6.3 Domain and Range

KEY IDEAS

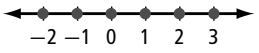
- The domain of a relation is the set of all numbers for which the independent variable is defined.

- The domain of a relation may also be described as:
  – the set of first coordinates in a set of ordered pairs
  – the possible values in the first column of a table of values
  – the possible values on the horizontal axis of a graph.

- The range of a relation is the set of all numbers for which the dependent variable is defined.

- The range of a relation may also be described as:
  – the set of second coordinates in a set of ordered pairs
  – the possible values in the second column of a table of values
  – the possible values on the vertical axis of a graph.

- The domain and range can be expressed in different ways.

Words	All integers equal to or greater than -2 and less than or equal to 3
Number Line	
Interval Notation	$[-2, 3]$
Set Notation	$\{n \mid -2 \leq n \leq 3, n \in \mathrm{I}\}$
A List	$\{-2, -1, 0, 1, 2, 3\}$

Example

The annual ecoENERGY for Vehicles Awards, administered by Natural Resources Canada's Office of Energy Efficiency, are presented for the most fuel-efficient vehicles for the current model year. One popular midsized car has a 77 L fuel tank. Its fuel efficiency is listed as 7.0 L/100 km in the city, and 5.5 L/100 km on the highway.

a) State the domain for the distance this car can travel for both city and highway driving. Express the domains in words and in set notation.

b) State the range of the fuel tank.

c) Draw one graph with two consumption lines on it: one for city driving and another for highway driving. Provide appropriate labels and scaling for the axes.

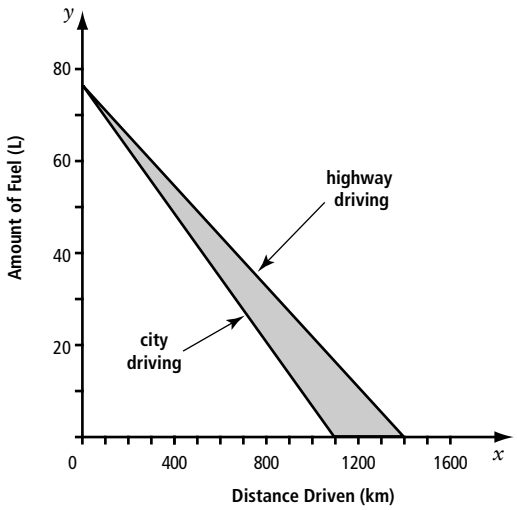
d) What does the portion of the graph between the two lines represent?

Solution

a) For city driving, the vehicle can travel 100 km for every 7.0 L of fuel. On a full tank of fuel it can travel $\frac{77}{7} \times 100$, or 1100 km. So, the domain of distance travelled for city driving is from 0 km, if the car is simply left idling, to 1100 km, or $\{d \mid 0 \leq d \leq 1100\}$. For highway driving, the vehicle can travel 100 km for every 5.5 L of fuel. On a full tank of fuel it can travel $\frac{77}{5.5} \times 100$, or 1400 km. So, the domain of distance travelled for highway driving is from 0 km to 1400 km, or $\{d \mid 0 \leq d \leq 1400\}$.

b) The range of the fuel tank is from 0 L to 77 L of fuel, or $\{L \mid 0 \leq L \leq 77\}$.

c) The independent variable, plotted on the x-axis, is distance driven. The dependent variable, plotted on the y-axis, is the amount of fuel used.



d) The shaded area between the two lines represents all of the possible distances and litres of fuel consumed by a mixture of city and highway driving, if the car is always kept within the predicted consumption rates.

A Practise

1. Draw a number line to represent each set of numbers. Hint: On a number line a solid dot means that the value is part of the set and an open circle means that the value is not part of the set.

a) $\{-2, 0, 2, 4, 6, 8, 10\}$

b) $\{x \mid x < 5\}$

c) your age from grade 1 until now

d) all the factors of 15

e) the square roots of all perfect squares from 1 to 100

f) $(-3, 4]$

2. For each relation, state the domain and range.

a) buying less than 5 cans of soup that cost \$0.38 each

b) listing all coin names from \$0.01 to \$2.00 and their value

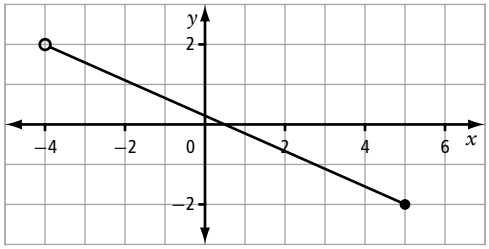
c) the squares of numbers 1 through 10

d) the cost for you and up to 5 of your friends to attend a concert at \$35.00 per ticket

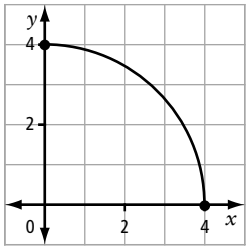
e) the granola bars you can buy with a \$10.00 bill, at \$1.50 per bar

3. Give the domain and range of each graph. Use both set notation and interval notation.

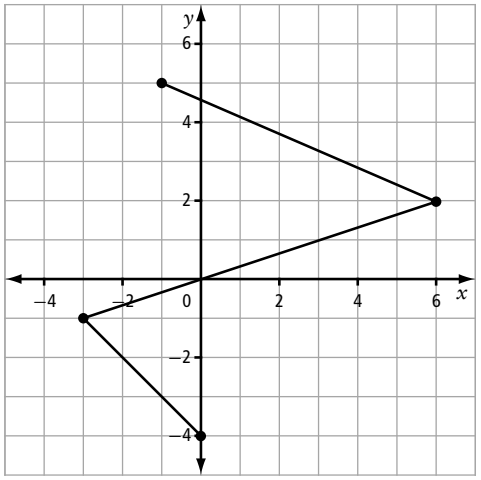
a)



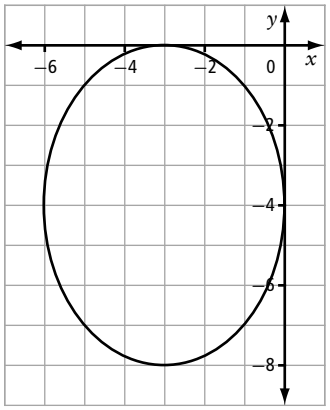
b)



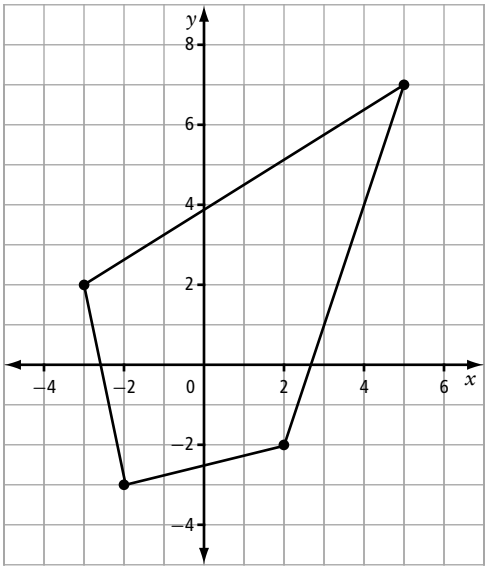
c)



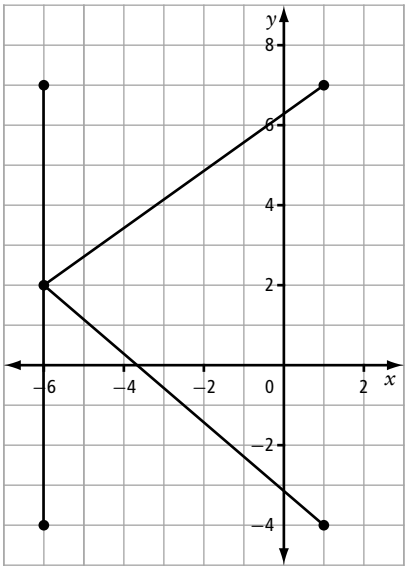
d)



e)



f)



☆**4.** The cost, C, of filling up a car with gasoline and buying an \$8.00 car wash can be expressed by the equation $C = 0.92n + 8.00$, where n is the number of litres of gasoline purchased. The car has a gas tank capacity of 40 L.

a) What is the domain of this equation?

b) What is the range of costs for this problem?

c) Which is the independent variable? Explain why.

B Apply

5. The table presents the internal temperatures that particular foods must reach in order to achieve the specified doneness and be considered safe to eat.

Food	Internal Temperature
Beef, veal, and lamb—medium-rare	63 °C (145 °F)
Beef, veal, and lamb—medium	71 °C (160 °F)
Beef, veal, and lamb—well done	77 °C (170 °F)
Pork (pieces and whole cuts)	71 °C (160 °F)
Poultry—pieces	74 °C (165 °F)
Poultry—whole	85 °C (185 °F)
Egg dishes	74 °C (165 °F)

a) State the range of temperatures for safe cooking of the foods in the table.

b) Is it better to use set notation or a list for the range, considering the context of the information?

c) What would be the danger of using either set notation or a list for the temperatures?

6. Draw a graph with each domain and range in the table. The graph can be made up of line segments or curves.
Hint: Remember that a round bracket means that the value is not part of the set, while a square bracket means that the value is part of the set.

	Domain	Range
A	(−3, 5)	(1, 5)
B	(−3, 5]	[1, 5)
C	[−3, 5]	[1, 5]
D	[−3, 5)	(1, 5]

7. The domain of a relation is given as (−8, 6), while its range is $\{y \mid -4 \leq y < 5\}$. Set up a grid with the x-axis and y-axis marked from −10 to 10. Draw a rectangle that the relation would lie within when graphed. When drawing the rectangle, use a solid line if the graph could be on it, and a dashed line if the graph only comes up to it, but does not include it.

8. Which among the items listed could have a domain or range of either [−12, ∞) or (−∞, 0]?
Hint: Remember that ∞ is used when there is no end point.
- a line
- a line segment
- a ray
- an oval

C Extend

9. Draw a graph of any relation that has

a) an identical domain and range

b) only one domain element

c) only one range element

10. Does a relation always need a domain? Explain.

D Create Connections

11. Create a relation where the range is $\{0 \text{ cm} \leq y \leq 20 \text{ cm}\}$.

12. In professional sports, there are many relationships that have an independent and dependent connection. For example, the players in a golf tournament represent a domain, and the scores they post for each round or for the tournament are the range. Provide two or more different examples, stating what the domain and range would be for each example.