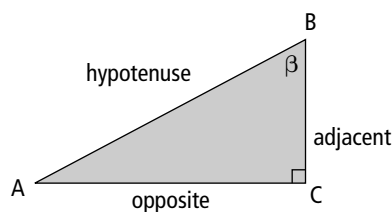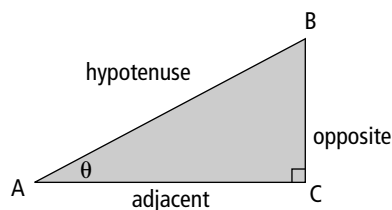# Chapter 3 Right Triangle Trigonometry

3.1 The Tangent Ratio

> ### « KEY IDEAS »
>
> • In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal.
>
> • The sides of a right triangle are labelled according to a reference angle.
>
>
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>
>
> • The tangent ratio compares the length of the side opposite the reference angle to the length of the side adjacent to the angle in a right triangle. For the reference angle θ in the upper triangle,
>
> $$\tan \theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta}$$
>
> For the reference angle β in the lower triangle,
>
> $$\tan \beta = \frac{\text{length of side opposite } \beta}{\text{length of side adjacent to } \beta}$$
>
> • You can use the tangent ratio to
>   – determine the measure of one of the acute angles when the lengths of both legs in a right triangle are known
>   – determine a side length if the measure of one acute angle and the length of one leg of a right triangle are known

Example

A housing contractor is required to build a roof on a house with a 38° slope from the horizontal. Additionally, the peak of the roof is to align with the centre of the end walls. The house is 16 m wide and has that are 4 m high. To the nearest tenth of a metre, how high will the peak be above the ground once the house is completed?

Solution

Organize the information and sketch a diagram to illustrate the problem.
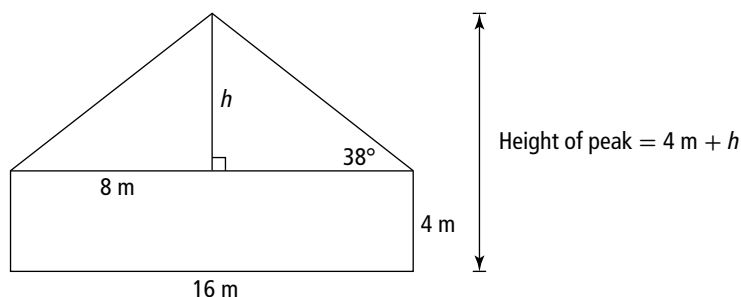
Since the peak must be in the centre of a wall that is 16 m wide, it means that any point directly below the peak is 8 m from either side. Create a right triangle (two right triangles, actually) using the top of the wall, the height of the roof, and the roofline.

Let h represent the height, in metres, of the roof above the top of the wall.
Identify the sides in terms of the given angle of 38°: opposite $= h$ and adjacent $= 8$ m.
Apply the tangent ratio using $\theta = 38°$:

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 38° = \frac{h}{8}$$

Isolate the value h and solve. (Remember to set your calculator to degrees.)
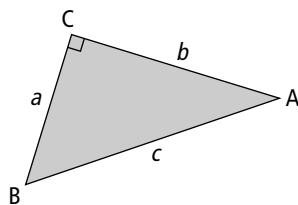
$8(\tan 38°) = h$
$8(0.7812) = h$
$\quad 6.2496 = h$

The height, h, of the roof above the wall is 6.25 m, to the nearest tenth of a metre.
Therefore, the height of the peak above the ground once the house is completed is
6.25 m + 4 m (height of wall) = 10.25 m.

A Practise

1. For $\angle$B in right $\triangle$ABC, identify

  a) the hypotenuse

  b) the adjacent side
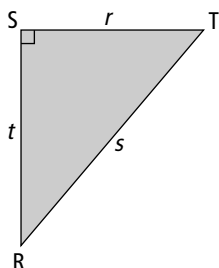
  c) the opposite side

2. Refer to right $\triangle$ABC in question 1.

  a) State the tangent ratio of $\angle$A.

  b) State the tangent ratio of $\angle$B.

3. Refer to right $\triangle$ABC in question 1.

  a) If $a = 10$ cm and $b = 12$ cm, what is the value of tan A?

  b) If $a = 1.9$ m and $b = 2.4$ m, what is the value of tan B?

  c) If $\tan A = \frac{5}{6}$ and $a = 15$, what is the value of b?

**4.** In right △RST, determine the length of side t if $\angle R = 39°$ and $r = 4.3$ m.

S ___ r ___ T

t

s

R

☆**5.** Use the tangent ratios and a calculator to determine the measure of each angle, to the nearest tenth of a degree.

X

1.6 m

Z

1.85 m

Y
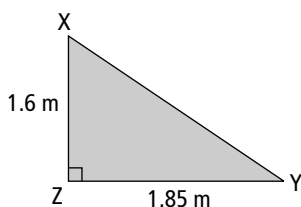
a) $\angle X$

b) $\angle Y$

B Apply

6. Calculate $\angle A$ and $\angle B$ in right △ABC where $\angle C = 90°$, side a measures 3.2 m, and side b measures 2.5 m. What is the measure of side c? Explain how you determined the length of this side.

7. Ms. Singh's design class is required to build model cars. The models are tested for efficiency by releasing them at the top of a ramp and comparing the distances they travel. The ramp extends over a horizontal distance of 375 cm and forms an angle of 38° with the floor. Calculate the height of the ramp to the nearest centimetre.
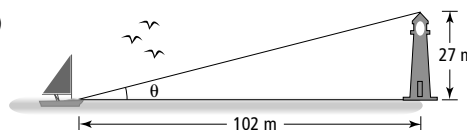
h

38°

375 cm
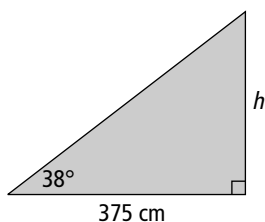
8. Determine the value of each variable. Express your answer to the nearest tenth of a unit.

a)

25°

27.6 m

x

b)

θ

102 m

27 m
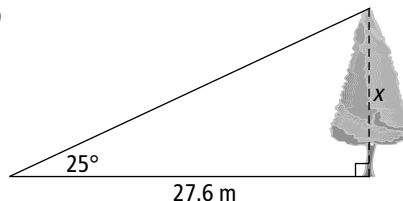
9. The length of a shadow cast by an elm tree is 6.2 m. At the same time, a woman who is 165 cm tall casts a shadow that is 285 cm in length. What is the height of the tree?

10. A wheelchair ramp on a bus forms an angle of 18° with the ground. If the floor of the bus is 65 cm above the ground, how much width does the ramp require beside the bus?

☆**11.** The minute hand of a clock has a length of 14 cm. The length of the hour hand is 11 cm.

11 12 1
10 2
14 cm
9 3
11 cm
8 4
7 6 5
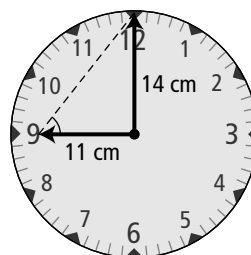
a) If a line is drawn between the ends of the two hands at the 9 o'clock position, what angle is formed between the line and the minute hand?

b) What is the angle between the line in part a) and the hour hand?

C Extend

12. An access road runs 75 m from and parallel to a high voltage power line. Two maintenance crews working along the road at different positions can see the same transmission tower. Looking from the north, crew #1 sees the tower 28° to the west of the road. From the south crew #2 sights the tower 19° to the west of the road. How far apart are the crews?
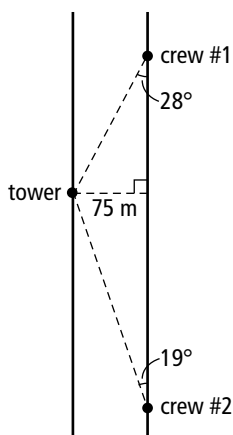
13. A plane flying at an altitude of 1200 m is directly over a small island. After a few minutes, the island is sighted at an angle 5.2° below and behind the plane. Determine the distance the plane travelled, to the nearest metre.

14. Different types of gravel will form different slopes when piled up. Coarse gravel can sustain an angle of 29° with the horizontal, whereas fine gravel can sustain an angle of 24°. Maria and Nathan are landscapers who need to create a circular mound 1.7 m high. To the nearest tenth of a metre, what is the minimum diameter of mound they can create using

a) coarse gravel?

b) fine gravel?

D Create Connections

15. Jasmine and Ivan are monitoring a pair of falcons nesting on a building ledge that is 112 m above the street. They position their telescope (on a 1.4-m tripod) at street level at a distance of 245 m from the building. At what angle does the telescope need to be set? Calculate your answer to the nearest tenth of a degree.

☆**16.** Guy wires are lengths of cord or cable used to support towers or poles. For greatest support, four guy wires should be used. They should be spaced evenly around a tower or pole and attached at least two thirds of the way up the structure. They should form an angle with the ground of 60° or less. Ramon wishes to erect a radio tower 6.5 m tall on a piece of property measuring 10 m by 4.2 m.

a) Does Ramon have enough space for proper guy wires?

b) How high a tower, to the nearest tenth of a metre, could Ramon build and support safely in the space?

17. $\triangle ABC$ is a right triangle with the right angle at vertex C.

a) If the triangle has a tangent ratio of 2, state two possible values for each of side a and side b.

b) If the triangle has a tangent ratio of $\frac{1}{2}$, state two possible values for each of side a and side b.

c) What do you notice about the values in parts a) and b)? Why is this?