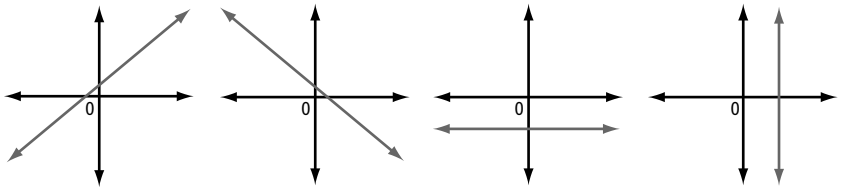
6.5 Slope

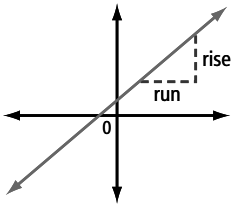
KEY IDEAS

- Positive Slope | Negative Slope | Slope is 0. | Slope is undefined.



- The slope of a line is the ratio of the rise to the run.

Positive Slope

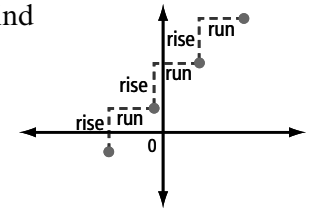


$m = \frac{\text{rise}}{\text{run}}$

- The slope of a line can be determined using two points on the line, (x_1, y_1) and (x_2, y_2).

  $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_2 \neq x_1$

- If you know one point on the line, you can use the slope to find other points on the line.



- The slope gives the average rate of change.

Time (s)	Distance (m)
1	4
2	7
3	10
4	13
5	16
6	19
7	22

Rate of change $= \frac{\Delta d}{\Delta t}$

Rate of change $= \frac{3}{1}$

The average rate of change is 3 m/s.

Time (s)	Distance (m)
1	4
3	10
5	16
7	22

Rate of change $= \frac{\Delta d}{\Delta t}$

Rate of change $= \frac{6}{2}$

Rate of change $= \frac{3}{1}$

The average rate of change is 3 m/s.

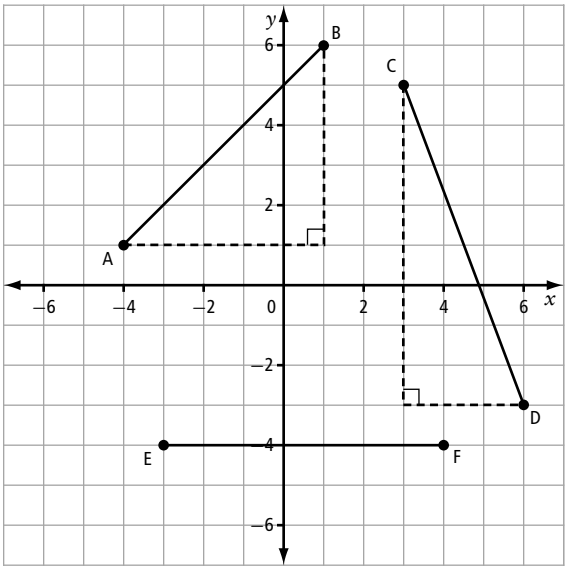
Example

Use two methods to determine the slope of each of the following line segments:

- A(−4, 1) and B(1, 6)
- C(3, 5) and D(6, −3)
- E(−3, −4) and F(4, −4)

Solution

Method 1: Graph each line segment. Then, for each segment, connect the two endpoints by drawing a right triangle.



Slope, m, is equal to $\frac{\text{rise}}{\text{run}}$. Determine the *rise* by counting the units along the height of the triangle, and the *run* by counting the units along the base of the triangle.

For AB:

$$m = \frac{5}{5}$$
$$= 1$$

For CD:

$$m = \frac{-8}{3}$$

For EF:

$$m = \frac{0}{7}$$
$$= 0$$

Method 2: Use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_2 \neq x_1$.

Substitute values to determine each slope.

For AB:

$$m = \frac{(6 - 1)}{(1 - (-4))}$$
$$= \frac{5}{5}$$
$$= 1$$

For CD:

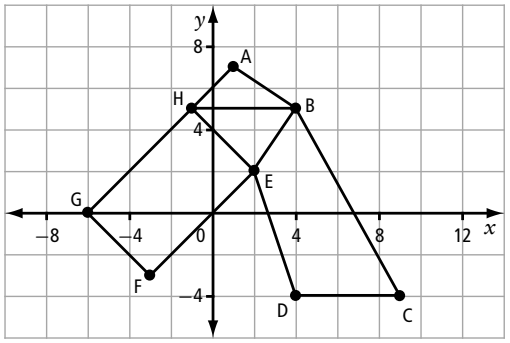
$$m = \frac{(-3 - 5)}{(6 - 3)}$$
$$= \frac{-8}{3}$$

For EF:

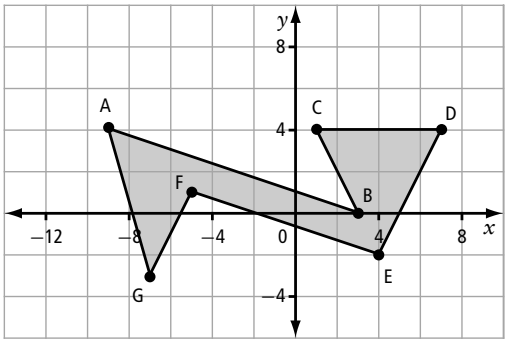
$$m = \frac{(-4 - (-4))}{(4 - (-3))}$$
$$= \frac{0}{7}$$
$$= 0$$

A Practise

1. Create a three-column table with the headings Positive Slope, Negative Slope, and Zero Slope. Place each line segment in the diagram in the appropriate column.



2. Determine the slope of each line segment in the diagram.



3. A 45° line has a slope of +1 or −1. Use the slope formula to determine the slope of a line passing through each pair of points. Is the line steeper or less steep than 45°?

a) (3, 5), (1, 4) **b)** (6, 5), (1, −3)

c) (2, −3), (−2, 6) **d)** (−4, 5), (−1, 4)

4. Graph a line from each given point back to the origin. Determine the slope of each line.

a) (4, −2) **b)** (−3, 0)

c) (7, 3) **d)** (−2, −5)

5. For each given point and slope, calculate what the next ordered pair *to the right* of the point will be. If possible, try to do this mentally, without graphing. The first one is done for you.

Given Point A(x, y)	Slope	Next Point to the Right of A
(3, 5)	$-\frac{1}{2}$	(5, 4)
(3, 5)	$\frac{2}{3}$	
(−3, 7)	$\frac{3}{7}$	
(2, −5)	$-\frac{4}{1}$	
(0, −4)	$\frac{5}{4}$	

B Apply

6. Building codes and safety concerns dictate slopes in structures. According to Canadian building codes, a wheelchair ramp cannot have a slope greater than $\frac{1}{12}$. When designing a mall, an architect has designed a central courtyard that is 84 cm higher than the corridor approaching it. How far away will a wheelchair ramp have to begin, in metres, if it is to have the steepest allowable slope?

7. Phoebe has asked her mother to help her save money for a vacation. At the end of each work week, Phoebe gives her mother the same amount of money to hold for her. At the beginning of last year, Phoebe had given her mother $255.00. At the end of the current year, the total had risen to $1035.00. Determine the average rate of change in her vacation savings. What does the number represent?

☆**8.** Snowcoach rides on the Columbia Icefields travel from a side moraine onto the glacier via a road that is constantly shifting due to ice movement and melting. The road is the steepest passenger route in the world, at a 32% grade.

a) If the elevation increase on this road is 130 m from start to finish, what would be the horizontal distance travelled (referred to as the shortest run)?

b) State the slope of the road in terms of $\frac{\text{rise}}{\text{run}}$.

c) Use the Internet to research how long the actual road is.

9. On the first day of school in September, Jodi measured the length of her hair. She found that her longest hair was 36 cm. At the end of the school year, in June, Jodi measured her hair again. Her longest hair at this time was 48.5 cm. What is the average rate of change of hair length per month?

10. Adult fingernails grow at a yearly rate of change. Lee Redmond set a Guinness world record when she grew her nails for about 29 years without cutting them. Added together, her nails were 8.65 m long. Assuming that she grew her nails for exactly 29 years, what was the average rate of change in her nails' length, per month, to the nearest centimetre?

C Extend

11. In 1981, the population of Saskatoon, Saskatchewan, was 154 210. In 2006, its population had grown to 202 340. Determine the average annual rate of growth and, assuming this rate of growth will continue, project Saskatoon's population in 2021. Research the most recent population of Saskatoon that you can, and see if it matches your projection.

☆**12.** A business wants to decide whether employees should use their own vehicle for company business or use a rental car. An employee uses a rental company that charges a daily rate plus a mileage charge, with no free kilometres.

	Trip A	Trip B
Days	3	3
Distance Driven (km)	425	680
Rental Charge	$301.25	$365.00

a) Determine the rate the car rental company charges per kilometre.

b) Determine the daily rate for a car rental.

c) Determine if it would be cheaper for the company to pay employees $0.50/km to use their own car.

d) Will your answer for part c) ever change if the trip is always 3 days long? Explain.

D Create Connections

13. Slope is defined as rise over run. Draw three different positive-sloped line segments on a piece of graph paper, with each one steeper than the previous. Make sure your lines go through ordered pairs that you can identify. For each segment, calculate the slope and then calculate the fraction $\frac{\text{rise}}{\text{run}}$. Explain why, as the line gets steeper, it becomes more apparent why the fraction is in the form $\frac{\text{rise}}{\text{run}}$.

14. Graph a series of lines that start at the origin and have the slope ratios found in the table. Notice that the numerator (rise) is consecutive even numbers and the denominator (run) is consecutive prime numbers. If you kept this pattern going, would you ever draw a line with a slope of 0? Explain.

Rise	2	4	6	8	10	12	14	16	18
Run	2	3	5	7	11	13	?	?	?