Chapter 4 Exponents and Radicals

4.1 Square Roots and Cube Roots

KEY IDEAS

- A perfect square is the product of two equal factors. One of these factors is called the square root.
  36 is a perfect square: $\sqrt{36} = 6$ because $6^2 = 36$. The symbol for square root is $\sqrt{\ }$.

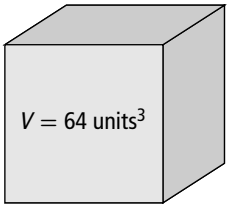
- A perfect cube is the product of three equal factors. One of these factors is called the cube root.
  – 125 is a perfect cube: $\sqrt[3]{-125} = -5$ because $(-5)^3 = -125$. The symbol for cube root is $\sqrt[3]{\ }$.

- Some numbers are both perfect squares and perfect cubes.
  15 625 is a perfect square: $125^2 = 15\,625$
  15 625 is a perfect cube: $25^3 = 15\,625$

- You can use diagrams, prime factorization, or a calculator to solve problems involving square roots and cube roots. Prime factorization involves writing a number as the product of its factors.

Determine the cube root of 64.

– Use a diagram.

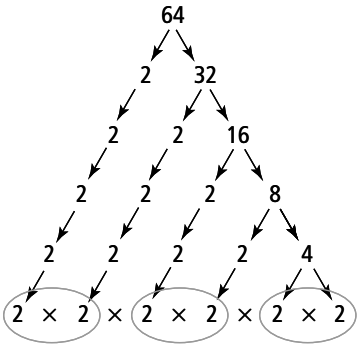


The edge lengths represent the cube root: (4)(4)(4) = 64.

– Use a calculator.

C 64 2nd $\sqrt[x]{y}$ 3 = 4.

– Use *prime factorization*.



There are three equal groups of 4.
Therefore, the cube root of 64 is 4.

Example

Identify each number as a perfect square or a perfect cube.

a) 256

b) 3375

Solution

Method 1: Use Guess and Check

a) Perfect square:
Since (13)(13) = 169, you could try 14.
$14^2 = (14)(14) = 196$ Too low
$16^2 = (16)(16) = 256$ Correct!
Therefore, 256 is a perfect square.

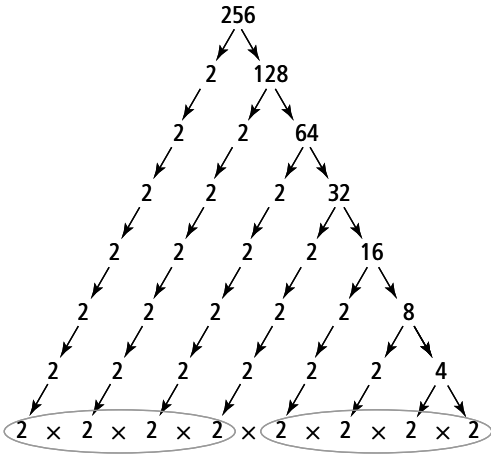
Perfect cube:
Since 256 ends with a 6, you could try 6.
$6^3 = (6)(6)(6) = 216$ Too low
$7^3 = (7)(7)(7) = 343$ Too high
No whole number cubed results in a product of 256.
Therefore, 256 is not a perfect cube.

b) Perfect square:
Since 3375 ends with a 5, you could try numbers that end with a 5.
$55^2 = (55)(55) = 2025$ Too low
$65^2 = (65)(65) = 4225$ Too high
No whole number squared results in a product of 3375.
Therefore, 3375 is not a perfect square.

Perfect cube:
Since 3375 ends with a 5, you could try numbers that end with a 5.
$25^3 = (25)(25)(25) = 15\,625$ Too high
$15^3 = (15)(15)(15) = 3375$ Correct!
Therefore, 3375 is a perfect cube.

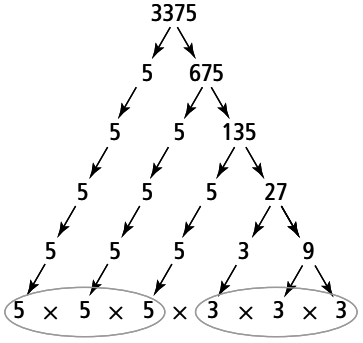
Method 2: Use Prime Factorization

a)



There are two equal groups of 2s.
Therefore, the square root of 256 is (2)(2)(2)(2) = 16.

b)



There is one group of 5s and one group of 3s.
Therefore, the cube root of 3375 is (5)(3) = 15.

Method 3: Use a Calculator

a) C 256 $\sqrt{x}$ = 16.
perfect square

C 256 2nd $\sqrt[x]{y}$ 3 = 6.349604
Since the cube root is not an integer, 256 is not a perfect cube.

b) C 3375 $\sqrt{x}$ = 58.09475
Since the square root is not a whole number, 3375 is not a perfect square.

C 3375 2nd $\sqrt[x]{y}$ 3 = 15.
perfect cube

A Practise

1. What is the value of each expression? Express the answers as integers or fractions.

a) 9^2 **b)** $(-15)^2$

c) -25^2 **d)** $\frac{4}{3^2}$

e) $-\frac{5^2}{8}$ **f)** $\left(\frac{-6}{7}\right)^2$

2. Evaluate. Express the answer as an integer or a fraction.

a) 9^3 **b)** $(-3)^3$

c) -6^3 **d)** $\frac{4^3}{8}$

e) $\frac{-9}{3^3}$ **f)** $\left(\frac{5}{7}\right)^3$

3. What is the value of each expression?

a) $\sqrt{25}$ **b)** $\sqrt{196}$

c) $\sqrt{(49)(16)}$ **d)** $\frac{18}{\sqrt{81}}$

e) $\frac{\sqrt{64}}{12}$ **f)** $\sqrt{\frac{64}{196}}$

g) $\frac{\sqrt{16}}{\sqrt{144}}$ **h)** $\sqrt{36x^2}$

i) $\frac{\sqrt{49a^2}}{\sqrt{169b^2}}$

4. Evaluate.

a) $\sqrt[3]{8}$ **b)** $\sqrt[3]{27}$

c) $\sqrt[3]{1728}$ **d)** $\sqrt[3]{(64)(125)}$

e) $\frac{\sqrt[3]{216}}{2}$ **f)** $\frac{15}{\sqrt[3]{15\,625}}$

g) $\sqrt[3]{\frac{8}{343}}$ **h)** $\sqrt[3]{125y^3}$

i) $\sqrt[3]{729a^3}$

☆**5.** Identify each number as a perfect square, a perfect cube, or both. Support your answer using a diagram or a factor tree.

a) 8 **b)** 512

c) 15 625 **d)** 196

e) 46 656 **f)** 729

6. State whether each number is a perfect square, a perfect cube, both, or neither.

a) 169 **b)** 225

c) 64 **d)** 256

e) 117 649 **f)** 133 642

7. Determine if each number is a perfect square or a perfect cube using prime factorization. Explain the process.

a) 16 **b)** 27

c) 1000 **d)** 324

e) 441 **f)** 2917

8. Calculate.

a) $\sqrt{289}$ **b)** $\sqrt{529}$

c) $\sqrt[3]{2744}$ **d)** $\sqrt[3]{10\,648}$

e) $\sqrt[3]{29\,791}$ **f)** $\sqrt[3]{19\,683}$

9. Bill is designing a cube-shaped storage container to store his hockey equipment. The container will have a volume of 2.744 m^3. What will the dimensions of the container be?

10. Sharon plans to build a square patio in a sunny area in her yard. If the patio has an area of 529 ft^2, what is its side length?

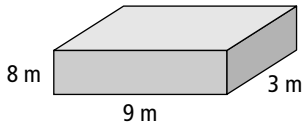
B Apply

☆**11.** Belle noticed that the water tap in the kitchen leaked. She decided to use a cylinder and collect the water drips for 24 h. She collected 5.88 cm^3 of water. Belle determined that at this rate her family would waste 2146.2 cm^3 of water per year. What would be the edge length of a cube that would contain this amount of water? Express the answer to one decimal place.

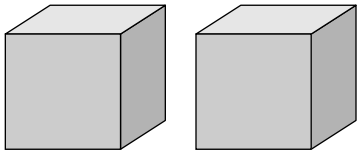
12. The Henderson family plans to build a square double garage. The floor plan shows that the garage will have an area of 576 ft^2. What are the side lengths of the garage?

13. If the area of John's square bedroom is 156.25 ft^2, what do the side lengths measure?

14. A right prism is shown. What would be the dimensions of a cube with the same volume?



15. The surface area of two dice is 1452 mm^2. What is the volume of each die?

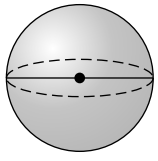


16. A grade 10 class collects scrap metal as a fundraiser. The students calculate that the scrap metal they collected occupies a volume of 238 m^3. If this metal were compressed into a cubic bale, what would its edge lengths be? Express the answer to the nearest tenth of a metre.

17. The Dice House is a zero carbon home designed by Sybarite, a British architecture firm. This cubic house has a volume of 729 m^3. What are the dimensions of the Dice House?

18. A sphere has the surface area shown. What is the length of the diameter of the sphere? Hint: $SA = 4\pi r^2$.

$SA = 803.84\ cm^2$



C Extend

19. Given the equation $y = x^2 - 4$, determine the value of y when

a) $x = 8$

b) $x = 14$

20. Given the equation $y = x^2 - 4$, determine the value of x when

a) $y = 32$

b) $y = 525$

☆**21.** Sonja owns a helium tank that holds 54 ft^3 of gas. She rents out the helium tank for parties and sells balloons with a 6-in. radius. How many balloons will a full helium tank inflate?

22. A sphere has a volume of 1296 cm^3. Determine the surface area of the sphere. Express the answer to the nearest square centimetre.

Hint: $V = \frac{4}{3}\pi r^3$.

23. Evaluate each square root.

a)

$\sqrt{25}$	
$\sqrt{2.5}$	
$\sqrt{0.25}$	
$\sqrt{0.025}$	
$\sqrt{0.0025}$	
$\sqrt{0.00025}$	

b)

$\sqrt{81}$	
$\sqrt{8.1}$	
$\sqrt{0.81}$	
$\sqrt{0.081}$	
$\sqrt{0.0081}$	
$\sqrt{0.00081}$	

c) What can you conclude about the square root of decimal numbers?

D Create Connections

☆**24.** Explain why $\sqrt{-25}$ has no solution and $\sqrt[3]{-27}$ has a solution.

25. a) What happens to the area of a square when you double the length of each side? triple the length of each side?

b) What happens to the volume of a cube when you double the length of each edge? triple the length of each edge?