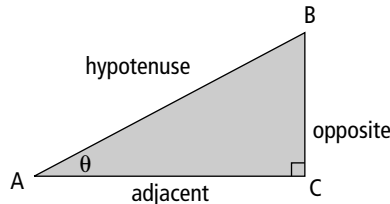## 3.2 The Sine and Cosine Ratios

<div style="border:1px solid #000;">

KEY IDEAS

- The sine ratio and cosine ratio compare the lengths of the legs of a right triangle to the length of the hypotenuse.

- The sine ratio compares the length of the side opposite an acute angle to the length of the hypotenuse.

$$\sin \theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}}$$

- The cosine ratio compares the length of the side adjacent to an acute angle to the length of the hypotenuse.

$$\cos \theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}}$$

- The sine and cosine ratios can be used to calculate side lengths and angle measures of right triangles.

- Visualizing the information that you are given and that you need to find is important. It helps you determine which trigonometric ratio to use and whether to use the inverse trigonometric ratio.

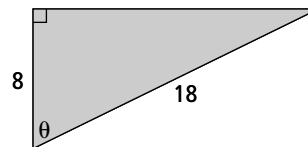- Determine the value of θ, to the nearest degree.

$$\cos \theta = \frac{\text{length of side adjacent } \theta}{\text{length of hypotenuse}}$$

$$\cos \theta = \frac{8}{18}$$

$$\theta = \cos^{-1}\left(\frac{8}{18}\right)$$

$\theta = 63.6122...°$
Angle θ is approximately 64°.

</div>

Example

Traditionally, ships were constructed in docks on land and then launched by sliding them down a ramp into the water. If the ramp was too steep, there was a risk of capsizing the ship; if it was not steep enough, the ship could get stuck. Engineers determined that a ramp angle of 32° was ideal. At a shipyard, the vertical distance from the construction dock to the water is 6.8 m. Calculate to the nearest tenth of a metre the length of ramp needed to launch a ship safely.

Solution

Organize the information by sketching a diagram to illustrate the problem.
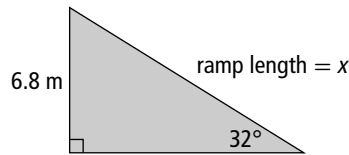
Choose the appropriate trigonometric ratio.
In relation to the 32° angle, the side that measures 6.8 m is the opposite side. The side of unknown length, x, is the hypotenuse. Therefore, use the sine ratio.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 32° = \frac{6.8 \text{ m}}{x}$$

Isolate the value of x and solve.

$$\sin 32°(x) = 6.8 \text{ m}$$

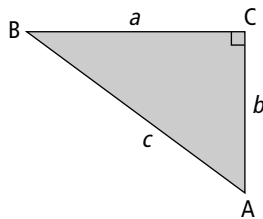$$x = \frac{6.8 \text{ m}}{\sin 32°}$$

$$x = \frac{6.8 \text{ m}}{0.5299}$$

$$x = 12.83 \text{ m}$$

The length of the ramp must be 12.8 m, to the nearest tenth of a metre.

A Practise

1. Express each trigonometric ratio in relation to right $\triangle ABC$.

   a) $\sin A$

   b) $\cos A$

   c) $\sin B$

   d) $\cos B$

2. Using right $\triangle ABC$ in question 1 and the stated values, determine each unknown value.

   a) If $b = 12$ cm and $c = 17$ cm, what is the value of $\cos A$?

   b) If $a = 10$ cm and $c = 15$ cm, what is the value of $\sin A$?

   c) If $b = 1.9$ m and $c = 2.4$ m, what is the value of $\sin B$?

   d) If $a = 2.6$ mm and $c = 3.9$ mm, what is the value of $\cos B$?

   e) If $\sin A = \frac{5}{6}$ and $c = 15$, what is the value of a?

   f) If $\cos B = \frac{9}{45}$ and $c = 15$, what is the value of a?

3. Use the sine and cosine ratios and a calculator to determine the measure of each angle, to the nearest tenth of a degree.

   a) $\angle R$

   b) $\angle S$
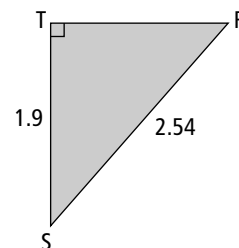
4. Use a calculator to determine the measure of each angle, to the nearest tenth of a degree.

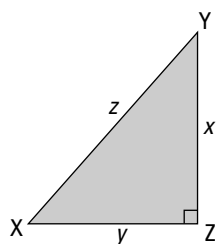   **a)** cos A = 0.5835

   b) sin B = 0.8358

   c) sin θ = 0.2181

   d) cos θ = 0.0488

5. For right △XYZ, determine the length of side x to two decimal places if

   a) ∠Y = 38° and z = 2.35 cm

   b) ∠X = 59° and z = 5.12 m

6. For right △XYZ in question 5, calculate the length of side z to two decimal places if

   a) ∠Y = 41° and x = 54.7 mm

   **b)** ∠X = 52° and x = 7.64 m

B Apply

7. Some students in Mr. Pang's wood shop class are building a shelf. The shelf is 28 cm deep and is supported by a 32-cm diagonal brace on each side. Calculate the angle that the brace forms with the backing piece, to the nearest degree. Calculate the required height of the backing piece, to the nearest millimetre.
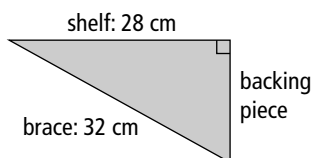
8. A 14-ft ladder is leaning against a wall 9 ft high in such a way that the top 3 ft of the ladder extend above the top of the wall.
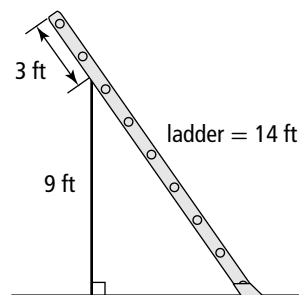

   a) What is the horizontal distance from the bottom of the ladder to the wall? Round your answer to the nearest tenth of a foot.

   b) What angle does the ladder form with the ground, to the nearest tenth of a degree?

9. A zip line is to be set up from a tree to the ground in the Cortez family's backyard. To prevent people from zipping down too quickly, the line should form an angle of 35° with the ground. To the nearest metre, how long can the zip line be if it is anchored 28 m from the base of the tree?

★10. In areas with extreme winds, houses are constructed with steel rods that run diagonally inside walls to help keep walls square. Rods are sold in standard lengths of 4 m and 5 m.
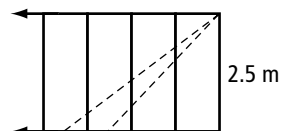

   If standard walls are 2.5 m high, what angle does the 5-m rod make with the floor? What angle with the floor does the 4-m rod make? State your answers to the nearest degree.

C Extend

11. A boat ramp with a cable winch is to be constructed to allow the McKenzie family to pull their boat from the water for the winter. If the shore has a slope of 39° and the front of the boat must be lifted 3 m above the water, what length does the ramp have to be?

☆**12.** A 2″ by 4″ length of lumber actually measures 1.5″ by 3.5″. To make a top rail for a deck, a carpenter stacks two pieces of lumber together to create a rail that is 3″ thick. However, the carpenter has only 3.5″ wood screws available. At what angle must he drive the screws in from underneath so that none of the points sticks through the top rail? Round your answer to the nearest degree.
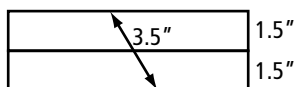

13. A water gun at Splasher's water park is positioned at the centre of the long side of a 10-m by 6-m rectangular pool. The gun can spray to a maximum distance of 7 m.
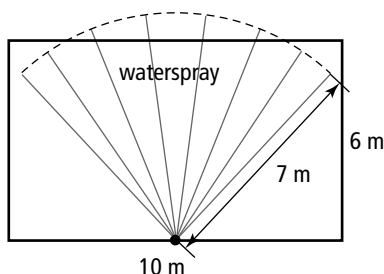

a) How much of the opposite side of the pool can be sprayed?

b) How much of the opposite side cannot be sprayed?

c) What spray distance would the gun need for the spray to reach the entire opposite side? Round each answer to the nearest tenth of a metre.

D Create Connections

14. Determine the length of CD, to the nearest tenth of a centimetre.
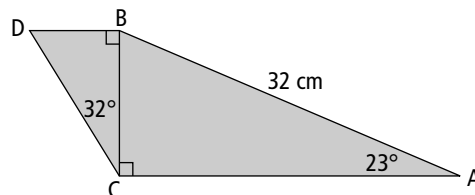
☆**15.** A traditional teepee uses 13 poles to form its conical shape. The poles are inclined at an angle of 65° to the ground and tied together 4.8 m up their length. Calculate the diameter of this structure, to the nearest tenth of a metre.
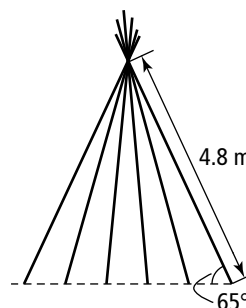
16. a) Copy the table and determine the trigonometric values for each stated value of θ , to four decimal places.

θ	tan θ	sin θ	cos θ
15°			
30°			
45°			
60°			
75°			

b) Describe any pattern you see in each column.

c) Do you see any relationship between the sine and cosine values? Explain.