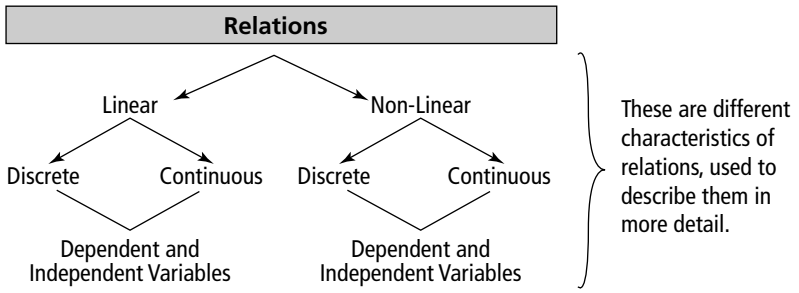
6.2 Linear Relations

KEY IDEAS

- Relations can be represented in a variety of ways. You can use words, equations, tables of values, ordered pairs, or graphs.



Example

Consider the following two relations:

a) Amir's height above the water after springing from a diving board

b) the cost for downloading music from a site that charges $1.25 per song

For each relation, decide
- if it is linear or non-linear
- if it is discrete (made up of values that are not connected on a graph) or continuous (made up of values that are connected by a line on a graph)
- what the dependent and independent variables are
- the best way to represent it (words, an equation, an ordered pair, a table of values, or a graph)

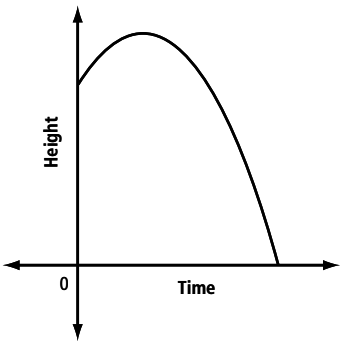
Solution

a) Amir's height above the water is non-linear over time. His height increases when he springs from the board, and then decreases as he plunges toward the water.

The data for this relation is continuous because for every instant of time that Amir is in the air there is a corresponding positive height above the water.

Amir's height above the water depends on how long he has been in the air. So, time is the independent variable and height is the dependent variable.

A graph is the best means of representing this situation. The dependent variable is always on the y-axis, so height is on the y-axis and time is on the x-axis.



b) The cost per song is constant, so this is a linear relationship.

This is a discrete relation because you cannot purchase part of a song. Each additional song purchased results in a unique purchase price.

The cost for the songs depends on the number of songs you download. So, the independent variable is number of songs, and the dependent variable is cost.

You could represent this relationship with a series of ordered pairs, a table of values, or a graph. Each would be clear. The equation $C = 1.25(n)$, where C is the cost and n is the number of songs downloaded, is probably most effective in this case. This equation makes it easy to determine the cost of any number of downloads.

A Practise

1. Convert each relation to a set of ordered pairs and a graph

a)

x	2	3	5	8	12
y	0	2	4	6	8

b)

a	10	8	6	4	2
b	2	4	6	8	10

☆2. Convert each relation to an equation and a table with 4 pairs of values. Is the relation discrete or continuous? Explain why.

a) the cost of buying concert tickets priced at $23.00 each

b) two positive numbers whose product is 24. Hint: Don't forget decimals. How would your answer change if you considered whole numbers only?

3. Given the following tables of values, determine which relations are linear and which are non-linear. Describe each relation in words.

a)

x	−3	−2	−1	0	1	2	3
y	6	2	0	0	2	6	12

b)

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7

c)

x	−3	−2	−1	0	1	2	3
y	6	5	4	3	4	1	0

d)

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

☆4. Determine if each formula is linear or non-linear by representing it in any of the other forms of a relation. Indicate which variable is independent, and which is dependent.

a) the area, A, of a circle with radius r: $A = \pi r^2$

b) the perimeter, P, of an equilateral triangle with side s: $P = 3s$

c) the number of diagonals, d, of a polygon with n sides: $d = \frac{n(n-3)}{2}$

5. Predict which of the following are linear relations. Use a graphing calculator or graphing software to check your prediction.

a) $y = \pi x$

b) $y = \frac{x}{9}$

c) $y = x\sqrt{x}$

d) $y = (x + 1)(x - 1)$

B Apply

6. A video store charges $4.50 to rent a new release movie. The store's owner wants to put up a poster to make it easy for customers to determine the cost of renting multiple movies.

a) Name the independent and dependent variables in this situation.

b) Describe the pricing policy in words.

c) Write an equation to represent the cost of renting 1 through 5 movies.

d) Show a set of ordered pairs for renting 1 through 5 movies.

e) Make a table of values that shows the cost of renting 1 through 5 movies.

f) Make a graph for renting 1 through 5 movies. Does it make sense to show the cost of renting zero movies?

g) From the 5 ways you represented the relation, which do you think would be the best way for the owner to present the information on the poster? Explain.

7. A pizza restaurant sells 4 sizes of pizza: 8 in., 10 in., 12 in., and 14 in. All the pizzas are round and the size is the pizza's diameter.

a) Make a table of values comparing diameter to the area of the pizza.

b) Which is the independent variable?

c) Graph the four ordered pairs.

d) Is this data linear or non-linear?

e) Is this data continuous or discrete?

8. The pizza restaurant owner from question 7 charges $10 for his 8 in. pizza. He wants to price the others so that the relation between the *area* of the pizza and the price is linear.

a) What are the radii of the four different pizzas?

b) Determine the areas of the four pizzas, rounded to the nearest square inch. Hint: Remember that $A = \pi r^2$.

c) What is the price per square inch of the 8 in. pizza?

d) What should the owner charge for each of the other pizzas, rounded to the nearest $0.10?

e) Do you think it is a good idea for the restaurant owner to price his pizzas this way?

f) Is the relation between the area of the pizza and the price linear or non-linear? discrete or continuous?

g) Is the relation between the diameter of the pizza and the price linear or non-linear?

☆**9.** Jogging is usually defined as running at approximately 10 km/h, or 6 min/km.

a) Is this a linear or non-linear relation?

b) Make one table of values with time in hours versus distance for running up to 5 hours. Make another table of values with distance versus time in minutes for running up to 8 km. Which table gives a better representation of jogging speed?

c) What would be the independent variable for graphing a jogging speed of 10 km/h?

d) Make a graph for jogging 10 km at a 6 min/km.

e) Is your graph continuous or discrete? Why?

10. The advertising manager for a small Alberta computer company believes that in any given month there is a linear relation between the amount of money spent on advertising, a, and the number of computers sold, n. In September, the company spent \$8000 on ads and sold 12 000 computers. Then, in October, it increased the advertising budget to \$20 000 and sold 18 000 units.

a) Set up a grid and plot the two points in the question. Hint: Determine the independent and dependent variables to decide what each axis represents. Mark both axes from 0 to 40 000.

b) Draw a line through your two points and use the graph to predict the number of computers the company would sell if the advertising budget were increased to \$40 000.

c) How much would the company need to spend on ads to sell 21 000 units?

d) How many computers would the company sell if it did no advertising?

C Extend

11. The length from your wrist to your elbow is approximately the same as the length of your foot. Measure 5 of your friends' arms and feet to determine if this is true.

a) Place this data in a table of values.

b) Graph the 5 pairs of data.

c) In this question, which measurement is independent? dependent?

d) Is the data linear or non-linear?

e) Is the data discrete or continuous?

12. When grocery shopping, the costs of many items can be represented by a linear relationship.

a) List 5 grocery items having costs that are linear in nature.

b) What is the dependent variable in all of these examples?

c) Are your examples discrete or continuous?

d) Name one item from a grocery store for which the cost would always be discrete. Name one for which the cost would always be continuous.

e) Name a grocery item that may have non-linear pricing structure.

☆**13.** Make a table of values for each of the following using $x = \{0, 1, 2, 3, 4, 5\}$.

- $y = 3x(x - 2)$
- $y = 5(x - 2)$
- $y = \frac{(x - 2)}{2}$

Predict whether each equation is a linear function by considering whether the y-value is increasing or decreasing by a constant amount. Then, graph each equation using technology and explain how the graphs support your predictions.

D Create Connections

14. When looking at a given relation, describe a way that you can predict whether the relation is linear or non-linear if the relation is

a) an equation

b) a table of values

c) a set of ordered pairs

d) a graph

e) given in words

15. When you are graphing a relation, is there a simple way to decide whether to connect the data points to make the graph continuous or to leave the points discrete? Discuss with a classmate, and then explain in your own words.