5.4 Factoring Special Trinomials

KEY IDEAS

- Some polynomials are the result of special products. When factoring, you can use the pattern that formed these products.

Difference of Squares:
The expression is a binomial.
The first term is a perfect square.
The last term is a perfect square.
The operation between the terms is subtraction.
The two binomial factors will be the square roots of the squares, connected by "+" and "–" signs.

$x^2 - 25 = x^2 - 5$
$= (x - 5)(x + 5)$

Perfect Square Trinomial:
The first term is a perfect square.
The last term is a perfect square.
The middle term is twice the product of the square root of the first term and the square root of the last term.
The trinomial is of the form $(ax)^2 + 2abx + b^2$ or $(ax)^2 - 2abx + b^2$.

$x^2 + 16x + 64 = x^2 + 8x + 8x + 64$
$= x(x + 8) + 8(x + 8)$
$= (x + 8)(x + 8)$

Example

Factor $y^2 + 28y + 196$, if possible.

Solution

$\underline{1y^2} + \underline{28}y + \underline{196}$

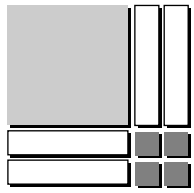
$\sqrt{1} = 1$
$\sqrt{196} = 14$
$14 + 14 = 28$
This trinomial is a "perfect square." The first and last terms are squares, and the middle term is double the product of their square roots.

$y^2 + 14y + 14y + 196$
$y(y + 14) + 14(y + 14)$
$(y + 14)(y + 14)$
$(y + 14)^2$

A Practise

1. State the factors of the polynomial shown by each algebra tile model.

a)



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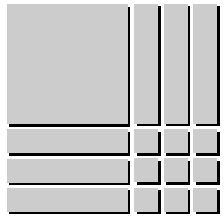

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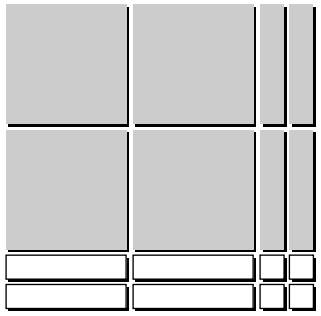

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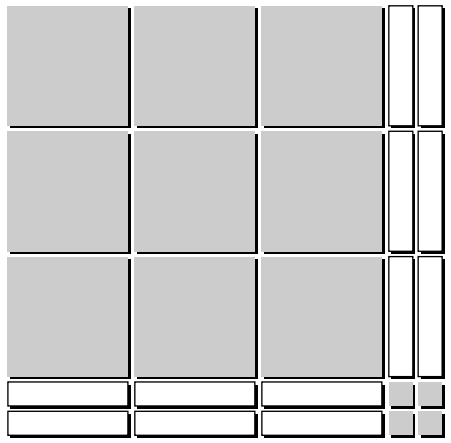


2. Solve.

a) $(x + 5)(x - 5)$

b) $(3r - 4)(3r + 4)$

c) $5(w - 6)(w + 6)$

d) $(2b - 7c)(2b + 7c)$

e) $4(2x - 3y)(2x + 3y)$

f) $2y(x + 3)(x - 3)$

3. Determine the product.

a) $(y + 5)^2$

b) $(3d + 2)^2$

c) $(4m - 5p)^2$

d) $2(e - 6f)^2$

e) $3(2z - 4)^2$

f) $(2x - 3y)^2$

4. Determine the missing terms that complete the factors or products.

a) $n^2 - \_\_\_ + 25 = (n - \_\_\_)^2$

b) $r^2 - \_\_\_ = (r + \_\_\_)(r - s)$

c) $-16d^2 = (3c - \_\_\_)(3c + \_\_\_)$

d) $4s^2 + \_\_\_ + 36 = (\_\_\_ + \_\_\_)^2$

e) $4x^2 + \_\_\_ + 4 = (2x + \_\_\_)^2$

f) $(4x - \_\_\_)^2 = 16x^2 - \_\_\_ + 4$

5. Factor each binomial, if possible.

a) $a^2 - 100$

b) $t^2 - 49$

c) $x^2 + 4$

d) $64 - h^2$

e) $50g^2 - 72h^2$

f) $9p^2 - 15r^2$

g) $s^2 + 144$

h) $72g^2 - 32h^2$

6. Factor each trinomial, if possible.

a) $y^2 + 12y + 36$

b) $x^2 - 6x + 9$

c) $2z^2 + 12z + 18$

d) $a^2 + 5a + 25$

e) $144 - 48b - 4b^2$

f) $9s^2 + 48s + 64$

g) $25n^2 - 110n + 121$

h) $5t^2 - 60t + 108$

7. Factor completely.

a) $16d^2 - 64e^2$

b) $27m^2 - 48$

c) $-2k^2 - 24k - 72$

d) $3c^3 + 51c^2 + 147c$

e) $100a^2 - 25b^2$

f) $s^3t - 18s^2t + 81st$

g) $81g^4 - 16$

h) $12lm^2 + 12lmn + 3ln^2$

B Apply

8. An error was made in factoring the following trinomials or binomials. Identify the error. Then, factor correctly.

a) $4a^2 - b^2 = (2a - b)(2a - b)$

b) $9x^2 + 6x + 1 = (3x + 1)(3x + 2)$

c) $216 - 9y^2 = (16 - 3y)(16 + 3y)$

d) $d^2 - 4e^2 = (d + 4e)(d - e)$

e) $49 - 14h + h^2 = (h - 7)^2$

9. Determine the value(s) of c so that each trinomial is a perfect square.

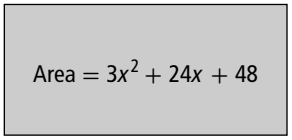
a) $w^2 + cw + 1$

b) $9b^2 + cb + 16$

c) $25 - cs + 36s^2$

d) $16g^2 + cgh + 36h^2$

☆**10.** The area of a rectangle can be represented by the trinomial $3x^2 + 24x + 48$.



a) Factor the trinomial completely.

b) If the length of the rectangle is triple the width, use the factors in part a) to represent the length and width.

c) If x represents 5 cm, what are the length and the width of the rectangle?

d) Calculate the area of the rectangle and check your answer.

11. Using the difference of squares model, $a^2 - b^2 = (a - b)(a + b)$, use mental math to make the following calculations. Record your reasoning.

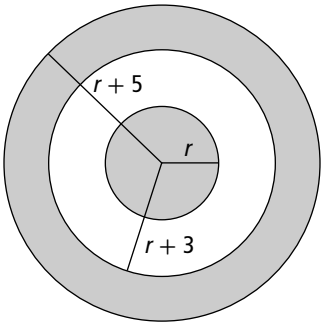
a) $16^2 - 4^2$

b) $7^2 - 27^2$

c) $45^2 - 15^2$

d) $113^2 - 13^2$

12. The diagram shows three concentric circles with radii r, $r + 3$, and $r + 5$.



a) Write an expression for the total area of the shaded regions.

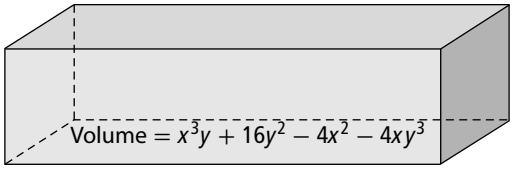
b) Factor this expression completely.

b) If $r = 2$ cm, calculate the total area of the shaded regions. Give your answer to the nearest tenth of a square centimeter.

C Extend

☆**13.** Factor $2r^5 - 4r^3 + 2r$ completely.

14. The volume of a rectangular prism is $x^3y + 16y^2 - 4x^2 - 4xy^3$.



Determine expressions for the dimensions of the prism.

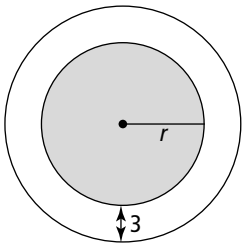
15. To determine the product of two numbers that differ by 6, square their average and then subtract 9.

a) Use this method to determine the following products.
(17)(23) = ____
(25)(31) = ____

b) Explain this method using a difference of squares.

16. The area of a square of side length a may be expressed as $A = 9b^2 - 12b + 4$. What is the area of a rectangle in terms of b if the length of the rectangle is $(a + 2)$ and the width of the rectangle is $(a - 2)$?

☆**17.** Many road intersections use roundabouts to handle traffic flow. Some roundabouts contain a circular area with plants surrounded by a cement walkway.



a) Write an expression that represents the area of the garden.

b) Write an expression that represents the area of the walkway and the garden. Then, expand that expression.

c) Using your answer from part a), write a simplified expression to determine the area of the walkway.

d) If $r = 8$ m, calculate the area of the walkway to the nearest tenth of a square metre.

D Create Connections

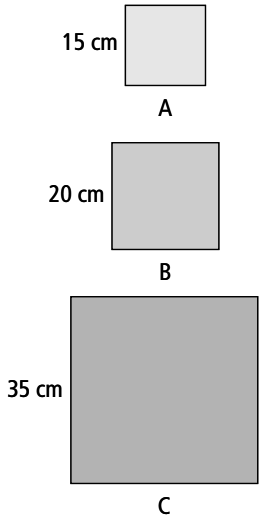
18. a) Continue the pattern to complete the table below.

$11^2 = 121$	$10 \times 12 = 120$
$12^2 =$ ____	$11 \times 13 =$ ____
$13^2 =$ ____	$12 \times 14 =$ ____
$14^2 =$ ____	$13 \times 15 =$ ____
_____	_____

b) How does the squared number compare with the product of the factors that are 1 less and 1 greater than the squared number?

c) Write and simplify algebraic expressions to show why this is the case.

19. Consider three squares having the dimensions shown.



a) How much greater is the area of square C than the combined areas of square A and square B?

b) Explain how the answer can be calculated mentally.