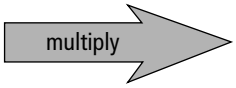
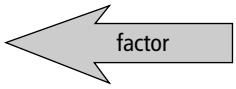
5.2 Common Factors

KEY IDEAS

- Factoring is the reverse of multiplying.



$5(x + 2) = 5x + 10$



To find the greatest common factor (GCF) of a polynomial, find the GCF of the coefficients and variables.

- To factor a GCF from a polynomial, divide each term by the GCF.
- Polynomials can be written as a product of the GCF and the sum or difference of the remaining factors.

$$2m^3n^2 - 8m^2n + 12mn^4 = 2mn(m^2n - 4m + 6n^3)$$

- A common factor can be any polynomial, such as a binomial.

$a(x + 2) - b(x + 2)$ has a common factor of $x + 2$.
The final factored form of this polynomial is $(a - b)(x + 2)$.

Example

Write $12x^4y - 28x^3y^2 + 8x^2y^3$ in factored form.

Solution

The GCF of 12, 28, and 8 is 4.
The GCF of x^4y, x^3y^2, and x^2y^3 is x^2y.
So the GCF for the whole polynomial is $4x^2y$.
Mentally divide the polynomial by the GCF to determine the other factor.
So the polynomial in factored form is $4x^2y(3x^2 - 7xy + 2y^2)$. Note that this GCF is a monomial. A GCF *may* be a binomial.

A Practise

1. List the factors of each number in each pair. Then, identify the GCF.

a) 10 and 15

b) 24 and 36

c) 16 and 48

d) 40 and 60

e) 18 and 45

f) 14 and 24

2. List the prime factors of the coefficients and of the variables for each term.

a) $6x^2$, $12x$

b) $20c^2d^3$, $30cd^2$

c) $4b^2c^3$, $6bc^2$

d) $18xy^2z$, $24x^2y^3z^2$

e) $5m^3n$, $20mn^2$

3. State the common prime factors and identify the GCF for each pair of terms in question 2.

4. Determine the GCF of the following sets of terms.

a) $14a$, $21b$

b) $-5n^2$, $-10n$

c) $3rs$, $7t$

d) $12f^2g^3$, $16fg^2$, $32f^3g^2$

e) $-15d^2e^3$, $-30cd^2e$, $-45cde$

f) $-18j^3k$, $27j^2kl$, $36j^2k^2l^2$

5. Identify the least common multiple for each of the following sets of numbers or terms.

a) 16 and 20

b) 15, 30, and 40

c) $6x$ and $9x$

d) $2t$, $3t^2$, and $4t^3$

e) $4ab^3$, $6a^2b^2$, and $10a^3b$

f) $8cde$, $14c^3de^2$, and $18c^2d^2e^3$

6. Factor the following polynomials. Then, use multiplication to check each answer.

a) $6s + 30$

b) $4t + 28$

c) $5a - 5$

d) $16r^2 - 12r$

e) $7xy + 14xy - 49xz$

f) $3c^3 - 9c^2 - 27d^2$

7. State the missing factor.

a) $15w^2 - 5w = 5w$ (_____)

b) $4a^2 - 6a^3 =$ _____ $(2 - 3a)$

c) $10x^2y^2 - 50xy = 10y$ (_____)

d) $2g^2 + 4g = 2g$ (_____)

e) $35x^2y + 15x^2y^2 + 5xy =$ _____ $(7x + 3xy + 1)$

f) $2r^2 + 6r^3s - 4rs^2 =$ _____ $(r + 3r^2s - 2s^2)$

8. Identify the GCF for each pair of terms.

a) $x(x - 6)$ and $4(x - 6)$

b) $a(a + 3)$ and $-7(a + 3)$

c) $d(d - 9)$ and $-6(d - 9)$

d) $ab(b + 2)$ and $a^2b(b + 2)$

e) $(x^2 + 2x)$ and $xy(x + 2)$

f) $4m(n - 1)$ and $(2m^2n^3 - 2m^2n^2)$

9. Write each expression in factored form.

a) $s(s + 5) - 2(s + 5)$

b) $r(r - 7) - 4(r - 7)$

c) $g(g + 6) + 9(g + 6)$

d) $p^2 + 3p + 4p + 12$ (Hint: Group like terms to find a common factor.)

e) $b^2 - 7b - 3b - 21$

f) $r^2 - 3r + 2rs - 6s$

B Apply

☆**10.** The Mount Baker girls' basketball team is planning a spaghetti dinner fund raiser. Table decorations will include floral centrepieces. The team decides to use roses, tulips, and daffodils. The local florist has 36 roses for $2.50 each, 48 daffodils for $1.70 each, and 60 tulips for $1.50 each.

a) If each centrepiece is to have the largest possible number of each type of flower, with the same number of flowers in each centrepiece, how many of each type of flower will the centrepieces contain?

b) To recover the money spent on the flowers, the centrepieces will be sold for the cost of the flowers. What will each centrepiece cost?

11. State whether each polynomial is factored fully and correctly. If it is not, write the correct and fully factored form.

a) $12x - 6 = 3(4x - 2)$

b) $-10w^2 - 10w = 10w(-w - 1)$

c) $10c^4d - 20c^3d^2 + 15c^2d^2 = 5c^2d(2c^2 - 4cd + 3d)$

d) $x^2 + 2xy + 3x + 6y = x(x + 2y) + 3(x + 2y)$

e) $a^2 - 3ab - 2ab + 6b^2 = (a - 2b)(a - 3b)$

f) $t^2 - 4t + 5t - 5 = t(t - 4) + 5(t - 1)$

12. a) Write a polynomial with two terms that have a GCF of $4x$.

b) Write a polynomial with two terms that have a GCF of $3rs$.

c) Write a polynomial with two terms that have a GCF of $5m^2n^2$.

d) Write a polynomial with three terms that have a GCF of ab.

e) Write a polynomial with three terms that have a GCF of $2c^2d^2$.

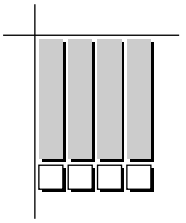
f) Write a polynomial with four terms that have a GCF of $2e$.

g) Write a polynomial with four terms that have a common factor of $c + 4$.

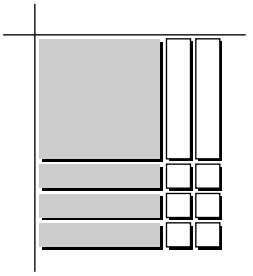
13. The models show rectangles of algebra tiles. Answer the following questions for each rectangle.

- What are the dimensions for each model?
- Write an expression for each model, using your dimensions.

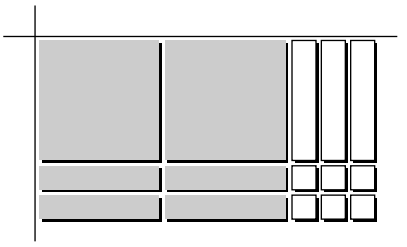
a)



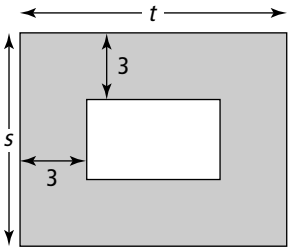
b)



c)



☆**14. a)** The white rectangle in the diagram is centred within the larger rectangle. State the dimensions of the white rectangle.



b) If $t = 10$ cm and $s = 8$ cm, express the area of the white rectangle as a product of binomials.

15. You have two dishes of lasagna. One dish measures 6″ by 15″. The second dish measures 9″ by 12″.

a) You want to cut the lasagna into equal-size servings with nothing left over. What is the largest size serving you can cut?

b) Suppose there are 20 students in your class. What is the largest size square you can cut to ensure that each student receives at least one serving and all servings are of equal size?

C Extend

16. List the pairs of numbers less than 100 that have a GCF of 15 and a product of 2700. How do you know that you have all of the possible pairs?

17. A school receives a shipment of notebooks. The unopened and equal packages of notebooks are put into three stacks. The three stacks have 365, 525, and 595 notebooks, respectively. What is the largest possible number of notebooks in each package?

18. The diameter of the bottom tier of a three-tiered round cake is 6 cm greater than the diameter of the middle tier and 12 cm greater than the diameter of the top tier.

a) Write an algebraic expression for the total top surface area of the three tiers.

b) Multiply and then simplify.

19. Write a polynomial that satisfies the following clues:
- It is a trinomial.
- Each term has the same variable.
- The exponents are odd integers.
- The GCF is $8x$.
- The greatest exponent is 5.
- The coefficients in one term of the polynomial's factored form are 1, 2, and 3.

20. The greatest common factor of two numbers is 487. Both numbers are even. Neither is divisible by the other. What are the smallest two numbers they could be?

D Create Connections

21. a) Draw a diagram to illustrate the largest circle that can be contained within a square with side length s.

b) Write an expression for the area of this circle. Use s as a variable in your expression.

c) Write the expression for the area of the square not contained by the circle. Develop an appropriate polynomial. Then, factor the polynomial.

22. The height of a basketball thrown vertically can be modelled by the expression $v_0t - 5t^2 + h_0$, where v_0 is the initial velocity of the basketball, t is the time, in seconds, that the ball is in the air, and h_0 is the initial height, in metres, of the ball.

a) If the initial height of the ball is 2 m, and the initial velocity of the ball is 15 m/s, how long will the ball be in the air before it comes back down to its initial height?

b) Factor the expression $15t - 5t^2$. At which times will the product be 0? Explain. Does this simplify the process for seeking the answer for part a)?

23. The height of a rectangular prism is $2x$. The width of the prism is 1 unit more than one and one half times the height.

a) Determine the length of the prism if the volume is $30x^3 + 10x^2$.

b) If the height is 6 cm, determine the width, length, and volume of the prism.