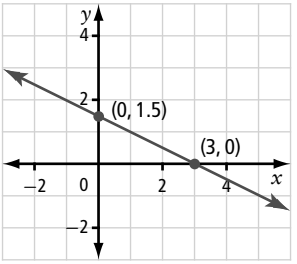
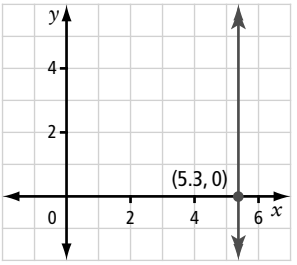
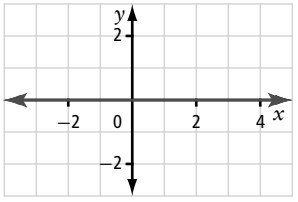
7.2 General Form

KEY IDEAS

- The general form of a linear equation is $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number.
- To graph an equation in general form, determine the intercepts, then draw a line joining the intercepts; or convert to slope-intercept form.
- To determine the x-intercept, substitute $y = 0$ and solve. To determine the y-intercept, substitute $x = 0$ and solve.
- A sketch of a linear relation may have one, two, or an infinite number of intercepts. A line that represents an axis has an infinite number of intercepts with that axis. A horizontal or vertical line that does not represent an axis has only one intercept.

Equation	x-Intercept(s)	y-Intercept(s)	Graph
$x + 2y - 3 = 0$	$x + 2y - 3 = 0$ $x + 2(0) - 3 = 0$ $x = 3$	$x + 2y - 3 = 0$ $(0) + 2y - 3 = 0$ $2y = 3$ $y = 1.5$	
$x = 5.3$	$x = 5.3$	no y-intercept	
$3y = 0$	infinite number of x-intercepts	$3y = 0$ $y = 0$	

Example

Consider the linear equation $y = -\frac{3}{2}x - 3$, which is in slope-intercept form.

a) Write the equation in general form, $Ax + By + C = 0$.
b) How many intercepts will this graph have? Explain how you know.
c) Sketch the graph using the x-intercept and y-intercept.
d) Explain how the equation $3x + 6 = 0$ differs from the linear equation in part a). Predict how this difference will be reflected in the graph of the line. Sketch the graph to check your prediction.
e) Explain how the equation $2y + 6 = 0$ differs from the linear equation in part a). Predict how this difference will be reflected in the graph of the line. Sketch the graph to check your prediction.

Solution

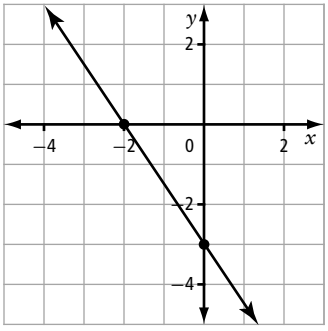
a)
$$y = -\frac{3}{2}x - 3$$
$$(2)y = (2)\left(-\frac{3}{2}x - 3\right) \qquad \text{Multiply by 2 to get rid of the fraction.}$$
$$2y = -3x - 6$$
$$2y + 3x + 6 = -3x + 3x - 6 + 6 \qquad \text{Move all terms to one side of the equal sign.}$$
$$3x + 2y + 6 = 0$$

b) This equation has both an Ax-term and a By-term, which means that it can be solved in terms of both x and y. Hence, the equation has two intercepts.

c) Determine the x-intercept and y-intercept and draw a line passing through the two.

To determine the x-intercept, replace y with 0, and solve for x.
$$3x + 2y + 6 = 0$$
$$3x + 2(0) + 6 = 0$$
$$3x + 0 + 6 = 0$$
$$3x + 6 = 0$$
$$3x + 6 - 6 = 0 - 6$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2$$
The x-intercept is $(-2, 0)$.

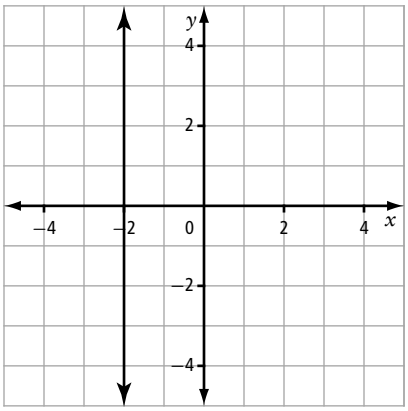
To determine the y-intercept, replace x with 0 and solve for y.
$$3x + 2y + 6 = 0$$
$$3(0) + 2y + 6 = 0$$
$$0 + 2y + 6 = 0$$
$$2y + 6 = 0$$
$$2y + 6 - 6 = 0 - 6$$
$$2y = -6$$
$$\frac{2y}{2} = \frac{-6}{2}$$
$$y = -3$$
The y-intercept is $(0, -3)$.



d) The equation $3x + 6 = 0$ is in general form, but has no By-term. Since the equation cannot be solved in terms of y, there is no y-intercept. This graph must be a vertical line passing through the x-intercept.

$$3x + 6 = 0$$
$$3x + 6 - 6 = 0 - 6$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2$$

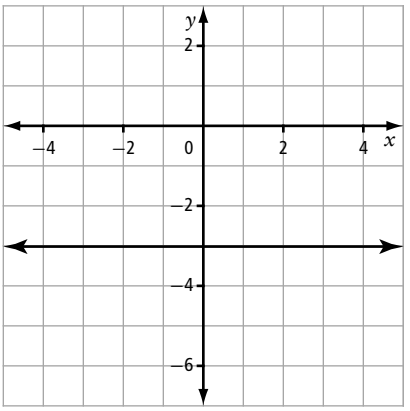
The x-intercept is $(-2, 0)$.



e) The equation $2y + 6 = 0$ is in general form, but has no Ax-term. Since the equation cannot be solved in terms of x, there is no x-intercept. This graph must be a horizontal line passing through the y-intercept.

$$2y + 6 = 0$$
$$3y + 6 - 6 = 0 - 6$$
$$2y = -6$$
$$\frac{2y}{2} = \frac{-6}{2}$$
$$y = -3$$

The y-intercept is $(0, -3)$.



A Practise

1. Write each equation in the general form, $Ax + By + C = 0$.

a) $y = \frac{1}{3}x + 5$ **b)** $y = \frac{-2}{7}x$

c) $y = \frac{1}{8}$ **d)** $y = -0.2x + 1.2$

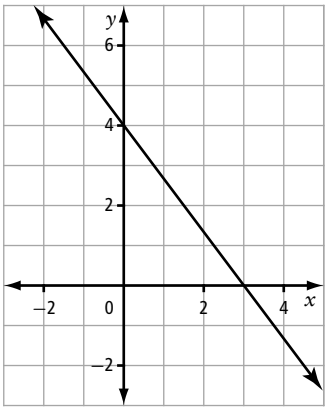
2. Determine the intercepts of each line. Graph each line.

a) $2x - y - 8 = 0$ **b)** $9x - 4y = 0$

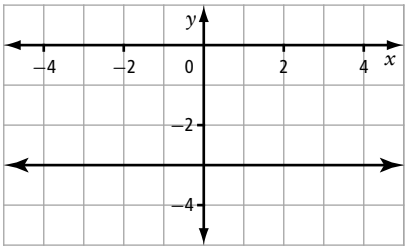
c) $5x - 20 = 0$ **d)** $8y + 4 = 0$

☆**3.** For each line, state the domain and range, slope, and any intercepts. Then, write the equation in general form.

a)



b)



B Apply

4. Write an equation in general form for each.

a) a line that does not have an x-intercept

b) a line that has an infinite number of intercepts

c) a line that does not have a y-intercept

d) a line for which both the x-intercept and y-intercept are 0

e) a line for which the x-intercept and y-intercept are the same, but are not 0

5. Determine the missing value, A, B, or C, in the following linear equations.

a) $6x - By + 1 = 0$, for the line that passes through the point $(-1, 5)$

b) $Ax + y - 10 = 0$, for the line that passes through the point $(3, -2)$

c) $9x - 5y + C = 0$, for the line that passes through the point $(0, 0)$

☆6. Josef is training for a race. His training consists of swimming and mountain biking. The table shows the number of calories burned per minute for a person of Josef's body mass.

Activity	Calories Per Minute
Swimming	14
Biking	12

a) Write a linear equation to show the number of minutes Josef would need to swim, x, and the number of minutes he would need to bike, y, to burn 4200 calories.

b) What are the intercepts of the line? What do they represent?

c) What are the graph's domain and range?

d) Suppose Josef bikes for 2 hours. How long would he need to swim to burn 4200 calories?

7. Jaden plants trees for the British Columbia government during the summer. The table shows how many trees she can plant per minute under different conditions.

Conditions	Number of Trees Planted Per Minute
Ideal	5
Rocky muskeg	2

a) If Jaden planted 2250 trees in one day, write an equation, in general form, showing the number of minutes she planted trees under ideal conditions, x, and the number of minutes she planted trees under rocky muskeg conditions, y.

b) For the linear equation, what are the slope, intercepts, and domain and range?

c) If Jaden planted trees under rocky muskeg conditions for 125 minutes, how long did she spend planting trees under ideal conditions?

C Extend

8. Graph the following on the same coordinate plane.
Line 1: $2x + 10 = 0$
Line 2: $4x - 5y + 30 = 0$
Line 3: $3x - 21 = 0$
Line 4: $-8x + 10y + 30 = 0$
Calculate the area of the region formed by their intersection.

D Making Connections

9. Write the coordinates of two points on a line that satisfies each given condition. Then, write the equation of each line in general form.

a) a line rises from left to right

b) a line is horizontal

c) a line falls from left to right

d) a line is vertical