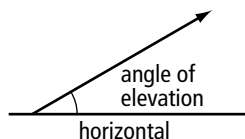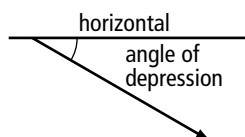## 3.3 Solving Right Triangles

> ### ⟪ KEY IDEAS ⟫
>
> • An angle of elevation is the angle between the line of sight and the horizontal when an observer looks upward.
>
>
>
> angle of elevation
>
> horizontal
>
> • An angle of depression is the angle between the line of sight and the horizontal when the observer looks downward.
>
>
>
> horizontal
>
> angle of depression
>
> • To solve a triangle means to calculate all unknown angle measures and side lengths.

Example

Two adjoining properties are bordered by three roads, as shown in the diagram. The property owners agree to put a fence around and between both lots. What total length of fencing is required, to the nearest metre? Explain why you should round up or down.

lot 1   lot 2

Hwy 7 z y x Hwy 231

61°   37°   Mollard Road

w 82 m
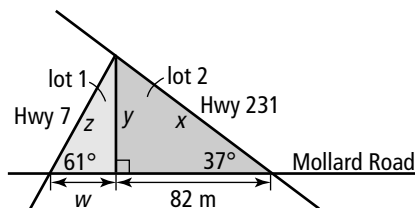
Solution

First, determine the length of fence around the first lot.
Let x represent the length, in metres, of fence required along Hwy 231. Let y represent the length, in metres, of fence required along the property line.
Use the given information and choose the appropriate trigonometric ratio to solve for each value.

For x:
$$\cos 37° = \frac{82 \text{ m}}{x}$$
$$\cos 37°(x) = 82 \text{ m}$$
$$x = \frac{82 \text{ m}}{\cos 37°}$$
$$x = \frac{82 \text{ m}}{0.7986}$$
$$x = 102.68 \text{ m}$$

For y:
$$\tan 37° = \frac{y}{82 \text{ m}}$$
$$(82 \text{ m})(\tan 37°) = y$$
$$(82 \text{ m})(0.7536) = y$$
$$y = 61.79 \text{ m}$$

Next, calculate the length of fence needed to complete the fence around the second lot.
Let w represent the length, in metres, of fence required along Mollard Road. Let z represent the length, in metres, of fence required along Hwy 7.
Use the given information and choose the appropriate trigonometric ratio to solve for each value.

For w:

$\tan 61° = \frac{61.79 \text{ m}}{w}$

$w = \frac{61.79 \text{ m}}{\tan 61°}$

$w = \frac{61.79 \text{ m}}{1.804}$

$w = 34.25 \text{ m}$

For z:

$\sin 61° = \frac{61.79 \text{ m}}{z}$

$z = \frac{61.79 \text{ m}}{\sin 61°}$
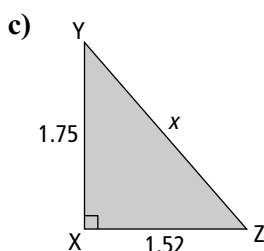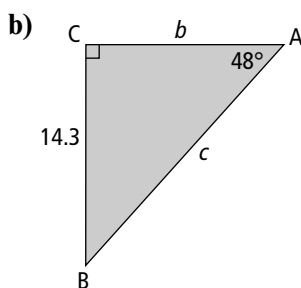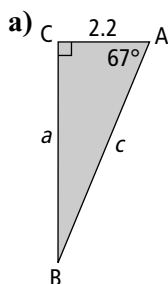
$z = \frac{61.79 \text{ m}}{0.8746}$

$z = 70.65 \text{ m}$

Total length of fence needed $= 82 \text{ m} + 102.68 \text{ m} + 61.79 \text{ m} + 34.25 \text{ m} + 70.65 \text{ m}$
$= 351.37 \text{ m}$

The two properties require a total of approximately 352 m of fencing. Round up to make sure that there is enough fencing.

A Practise
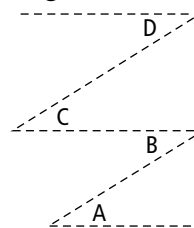
**1.** Solve each triangle. State each answer to the nearest tenth of a unit.

a)

b)

c)

d) Use a second strategy to solve part c).

2. Using the diagram, name

a) two angles of elevation

b) two angles of depression

c) two pairs of equal angles

3. Paolo and Chandra are on two balconies facing each other across a courtyard. Chandra sends a text message to Paolo to tell him that she sees him at an angle of depression of 23°. Paolo replies that Chandra is wrong and that Chandra is actually at an angle of elevation of 23°.

a) Who is right? Explain.

b) What is the relationship between angles of elevation and angles of depression?

4. For each figure, solve all variables. For side lengths, state your answers to the nearest tenth of a unit. For angle measures, give your answers to the nearest degree.

a)

b)
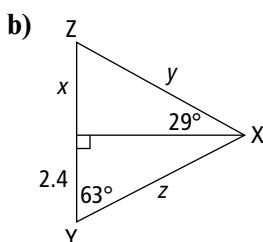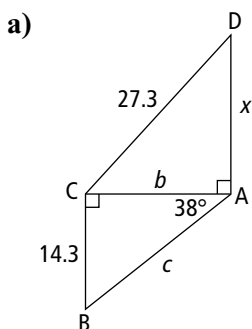
5. Determine the value of each variable. Express your answer to the nearest tenth of a unit.

a)

b)
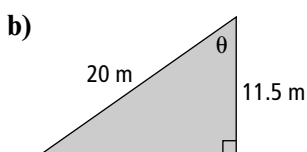
B Apply

★**6.** A car is parked on a street 4.8 m from the bottom of Ruthie's apartment building. From her window above the street, Ruthie views the car at an angle of depression of 73°. Kenneth lives directly across the street from Ruthie. From his window at exactly the same height as Ruthie's, Kenneth sees the car at an angle of depression of 59°. Determine the distance between Ruthie's and Kenneth's windows, to the nearest tenth of a metre.

7. There are two sails on the mast of a sailboat. The mast measures 8.5 m from the booms at the bottom of the sails to the top. The main sail meets the mast at an angle of 32° and the secondary sail meets the mast at an angle of 21°. Determine the combined length of the two booms, to the nearest tenth of a metre.
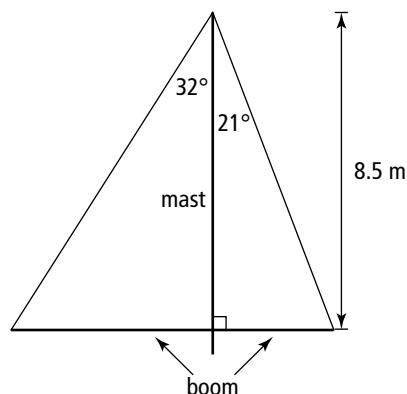

8. Pit mines are cone-shaped excavations often used in diamond mining. The side of one pit mine has an angle of depression of 35° so that it will not collapse.

a) If the mine has a diameter of 576 m, how deep is it?

b) If the mine is required to extend down 250 m, then how wide should it be at the top?

c) If the bottom of the mine is 250 m below the surface, what length does a conveyor belt to the top need to be if it follows the slope of the excavation?

In each case, round your answer to the nearest metre.

C Extend

9. A footbridge across a river is 12 m above the water. On one side, a ramp slopes to the bridge at an angle of 7°. On the other side, there is a set of stairs. The bottom of the stairs is 10 m from the bridge.

a) What is the distance from the bottom end of the ramp to the bridge?

b) At what angle do the stairs climb to the bridge?

10. A box measures 55 cm deep, 28 cm high, and 105 cm long.

28 cm
55 cm
105 cm
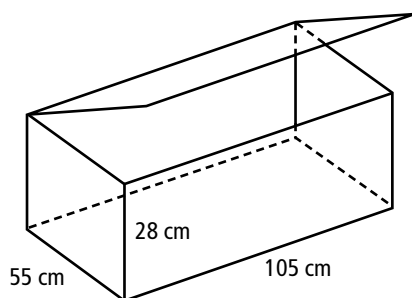
José wishes to use this box to hold his great-grandfather's cane, a family heirloom. The cane has a length of 120 cm.

a) Can the cane fit flat in the box?

b) Can José put the cane completely in the box without interfering with the lid? Explain.

☆**11.** A section of dike is to be constructed to hold water in a reservoir for a hydro-electric power dam. The dike needs to be built to a height of 36.5 m, with a slope of 55° on the reservoir side and a slope of 42° on the outside.


water side
36.5 m
55°
42°
d
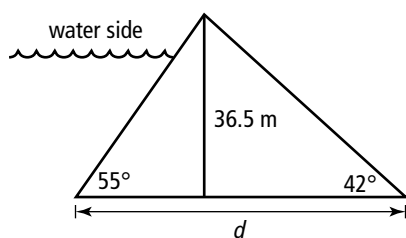
a) How wide is the dike at its base?

b) A wire mesh is to be attached to the outside slope of the dike to prevent rock slides.

Determine the length required for the wire mesh.

c) A different section of dike requires a height of 39 m. What is the width of its base?

d) A student engineer believes that for a 31-m-high section, the base needs to be 54.8 m wide. Is she correct? Explain.

Express each answer to the nearest tenth of a metre.

D Create Connections

☆**12.** In order to accurately measure the height of a mountain that cannot be climbed, two right triangles can be used. One lies horizontally along the ground and the other stands vertically with a vertex at the mountain's peak.

h
surveyor #1
43°
3.75 km
78°
surveyor #2
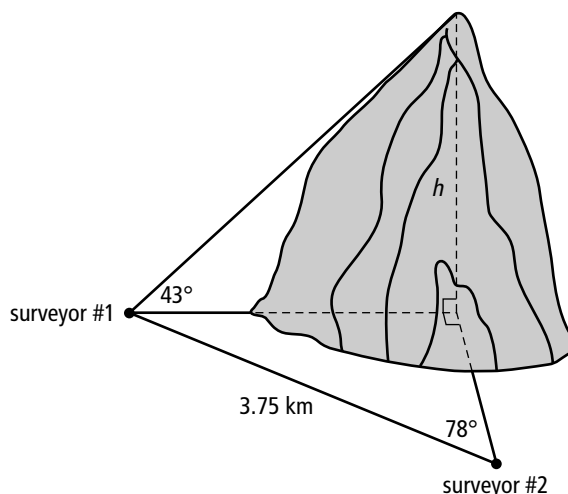
Two surveyors are 3.75 km apart. Surveyor #1 is directly west of the peak and surveyor #2 is directly south. From the position of the second surveyor, the first is at an angle of 78° west of north. Surveyor #1 can see the peak at an angle of elevation of 43°.

a) Determine the height of the mountain, to the nearest metre.

b) How long would a cable need to be, to the nearest metre, in order to connect the peak with the position of surveyor #1?

c) Solve this problem using a second set of strategies.