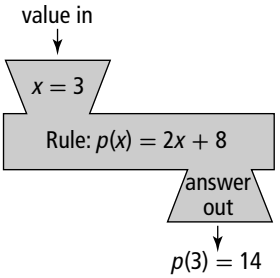
6.4 Functions

KEY IDEAS

- All functions are relations, but not all relations are functions.
- A relation is classified as a function if each value in the domain corresponds to exactly one value in the range.
- Each function has its own formula, or rule, which is often given using a special notation, called function notation. For example, $p(x) = 2x + 8$ shows that the function p takes an input value, multiplies it by 2, adds 8, and outputs the answer.



Example

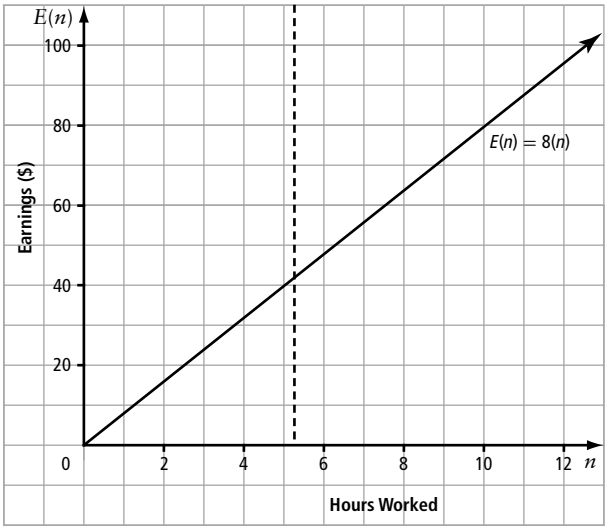
To distinguish between relations and relationships that are functions, and those that simply relate in an input/output manner, you must consider all the possibilities of the domain. If it is possible to have more than one output for one unique input, the relation is not a function.

Carefully consider the relationships listed in the table. State whether the relation is a function or not, and explain why. Sketch a graph of a function and a non-function from the questions. Show how the vertical line test supports your answer.

Relationship	Domain Input	Range Output
a) Working for $8/h	Hours worked in one week	Amount earned in that week
b) Putting footwear on before leaving the house	Choosing a pair of shoes	Leaving with something on your feet
c) Baking bread	Temperature of the oven	Height loaf rises
d) Studying for a math exam	Number of hours	Mark on exam
e) Squaring a number, and then adding 3	Any number you choose	Example: 28

Solution

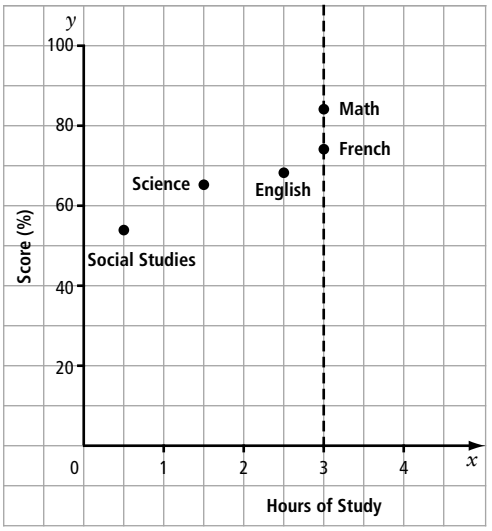
a) Function: it can be represented by $E(n) = 8(n)$, where E is earnings, and n is the number of hours worked. Each unique number of hours worked generates only one amount of money earned. This is supported by the vertical line test. Any vertical line, such as the one shown, will pass through one and only one point on the graph.



b) Function: you could wear runners, sandals, or any other footwear, and you would have something on your feet. Having footwear on produces only one output.

c) Not a function: the loaf may rise the same amount using different oven temperatures. Also, baking at the same temperature could produce a loaf of different heights.

d) Not a function: you could have studied for 3 hours for several exams and received different marks on some or all of them. The vertical line test shows that this is not a function because the vertical line at $x = 3$ passes through the mark received in both Math and French.



e) Function: for $S(n) = n^2 + 3$, both 5 and −5 produce 28. It is still a function if two different inputs produce the same output. It is not a function only if one input produces more than one output.

A Practise

☆**1.** For each relation, state whether it is a function. For those that are not functions, indicate where or explain why it is not a function. Where possible, use the vertical line test as part of your explanation.

a) (1, 3) (2, 4) (3, 5) (4, 3) (2, 1)

b) (5, 1) (4, 1) (3, 1) (2, 1) (1, 1)

c) (9, 3) (4, 2) (1, 1) (9, −3) (4, −2) (1, −1)

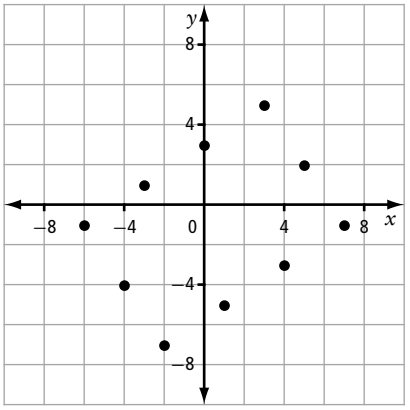
d)

Name	Shoe Size
Andrew	10
Nathan	11
Joel	12
Aaron	13
Simeon	12

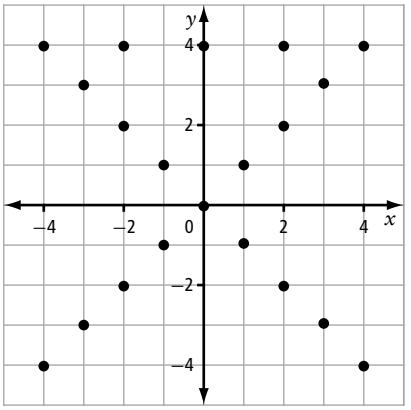
e)

Name	Sibling
Anika	Jared
Anika	Joel
Anika	Nathan
Carolyn	Aaron
Carolyn	Simeon

f)



g)



2. The formula for calculating the value of $500.00 deposited into an account earning 8% compounded annually for n years is $A = 500(1 + 0.08)^n$. Write this formula using function notation.

3. Anika is helping her parents make plans for her grandparents' 50th wedding anniversary. The cost for the banquet is given by the function $W(p) = 26p + 1200$, where W is the cost in dollars, and p is the number of people attending. Write this function as a formula in two variables.

4. If $z(a) = -3a + 7$, determine the following:

a) $z(-3)$

b) $z(2)$

c) a, if $z(a) = 7$

5. If $t(n) = 5 + (n - 1)(4)$, determine the following:

a) $t(1)$

b) $t(20)$

c) n if $t(n) = 41$

B Apply

☆**6.** For a single membership to FITFIT Health Club, you pay a $55 initiation fee upon enrollment and then $35 a month. The cost of belonging to the club is represented by the function $P(m) = 35m + 55$.

a) What is the independent variable in this relation and what does it represent?

b) What would it cost for you to belong to this health club for one year?

c) After how many months of membership would you have spent $1000?

d) The cost of belonging to one of FITFIT's competitors is represented by the function $P(w) = 10w + 100$, where w represents the number of weeks you are enrolled. Which club would be cheaper to belong to for one year?

7. For each function, calculate $f(5)$.

a) $f(x) = 3x - 6$

b) $f(x) = -2x + 11$

c) $f(x) = \frac{(x + 5)}{2}$

d) $f(x) = \frac{1}{4}(3 - x)$

8. Determine the value of x for the functions in question 7, when $f(x) = -15$.

9. Graph the following functions for the given domain. For what x-value does the graph go through $(x, 7)$?

a) $g(x) = 3x - 5$, for the domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

b) $h(x) = -2x + 7$, for the domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

c) $j(x) = 7(x - 4)$, for the domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

C Extend

10. Graphing calculators or software can find many function values quickly. Enter the function $f(x) = 3x - 11$ into either of these technologies. Determine the smallest value of x that produces

a) a prime number

b) a multiple of 8

c) a number larger than 100

d) the largest negative number

11. A science teacher asks students to write an equation to represent the temperature, T, measured in °C, of a liquid cooling on a laboratory table. The equation is to be in the form $T(m) = (A)m + (B)$, where m is the number of minutes since the liquid was placed on the table. Determine the values for A and B so that the equation produces the values in the chart.

Time (min)	Temperature (°C)
4	74
7	62
13	38
16	26

D Create Connections

12. Using the Internet, find the addresses in Canada associated with the following postal codes:

- V0B 2P0
- R2J 3E7
- T0H 2P0

a) Are any of these postal codes and the address(es) associated with them a function? Explain.

b) Use the Internet to search your own postal code. Is your postal code and address a function? Explain.