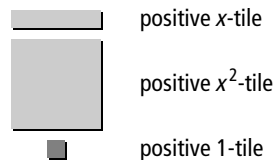# Chapter 5 Polynomials

5.1 Multiplying Polynomials

<div style="border:1px solid black; padding:1em;">

KEY IDEAS

- You can use the distributive property to multiply polynomials. Multiply each term in the first polynomial by each term in the second polynomial. Then, collect like terms.

- You can use algebra tiles to model algebraic expressions.

  positive x-tile

  positive x^2-tile

  positive 1-tile

  The same tiles not shaded represent negative quantities.

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Example

Use the distributive property to determine the product of $(2a + 4)$ and $(a^2 + 5a + 7)$.

Solution

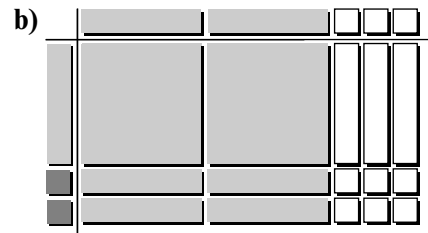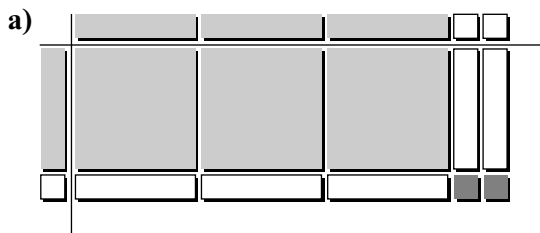Multiply each term in the first polynomial by each term in the second polynomial.
$(2a + 4)(a^2 + 5a + 7) = (2a)(a^2 + 5a + 7) + (4)(a^2 + 5a + 7)$
$\qquad\qquad\qquad\qquad = 2a^3 + 10a^2 + 14a + 4a^2 + 20a + 28$

Collect the like terms and arrange them in descending order of the power of a.
$\qquad\qquad\qquad\qquad = 2a^3 + 10a^2 + 4a^2 + 14a + 20a + 28$
$\qquad\qquad\qquad\qquad = 2a^3 + 14a^2 + 34a + 28$

A Practise

1. Determine the product that each algebra tile model shows. Then, state the dimensions.

a)

b)

2. Determine each product using algebra tiles.

   a) $(x + 3)(x + 4)$

   b) $(x - 2)(x - 5)$

   c) $(2x - 1)(3x + 2)$

   d) $(4x + 3)(x + 2)$

   e) $(x - 4)^2$

   f) $(x - 3)(3x + 4)$

3. Multiply.

   a) $(x - 4)(2x + 4)$

   b) $(t + 5)(t + 4)$

   c) $(3w + 2)(2w - 9)$

   d) $(z + 2)(z - 2)$

   e) $(a + b)^2$

   f) $(5e + 5)(6e - 1)$

4. Match each binomial on the left with a trinomial on the right.

   a) $(x - 2)(x - 1)$ **A** $x^2 + 10x + 21$

   b) $(x + 5)^2$ **B** $x^2 - 8x + 16$

   c) $(x + 7)(x + 3)$ **C** $x^2 - x - 56$

   d) $(x + 6)(x - 2)$ **D** $x^2 - 7x + 12$

   e) $(x - 4)^2$ **E** $x^2 - 3x + 2$

   f) $(x - 8)(x + 7)$ **F** $x^2 + 11x + 10$

   g) $(x - 3)(x - 4)$ **G** $x^2 + 4x - 12$

   h) $(x + 1)(x + 10)$ **H** $x^2 + 10x + 25$

5. Choose the trinomial that is the product of the binomials.

   a) $(x + 2)(x - 3)$

      A $x^2 - x - 6$

      B $x^2 + x - 6$

      C $x^2 + x - 1$

      D $x^2 - x + 1$

   b) $(x - 4)(x - 1)$

      A $x^2 - 4x - 4$

      B $x^2 - 4x + 4$

      C $x^2 + 4x + 5$

      D $x^2 - 5x + 4$

   c) $(x + 5)^2$

      A $x^2 + 10x + 10$

      B $x^2 + 10x + 5$

      C $x^2 + 10x + 25$

      D $x^2 + 5x + 25$

   d) $(x - 6)(x + 3)$

      A $x^2 + 3x - 18$

      B $x^2 - 3x - 18$

      C $x^2 - 6x - 18$

      D $x^2 - 6x + 18$

   e) $(x - 7)^2$

      A $x^2 - 14x - 14$

      B $x^2 - 14x + 49$

      C $x^2 - x + 49$

      D $x^2 - 7x + 49$

   f) $(x + 1)(x + 10)$

      A $x^2 + 11x + 10$

      B $x^2 + 10x + 11$

      C $x^2 + x + 10$

      D $x^2 + x + 1$

6. Use the distributive property to determine each product.

   a) $(d + 3)(2d^2 + 5d - 2)$

   b) $(4s - 5)(s^2 - 9s - 1)$

   c) $k(5k^2 - k + 7)$

   d) $(3c + 6)(c^2 + 4c + 7)$

   e) $(5y^2 - y)(2y^2 + 2y - 6)$

  ☆**f)** $(r^2 - 5r - 3)(3r^2 - 4r - 5)$

7. Simplify.

   a) $4(5y + 3)(2y - 3)(3y + 1)$

   b) $(3a + 9) + (2a - 5)(4a - 7) + (6a + 3)$

   c) $(2d - e)(3d - 5e) + (6d + 5e)(d - 4e)$

   d) $(5n + 4)^2 - (2n + 7)(8n - 6)$

   e) $(3w^2 + w + 4)(2w^2 - 5w - 6)$

  ☆**f)** $2(4t + 5s)(2t - 3s) - (5t - s)$

8. Multiply. Then, combine like terms.

a) $(3a + 7) + (4a - 3)(2a + 2)$

b) $(b + 2)(3b + 6) + (b - 3)^2$

c) $(2x - y)(x - 4y) + (x + y)(3x + y)$

d) $4(6a + 2c)(a - 3c) - (a + 2c)^2$

e) $(x^2 - 2x + 3)(2x^2 + 3x - 4)$

f) $(4b - d)^2 - 2(2b + 3d)(b + d)$

B Apply

9. An error was made in each of the following solutions.

- Write the step number that contains the error.

- Determine the correct solution.

a) $(4t - 5)(7t + 2) - (6t - 3)$

Step 1: $4t(7t + 2) - 5(7t + 2) - (6t - 3)$

Step 2: $28t^2 + 8t - 35t - 10 - 6t - 3$

Step 3: $28t^2 + 8t - 35t - 6t - 10 - 3$
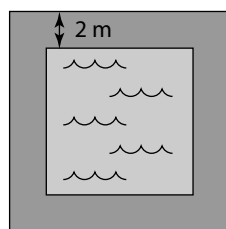
Step 4: $28t^2 - 33t - 13$

b) $x(2y^2 + y - 3) + x^2y$

Step 1: $2xy^2 + xy - 3x + x^2y$

Step 2: $2xy^2 + x^2y + xy - 3x$

Step 3: $4x^2y + xy - 3x$

10. The width of a deck that surrounds a square wading pool is 2 m.

a) Write a polynomial expression that represents the area of the pool and the pool deck.

b) What is the area of the deck and the pool, if the pool has an area of 49 m²?

11. A mirror mounted horizontally on a wall has a width to height ratio of 5:2. The mirror frame adds 6 in. to the width and 4 in. to the height.

a) Write a polynomial expression that represents the total area of the mirror, including the frame. Multiply and combine like terms.

**b)** If the dimensions of the mirror are $5x$ by $2x$, calculate the total area when $x = 8$ in.

12. A diamond is drawn within a rectangle that has a width of $(x + 6)$ units and a length of $(2x + 4)$ units. The diamond touches the centre point of each side of the rectangle, as shown.
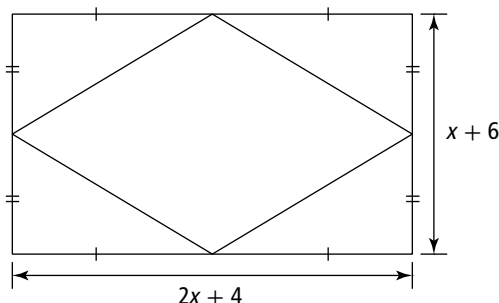
a) Write an expression to represent the area of the diamond. Multiply and combine like terms.

**b)** What is the relationship between the area of the diamond and the area of the rectangle?

C Extend

13. **a)** Write the expressions to calculate the area of the larger rectangle.
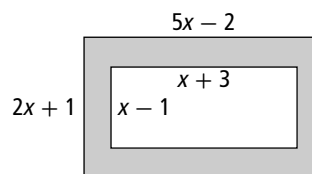
b) Write the expression to determine the area of the smaller rectangle.

**c)** Write the expression(s) for the area of the shaded region. Then, simplify.

14. An open-top box is made from a rectangular sheet of thin cardboard. The corner pieces are cut out as shown in the diagram. A metal corner piece reinforces the corner.
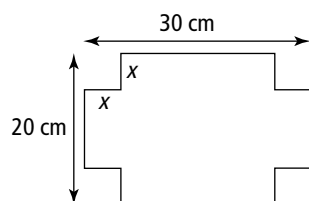
a) Write the expressions for the dimensions of the box.

b) Write the expression to calculate the volume of the box.

c) Simplify this expression.

D Create Connections

15. The diagram shows the first three rectangular prisms in a pattern.
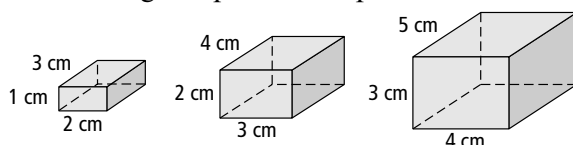
a) Write the dimensions and the volume of the 4th prism.

b) Write the unsimplified expression that shows the volume of the nth prism.

c) Show two different ways to determine the volume of the 10th prism.

16. a) Choose three consecutive even numbers. Multiply the first and third numbers. Then, calculate the square of the middle number. Repeat this multiplication and squaring with several different groups of three consecutive even numbers. What pattern do you notice?

b) Let x represent your middle number. What algebraic expressions represent the first and last numbers?

c) Use algebraic multiplication to show that your pattern in part a) is always true.

17. a) Copy and complete the tables.

Table A	
Numbers	**Total**
6, 7	42
–	56
–	–
–	–
–	–

Table B				
Numbers				**Total**
5	25	15	2	42
–	36	–	2	–
7	–	–	–	72
–	–	–	–	–
–	–	–	–	–

b) Write two binomials and their equivalent trinomial to explain why the totals in Table A are equal to the totals in Table B.

18. Consider the following list of expressions.

$(4 \times 3) - (2 \times 1)$
$(5 \times 4) - (3 \times 2)$
$(6 \times 5) - (4 \times 3)$
$(7 \times 6) - (5 \times 4)$

a) If the least number in each group of four numbers is n, write the unsimplified expression that determines the difference of the products in each case.

b) Simplify the expression.

c) Show how the simplified expression matches the differences of the products.