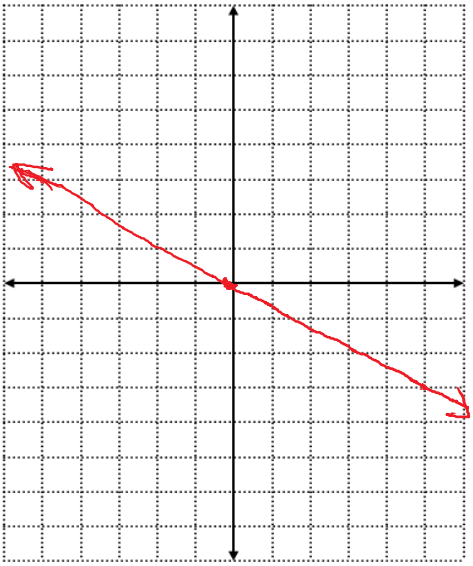
$\{(x,y) \mid y \geq 0\}$



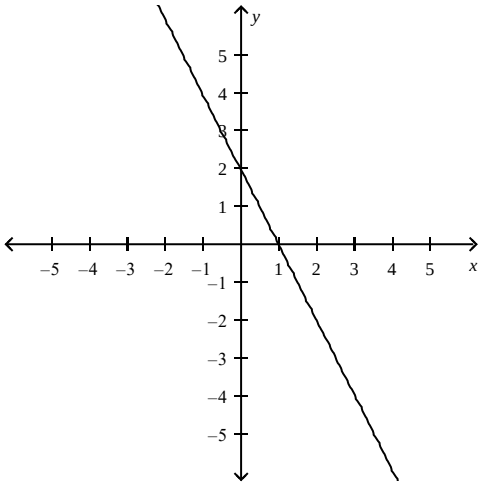
PRACTICE PROBLEMS

1. Is the point (0, 0) in the solution set for the linear inequality $10y - 12x > 5$?

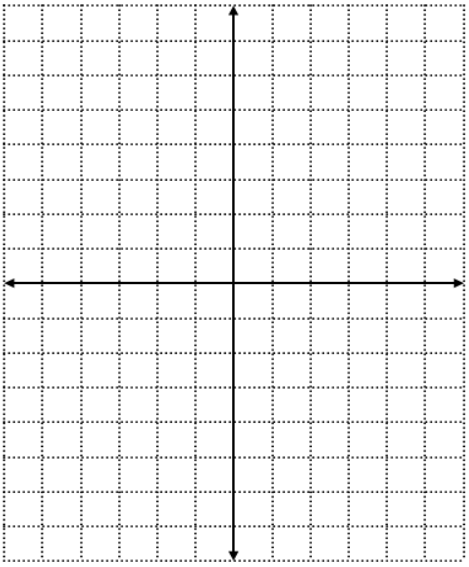
2. Which side of the boundary line is the solution set for the linear inequality $5y + 3x \leq 0$? Shade the solution area.



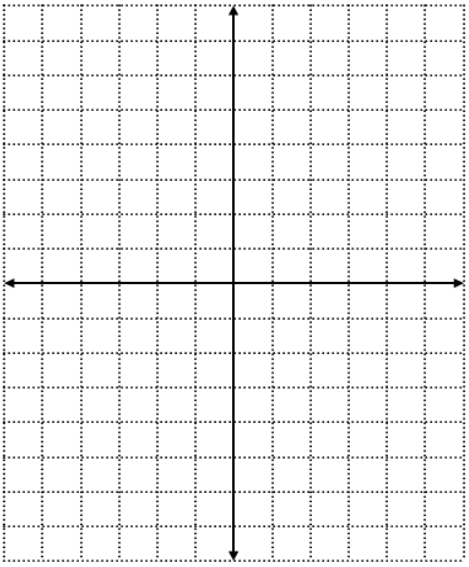
3. Which side of the boundary line is the solution set for the linear inequality $4y + 8x \leq 2$? Shade the solution area.



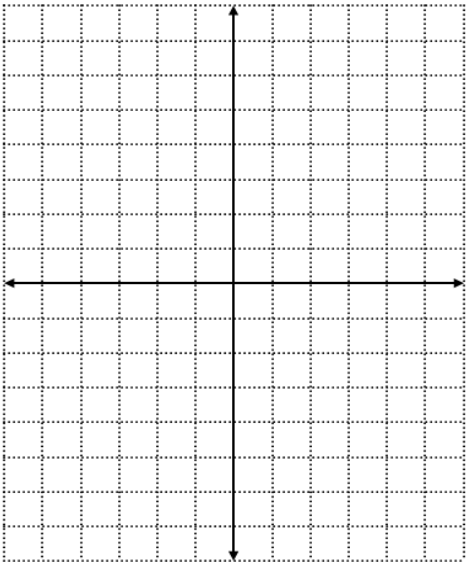
4. Graph the solution set for the linear inequality $x + y \geq 1$.



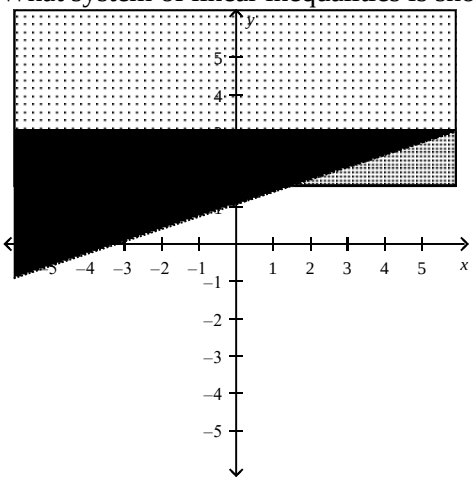
5. Graph the solution set for the linear inequality $3y - 6x < -1$.



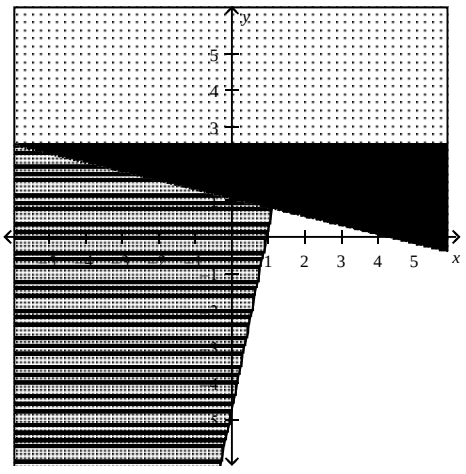
6. Graph the solution set for the linear inequality $5y - 2x \leq 15$.



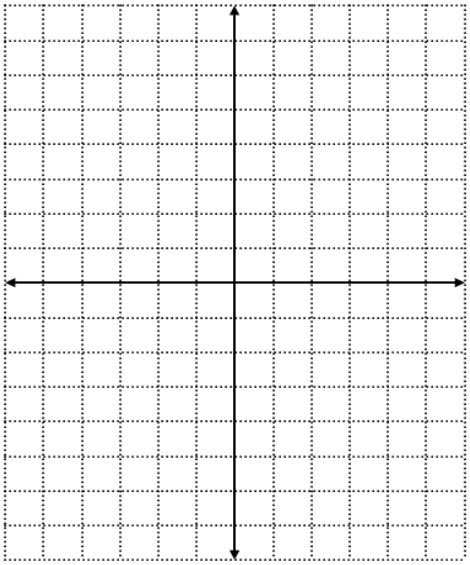
7. What system of linear inequalities is shown here?



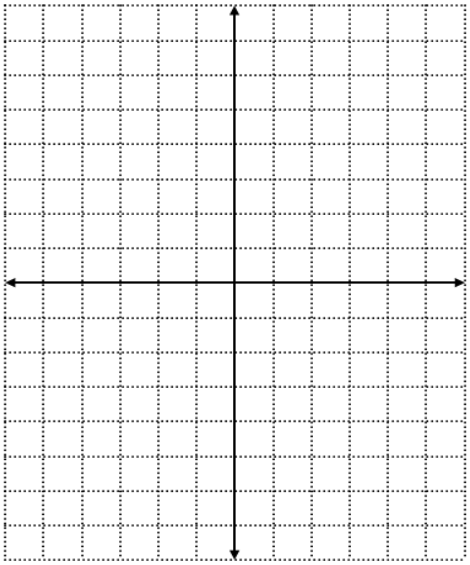
8. What system of linear inequalities is shown here?



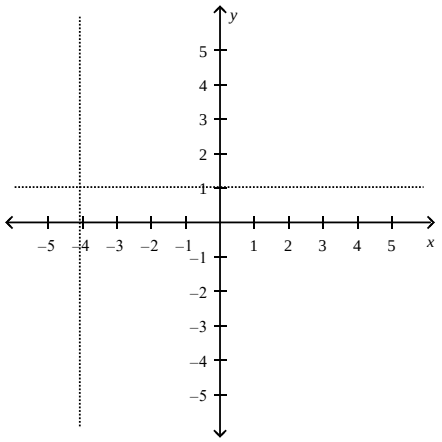
9. Graph the system of linear inequalities:
$\{(x, y) \mid x + y \leq 2, x > -3, x \in \mathrm{R}, y \in \mathrm{R}\}$



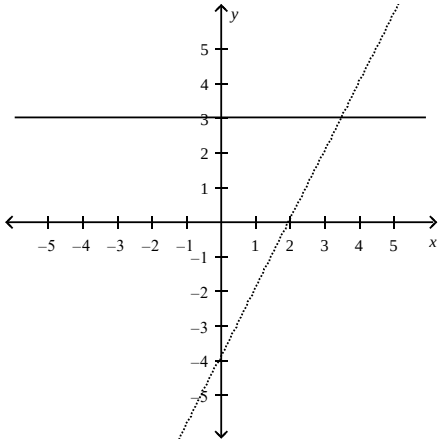
10. Graph the system of linear inequalities:
$\{(x, y) \mid x + y \leq 2, x > -3, x \in \mathrm{W}, y \in \mathrm{W}\}$



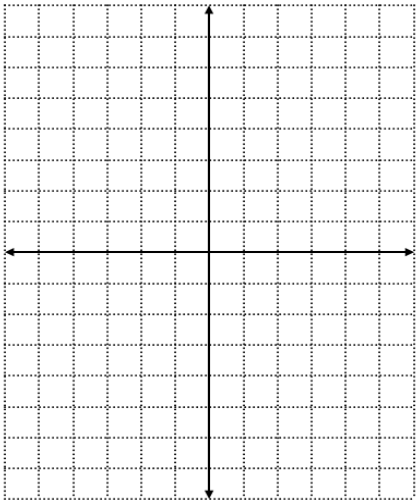
11. Complete the graph of the solution set for the following system of inequalities.
$\{(x, y) \mid y < 1, x > -4\}$



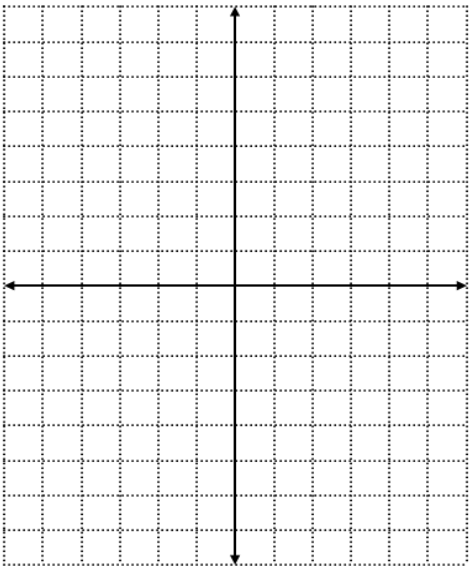
12. Complete the graph of the solution set for the following system of inequalities.
$\{(x, y) \mid y \geq 3, y > 2x - 4\}$



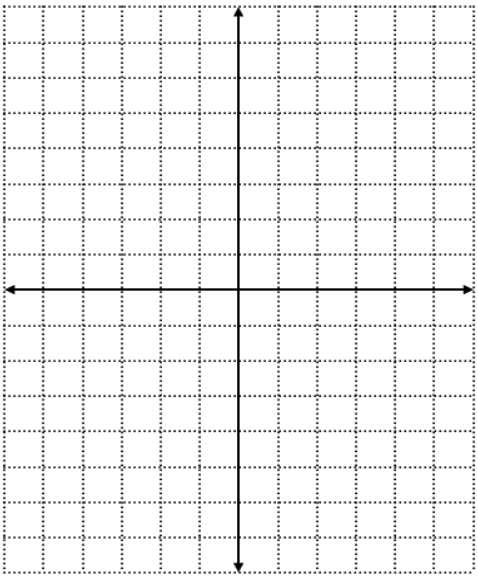
13. Complete the graph of the solution set for the following system of inequalities.
$\{(x, y) \mid y \geq 3x, 2x + 3y \geq -3\}$



14. Graph the solution set for the following system of inequalities.
$\{(x, y) \mid x \leq 4, y > x - 1, x \in \mathrm{R}, y \in \mathrm{R}\}$



15. Graph the solution set for the following system of inequalities.
$\{(x, y) \mid x + 2y \leq 2, y + 2 > x, x \in \mathrm{R}, y \in \mathrm{R}\}$



- Watch the following screencast video for the answers to the previous Practice Questions:
https://youtu.be/uqvMxdrUuQo
