5.3 Factoring Trinomials

KEY IDEAS

- To factor a trinomial of the form $x^2 + bx + c$, first find two integers with
  – a product of c
  – a sum of b
  For $x^2 + 12x + 27$, find two integers with
  – a product of 27
  – a sum of 12
  The two integers are 3 and 9.
  Therefore, the factors are $x + 3$ and $x + 9$.

- To factor a trinomial of the form $ax^2 + bx + c$ (where b and c are integers), first factor out the GCF, if possible. Then, find two integers with
  – a product of $(a)(c)$
  – a sum of b
  Finally, write the middle term as a sum. Then, factor by grouping.
  For $8k^2 - 16k + 6$, the GCF is 2, so
  $8k^2 - 16k + 6 = 2(4k^2 - 8k + 3)$
  Identify two integers with
  – a product of $(4)(3) = 12$
  – a sum of -8
  The two integers are -2 and -6. Use these two integers to write the middle term as a sum. Then, factor by grouping.
  $2(4k^2 - 2k - 6k + 3) = 2(2k - 3)(2k - 1)$

- You cannot factor some trinomials, such as $x^2 + 3x + 5$ and $3x^2 + 5x + 4$, over the integers.

Example

Factor $6a^2 + 11a - 10$, if possible.

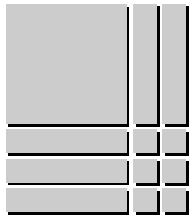
Solution

$\underline{6}a^2 + \underline{\underline{11}}a \underline{- 10}$	Ask: Are there two integers that when multiplied together equal -60 and when added together equal 11? Numbers that multiply to make 60. 1, 60 4, 15 2, 30 5, 12 3, 20 6, 10 Numbers that could add to make 11. 4, 15
$6a^2 + 15a - 4a - 10$	Then, break up the middle term with these integers.
$3a(2a + 5) - 2(2a + 5)$	Factor by grouping.
$(3a - 2)(2a + 5)$	

A Practise

1. State the trinomial represented by each rectangle of algebra tiles. Then, determine the dimensions of each rectangle.

a)



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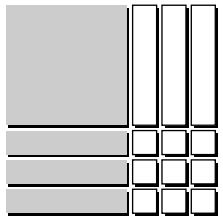

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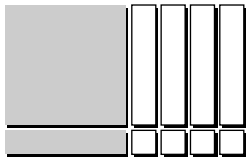


d)

2. Use algebra tiles or a diagram to factor each trinomial.

a) $2x^2 + 3x + 1$

b) $3x^2 + 5x - 2$

c) $6x^2 - 13x + 6$

d) $2x^2 + 5x - 12$

e) $4x^2 - 18x - 10$

f) $3x^2 + 17x - 28$

3. If possible, identify integers with the given product and sum.

	Product	Sum
a)	12	8
b)	15	−3
c)	−4	−3
d)	24	−11
e)	−42	19
f)	12	−10

4. Factor, if possible.

a) $y^2 + 8y + 12$

b) $x^2 + 10x + 21$

c) $a^2 - 19a + 90$

d) $y^2 - 4y - 6$

e) $m^2 - mn - 42n^2$

f) $b^2 + 19b + 34$

5. Factor, if possible.

a) $g^2 - 10g + 24$

b) $n^2 - 15n + 26$

c) $c^2 - 15c + 56$

d) $s^2 - 7st + 10t^2$

e) $f^2 - 6f + 12$

f) $3v^2 + v - 2$

6. Factor, if possible.

a) $2r^2 + 11r + 14$

b) $2l^2 + 11l + 12$

c) $3w^2 + 9w + 6$

d) $10b^2 + 8b + 2$

e) $y^2 + 5yz + 6z^2$

f) $12a^2 + 19a + 4$

7. Factor, if possible.

a) $2f^2 + 7f - 15$

b) $r^2 + r - 110$

c) $6b^2 + 6b - 3$

d) $10m^2 - 17mn + 3n^2$

e) $x^2 - x + 56$

f) $9g^2 - 9gf + 2f^2$

g) $6l^2 + 32l + 42$

h) $5a^2 - 52a + 63$

B Apply

8. Determine at least two values of d that allow each expression to be factored.

a) $a^2 + da + 6$

b) $w^2 + dw - 15$

c) $y^2 - dy + 18$

d) $r^2 - dr - 14$

9. Determine two values of h that allow each expression to be factored.

a) $6p^2 + hp - 1$

b) $d^2 + hd + 8$

c) $t^2 - ht + 56$

d) $s^2 - hs - 20$

10. Determine two values of p that allow each expression to be factored.

☆**a)** $c^2 - pc - 10$

b) $x^2 + pxy + 3y^2$

c) $a^2 + pab + 14b^2$

d) $v^2 - pvw + 35w^2$

11. Identify one value of r that will allow each expression to be factored.

a) $10b^2 + 14b - r$

b) $rs^2 + 19st + 3t^2$

c) $d^2 - 8de + re^2$

d) $5y^2 - 32y - r$

e) $2x^2 - 11x - r$

f) $3x^2 - 3x - r$

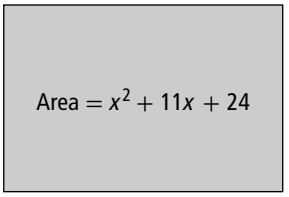
12. The penalty area on a soccer field can be represented by the trinomial $6x^2 - 2x - 48$.

a) Factor the trinomial to determine a binomial that represents the width and the length of the area.

b) The unit used for soccer fields is the yard. What are the dimensions of the area if $x = 12$ yd?

☆**13.** Determine the binomials that represent the width and length of each rectangle. Then, calculate the dimensions if $x = 12$ cm.

a)



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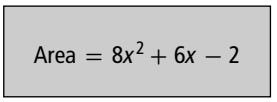

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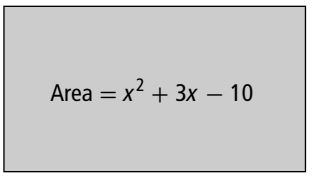


14. Carol throws a ball that will move through the air in a parabolic path due to gravity. The height, h, in feet, of the ball above the ground after t seconds can be modelled by the expression $h = -6t^2 + 27t + 15$.

a) Write the formula in factored form.

b) What is the height of the ball above the ground 4 s after it is thrown?

15. a) The area of a parallelogram is $A = x^2 + 13x + 42$. Determine the binomials that represent the height, h, of the parallelogram and the length, b, of its base. Then, calculate the dimensions of the parallelogram if $x = 18$ cm.

b) Suppose the area of the parallelogram in part a) is $A = 6x^2 + 7x - 3$. What are the binomials that represent the height and length of the parallelogram? Determine the dimensions if $x = 18$ cm.

C Extend

☆**16.** The area of a rectangle can be represented by the expression $35 - 8x - 3x^2$, where x represents a positive integer. What are the possible values for the width and the length of the rectangle?

17. Determine one value of c that allows the trinomial $cy^2 + 36y - 18$ to be factored over the integers.

18. a) What shape might have an area represented by the expression $16s^2 - 48s + 36$?

b) What in the expression indicates that shape?

c) What are the factors?

19. The area of a certain shape can be represented by the expression $x^2 + 6x + 9$.

a) Identify a possible shape.

b) Write expressions for the possible dimensions of the shape you identified in part a).

c) Suppose you have a second figure in the same shape as the shape you identified in part a) except that its area can be represented by the expression $4x^2 + 24x + 36$. Explain how you can use mental math to determine the dimensions of the second figure.

D Create Connections

20. A classmate is able to factor trinomials such as $n^2 + 7n - 44$ or $n^2 - 20n - 44$, but not trinomials such as $6n^2 + 13n - 5$ or $4n^2 - n - 3$. Explain the similarities and differences in factoring these two types of trinomials in sufficient detail that your classmate is then able to factor both types.

21. Write the completed statements after determining the answer in the blanks.

a) When factoring a trinomial of the form $x^2 + bx + c$, such as $x^2 + 5x + 6$, one can ask, "What two integers have a sum of ____ and a product of ____?"

b) The general form to show why part a) works, with each of m and n being any integer, is

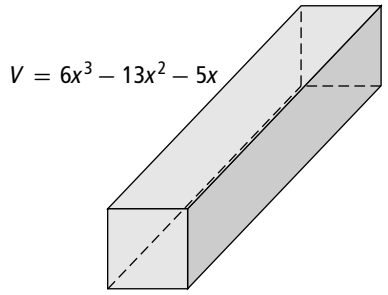
$$(x + m)(x + n) = x^2 + nx + \_\_ + mn$$
$$= x^2 + (n + \_\_)x + mn$$

c) Consider a trinomial of the form $ax^2 + bx + c$, such as $2x^2 + 13x + 15$. When factoring a trinomial of this form by grouping to break up the middle term, one can ask, "What two integers have a product of ____ and a sum of ____?"

d) The general form to show why part c) works, with each of a, m, and n being any integer, is

$$(ax + m)(x + n) = ax^2 + anx + \_\_ + mn$$
$$= ax^2 + (an + \_\_)x + mn$$

22. A rectangular prism has the volume as shown.



a) Factor the expression that represents the volume to determine the length of each of the sides of the prism.

b) If $x = 5$ cm, determine the lengths of the sides and the volume of the rectangular prism.