LEARNING GUIDE 5: SURFACE AREA

Watch the following instructional video. In your handout:
i) Copy down the given notes and examples
ii) Complete the assigned questions
https://youtu.be/_xN1Ms01jls

Review: Circles

radius
- the distance from the centre of the circle to the outside edge
- \( r \) shows the radius
- the radius is half the diameter: \( r = \frac{d}{2} \) or \( r = \frac{d}{2} \)

diameter
- the distance across a circle through its centre
- \( d \) shows the diameter
- the diameter is twice the radius: \( d = 2 \times r \) or \( d = 2r \)

circumference
- the distance around a circle (the perimeter)
- \( C \) shows the circumference
- \( C = 2 \times \pi \times r \) or \( C = \pi \times d \)

area
- \( A = \pi \times r^2 \) or \( A = \pi r^2 \)

1. Find the circumference of each circle to the nearest tenth (1 decimal place).

a)

\[
C = \pi \times d = 3.14 \times \frac{6}{1} = 18.8 \text{ cm}
\]

b)

\[
C = 2 \times \pi \times r = 2 \times \frac{2}{1} \times \pi = 12.6 \text{ cm}
\]
2. Find the area of each circle to the nearest tenth (1 decimal place).

a) \[ A = \pi \times r^2 \]
\[ A = \pi \times r \times r \]
\[ = _____ \times _____ \times _____ \]
\[ = _____________ \text{ cm}^2 \]

b) \[ \text{Area} = \pi \times r^2 \]
\[ \text{Area} = \pi \times r \times r \]

Area Formulas

Area of a rectangle = \( l \times w \)  
Area of a triangle = \( b \times h \div 2 \)

3. Find the area of this shape.

a) \[ A = b \times h \div 2 \]
\[ = _____ \times _____ \div 2 \]

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https://youtu.be/A5qMGrEBvwa
Warm Up

1. Draw a square and a rectangle. 
   a) square          b) rectangle

2. Use isometric dot paper to make it easier to draw 3-D shapes. 
   Follow the steps to draw each solid.
   a) cube          b) rectangular prism

3. Draw the top, front, and side view of your cube and rectangular prism.
   a) cube          b) rectangular prism
   top       front       side          top       front       side
5.1 Views of Three-Dimensional Objects

Example 1: Draw and Label Top, Front, and Side Views

Draw the top, front, and side views of this object.

A 3-D object has length, width, and height.
Example 2: Sketch a Three-Dimensional Object When Given Views

An object made of 6 blocks has these views. Sketch the object.

Solution

Sketch the object on isometric paper.

Show You Know

An object is made using 5 blocks. The top, front, and side views are shown. Sketch the object on isometric dot paper.
1. a) Circle the top view.
   b) Put a square around the front view.
   c) Put an X on the side view.

2. Draw each 3-D object using the views.

   a) top    front    side
   b) top    front    side
3. Draw the top, front, and side views.

You can make the shapes out of blocks before you draw them.

Nets of Three-Dimensional Objects

**rectangular prism**
- a box with dimensions that are not all equal

**net**
- a 2-dimensional shape that, when folded, creates a 3-D object
Example: Draw a Net for a Three-Dimensional Object

Draw a net for the umbrella stand.

Solution

Think, “What would the umbrella stand look like if you could cut it open and flatten it?”

The net has 1 ________________ and 1 rectangle.

The width of the rectangle is equal to the distance around the circle, which is called the ________________.

Draw a net for a soup can.

SOUP
Example: Build a Three-Dimensional Object from a Given Net

**triangular prism**
- a prism with 2 triangular bases
- each base is the same size and shape
- the sides are rectangles

Can this net be folded to form a tent?

---

**Solution**

Trace the net on a sheet of paper.
Cut along the outside edges.
Fold along the inside edges.
Tape the edges together to build a **triangular prism**.

---

Build a 3-D object from this net.
What object does it make?  

---
1. Draw the net for each object. Label the measurements on the net.

   a) \[ d = 30 \text{ mm} \]
   \[ 78 \text{ mm} \]

   b) Paper
   500 Sheets
   28 cm
   21.5 cm
   5 cm

2. Match each solid with its net. Write your answers on the blanks. Some solids are used more than once.

   - rectangular prism
   - cylinder
   - triangular prism

   [Net Diagrams for solids]
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https://youtu.be/z3zGCasqNVg

Warm Up

1. Find the area of the rectangle.

   a)

   \[
   \begin{array}{c}
   \text{8 cm} \\
   \text{9.1 cm}
   \end{array}
   \]

   \[
   A = l \times w \\
   = 9.1 \times 8 \\
   = \underline{72.8} \text{ cm}^2
   \]

2. Find the area of the triangle.

   a)

   \[
   \begin{array}{c}
   \text{h = 14 m} \\
   \text{b = 10 m}
   \end{array}
   \]

   \[
   A = (b \times h) \div 2 \quad \leftarrow \text{Formula} \\
   = (10 \times 14) \div 2 \quad \leftarrow \text{Substitute} \\
   = \underline{70} \div 2 \\
   = \underline{35} \text{ m}^2 \quad \leftarrow \text{Answer}
   \]

Surface Area of a Prism

   surface area
   - the number of square units needed to cover all the faces of a 3-D object
   - the sum of the areas of all the faces of an object
   - measured in square units (cm\(^2\), m\(^2\))
Example: Calculate the Surface Area of a Rectangular Prism

a) Draw the net of this right rectangular prism.

Solution

b) What is the surface area of the prism?

Solution

The right rectangular prism has 6 faces. There are 3 different sizes of faces.

\[
\begin{array}{ccc}
\text{front or back} & \text{top or bottom} & \text{ends} \\
6 \text{ cm} & 4 \text{ cm} & 4 \text{ cm} \\
10 \text{ cm} & 10 \text{ cm} & 6 \text{ cm} \\
\end{array}
\]

\[
\begin{align*}
A &= l \times w \\
&= 10 \times 6 \\
&= 60 \times 2 \\
&= 120 \text{ cm}^2
\end{align*}
\]

\[
\begin{align*}
A &= l \times w \\
&= 10 \times \underline{\phantom{6}} \\
&= 40 \times 2 \\
&= 80 \text{ cm}^2
\end{align*}
\]

\[
\begin{align*}
A &= l \times w \\
&= 6 \times \underline{\phantom{6}} \\
&= 24 \times 2 \\
&= 48 \text{ cm}^2
\end{align*}
\]

Surface Area = (area of front and back) + (area of top and bottom) + (area of ends)

\[
= 120 + 80 + 48
\]

\[
= 248 \text{ cm}^2
\]
Example: Calculate the Surface Area of a Right Triangular Prism

a) Draw the net of this triangular prism.

Solution

\[ A = l \times w \]
\[ = 9 \times \underline{\text{______}} \]
\[ = \underline{\text{______}} \text{ m}^2 \]

b) What is the surface area?

Solution

The bases of the prism are equilateral triangles. The sides of the prism are rectangles.

\[ A = (b \times h) \div 2 \]
\[ = (3 \times 2.6) \div 2 \]
\[ = \underline{\text{______}} \div 2 \]
\[ = \underline{\text{______}} \text{ m}^2 \]

The right triangular prism has 5 faces.

\[ A = \underline{\text{______}} \]
\[ A = \underline{\text{______}} \]
\[ A = \underline{\text{______}} \]
\[ A = \underline{\text{______}} \]
\[ A = \underline{\text{______}} \]
Surface Area = (3 × area of rectangle) + (2 × area of triangle)

= (3 × 27) + (2 × 3.9)

= ___________ + ___________

= ___________

The surface area of the right triangular prism is ___________ m².

Practise

1. Find the surface area of the rectangular prism to the nearest tenth of a square centimetre (1 decimal place).

   \[ A = l \times w \]
   \[ = \text{_________} \times \text{_________} \]
   \[ = \text{_________} \]

Area of front and back:

\[ A = \text{_________} \times 2 \]
\[ = \text{_________} \]

Area of top and bottom:

\[ A = \text{_________} \times 2 \]
\[ = \text{_________} \]

Area of 2 ends:

\[ A = \text{_________} \times \text{_________} \]
\[ = \text{_________} \]

Surface Area = (area of front and back) + (area of top and bottom) + (area of ends)
The surface area of the right rectangular prism is ____________ cm².

= ____________ + ____________ + ____________

= ____________ cm²
2. Find the surface area of this ramp in the shape of a right triangular prism.

\[ A = l \times w \]

\[ = \quad \times \quad \]

\[ = \quad \text{m}^2 \]

\[ A = (b \times h) ÷ 2 \]

\[ = (\quad \quad \times \quad \quad) ÷ 2 \]

\[ = \quad ÷ 2 \]

\[ = \quad \text{m}^2 \]

Surface Area = (area of 3 rectangles) + (2 x area of triangle)

\[ = (\quad \quad + \quad \quad + \quad \quad) + (2 \times \quad \quad) \]

\[ = \quad + \quad \]

\[ = \quad \]

The surface area of the ramp is \quad \text{m}^2.
3. Sometimes cheese is packaged in a triangular box. How much cardboard would you need to cover this piece of cheese?

The tick marks show that the sides are equal.

**side rectangle**

\[
A = l \times w \\
= \underline{3\text{ cm}} \times \underline{5.7\text{ cm}} \\
= \underline{17.1}\text{ cm}^2
\]

**end rectangle**

**triangle**

\[
A = \frac{b \times h}{2} \\
= \frac{\underline{3\text{ cm}} \times \underline{8\text{ cm}}}{2} \\
= \underline{12}\text{ cm}^2
\]

Surface Area = \((2 \times \text{area of side rectangle}) + (\text{area of end rectangle}) + (2 \times \text{area of triangle})

\[
= (2 \times \underline{17.1}) + (\underline{12}) + (2 \times \underline{12}) \\
= \underline{34.2 + 12 + 24} \\
= \underline{70.2}\text{ cm}^2
\]

Sentence: ___________________________________________________________________
4. The area of each face of a rectangular prism is shown. What is the surface area of the prism?

![Diagram of a rectangular prism with face areas: front 20 mm², top 15 mm², end 12 mm²]

Surface Area = (area of front and back) + (area of top and bottom) + (area of ends)

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https://youtu.be/ZA_9hF8xMqo

**Warm Up**

1. Calculate the diameter or radius.

   a) ![Image of a circle with a diameter of 2.6 mm]

   \[ d = 2 \times r \]

   \[ = 2 \times \underline{\phantom{0}} \]

   \[ = \underline{\phantom{0}} \text{ cm} \]

   b) ![Image of a coin with a diameter of 28 mm]

   \[ r = d \div 2 \]

   \[ = \underline{\phantom{0}} \div 2 \]

   \[ = \underline{\phantom{0}} \]
2. Calculate the circumference.

*Calculate:*

← Formula → \[ C = \pi \times d \]

← Substitute → \[ = 3.14 \times 2.2 \]

← Solve → \[ = \text{______________ cm} \]

Round your answer to 1 decimal place.

*Calculate:*

\[ r = \text{______________} \]

← Formula → \[ A = \pi \times r^2 \]

← Substitute → \[ = 3.14 \times \square \]

← Solve → \[ = 3.14 \times \text{_________} \times \text{_________} \]

\[ = \text{______________ cm}^2 \]

**Surface Area of a Cylinder**

cylinder

* a 3-D object with 2 parallel and congruent circular bases

*Congruent means the exact same size.*
Example: Determine the Surface Area of a Cylinder

a) Calculate the surface area of the can.
   Round your answer to the nearest hundredth of a square centimetre (2 decimal places).

Solution

Method 1: Use a Net

Step 1: Draw the net and label the measurements.

Step 2: Find the radius.
   diameter = 7.6 cm
   radius = 7.6 ÷ 2
   = ____________

Step 3: Find the area of 1 circle.
   \( A = \pi \times r^2 \)
   = 3.14 \times 3.8^2
   = 3.14 \times 3.8 \times 3.8
   = ____________

Step 4: Find the area of 2 circles.
   \( 2 \times 45.3416 = ____________ \)

Step 5: Find the area of the rectangle using the circumference.
   \( A = l \times w \)
   \( A = (\pi \times d) \times w \)
   \( A \approx 3.14 \times 7.6 \times 11 \)
   \( A \approx ____________ \)

Step 6: Total surface area = area of 2 circles + area of 1 rectangle
   = ____________ + ____________
   = ____________

The total surface area is approximately ____________ cm\(^2\).
**Method 2: Use a Formula**

The formula for the surface area of a cylinder is

\[
S.A. = 2 \times \left( \pi \times r^2 \right) + \left( \pi \times d \times h \right)
\]

- **2 circles**
- **circle area**
- **rectangle area**
  - length is the circumference of a circle (\( \pi \times d \))
  - width is the height of the cylinder (\( h \))

\[
d = 7.6 \text{ cm} \quad r = 7.6 \div 2 \quad h = 11 \text{ cm}
\]

\[
r = 3.8 \text{ cm}
\]

\[
S.A. = 2 \times (\pi \times r^2) + (\pi \times d \times h)
\]

\[
S.A. = 2 \times (3.14 \times 3.8^2) + (3.14 \times 7.6 \times 11)
\]

\[
S.A. = 2 \times (3.14 \times 3.8 \times 3.8) + (3.14 \times 7.6 \times 11)
\]

\[
S.A. = 2 \times \text{rectangle area} + \text{circle area}
\]

\[
S.A. = \text{rectangle area} + \text{circle area}
\]

\[
S.A. = \text{rectangle area}
\]

The surface area of the can is \( \text{rectangle area} \)\( \text{cm}^2 \), to the nearest hundredth (2 decimal places).
1. Draw a net for this cylinder.

2. Calculate the surface area of the cylinder. Then, calculate the surface area to the nearest tenth of a square centimetre (1 decimal place).

   *Calculate area of circle:*
   
   \[
   \text{Area} = \pi \times r^2
   \]

   *Calculate area of 2 circles:*

   *Calculate area of rectangle:*

   \[
   \text{Calculate surface area:}
   \]

   \[
   A = \text{____________} + \text{____________}
   \]

   \[
   = \text{____________}
   \]

3. Calculate the surface area of the cylinder. Round your answer to the nearest tenth of a square centimetre.
4. Use the formula $S.A. = 2 \times (\pi \times r^2) + (\pi \times d \times h)$ to calculate the surface area of each object.
   Round each answer to the nearest hundredth of a square unit (2 decimal places).

   a) 
   \[ d = 2.5 \text{ cm} \quad r = \quad h = \]

   Formula →
   Substitute →
   Solve →

   b) 
   \[ d = 5 \text{ cm} \quad r = \quad h = \]

   Formula →
   Substitute →
   Solve →

5. Which method do you like best for finding the surface area of a cylinder?
   Circle your answer.

   Using the sum of the area of each face, like in #3 and #4.
   or
   Using a formula, like in #5.
6. Kaitlyn and Hakim each bought a tube of candy. Both containers cost the same amount.

\[ d' = 8 \text{ cm} \quad \text{Kaitlyn} \quad CANDY \quad 122 \text{ cm} \]

\[ d' = 10 \text{ cm} \quad \text{Hakim} \quad CANDY \quad 85 \text{ cm} \]

a) How much plastic is needed to make Kaitlyn’s container?

b) How much plastic is needed to make Hakim’s container?

Sentence: ___________________________

Sentence: ___________________________

← Formula →

← Substitute →

← Solve →

c) Which container is made of more plastic?

_______________________________