7.3 Slope-Point Form

KEY IDEAS

- For a non-vertical line through the point (x_1, y_1) with slope m, the equation of the line can be written in slope-point form as $y - y_1 = m(x - x_1)$.

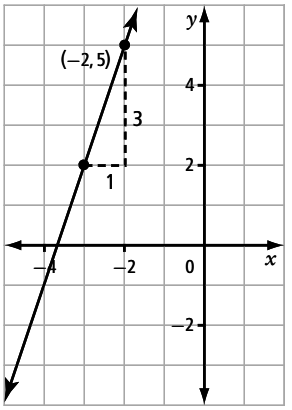
  A line through (−2, 5) has a slope of 3.

  The slope-point form of the equation of this line is

  $y - y_1 = m(x - x_1)$

  $y - 5 = 3[x - (-2)]$

  $y - 5 = 3(x + 2)$



- An equation written in slope-point form can be converted to either slope-intercept form or general form.
- Any point on a line can be used when determining the equation of the line in slope-point form.

Example

Consider a line passing through the points (−4, 5) and (6, 0).

a) Write the equation of this line in slope-point form.
b) Rewrite the equation in part a) in slope-intercept form.
c) Rewrite the equation in part a) in general form.
d) Sketch the graph.

Solution

a) Determine the slope.

$$m = \frac{y - y_1}{x - x_1}$$

$$m = \frac{0 - 5}{6 - (-4)}$$

$$m = \frac{-5}{10}$$

$$m = \frac{-1}{2}$$

Use either point (−4, 5) or (6, 0) to replace the point (x_1, y_1). Replace m with $\frac{-1}{2}$.

Using point (−4, 5):

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-1}{2}(x - (-4))$

$y - 5 = \frac{-1}{2}(x + 4)$

Using point (6, 0):

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-1}{2}(x - 6)$

$y = \frac{-1}{2}(x - 6)$

The slope-point form of the equation of the line passing through the points (−4, 5) and (6, 0) is $y - 5 = \frac{-1}{2}(x + 4)$ or $y = \frac{-1}{2}(x - 6)$.

b) The slope-point form can be changed to the slope-intercept form by solving for y.

For point $(-4, 5)$:

$$y - 5 = \frac{-1}{2}(x + 4)$$
$$y - 5 = \frac{-1}{2}x - 2$$
$$y - 5 + 5 = \frac{-1}{2}x - 2 + 5$$
$$y = \frac{-1}{2}x + 3$$

For point $(6, 0)$:

$$y = \frac{-1}{2}(x - 6)$$
$$y = \frac{-1}{2}x + 3$$

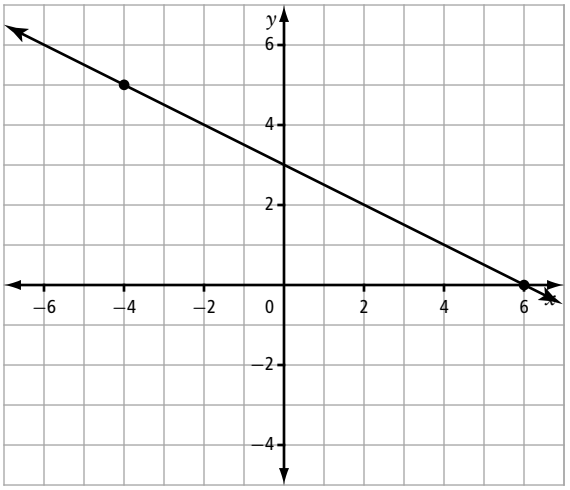
The slope-intercept form of the equation passing through the points $(-4, 5)$ and $(6, 0)$ is $y = \frac{-1}{2}x + 3$. The result is the same, regardless of which of the two points is used.

c) The slope-point form can be changed to general form.

$$y - 5 = \frac{-1}{2}(x + 4)$$
$$y - 5 = \frac{-1}{2}x - 2$$
$$2(y - 5) = 2\left(\frac{-1}{2}x - 2\right)$$
$$2y - 10 = -x - 4$$
$$2y - 10 + 4 = -x - 4 + 4$$
$$2y - 6 = -x$$
$$2y - 6 + x = -x + x$$
$$x + 2y - 6 = 0$$

The general form for the equation of the line passing through the points $(-4, 5)$ and $(6, 0)$ is $x + 2y - 6 = 0$.

d) Plot the points $(-4, 5)$ and $(6, 0)$ and draw a line passing through them. Or, plot the y-intercept, $(0, 3)$, and draw a line passing through it with a slope of $\frac{-1}{2}$.



A Practise

1. Identify the slope and a point on each line.

a) $y + 7 = 4(x - 3)$ **b)** $y - 5 = \frac{1}{3}(x + 5)$

c) $y = -2(x - 6)$ **d)** $y + 1 = x - 3$

2. Rewrite the following in slope-intercept form, $y = mx + b$, and general form, $Ax + By + C = 0$.

a) $y - 3 = \frac{2}{3}(x + 1)$ **b)** $y + 4 = -2(x - 1)$

c) $y = \frac{3}{4}(x - 4)$ **d)** $y - 1 = 3(x + 6)$

3. From the information given, write the equation of the line in slope-point form, slope-intercept form, and general form.

☆**a)** $(-1, -5)$; $m = \frac{4}{3}$

b) $\left(\frac{-1}{2}, -3\right)$; $m = 1$

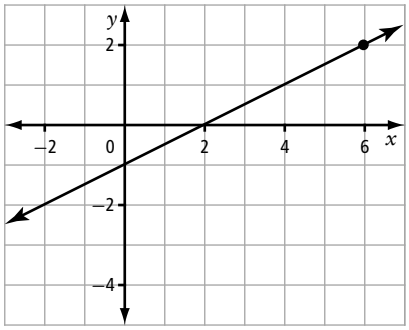
c) $(1, 4)$; $m = -1.5$

☆**d)** $(-5, -8)$ and $(-7, -9)$

e) $(-1, -2)$ and $(3, 0)$

4. Write an equation in slope-point form for each graph.

a)



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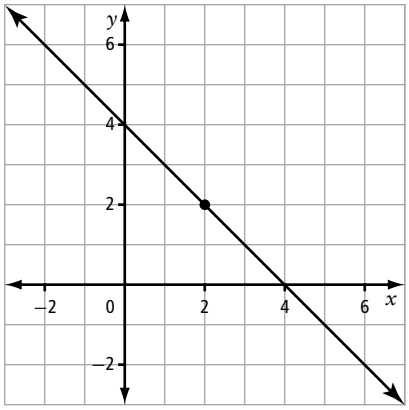

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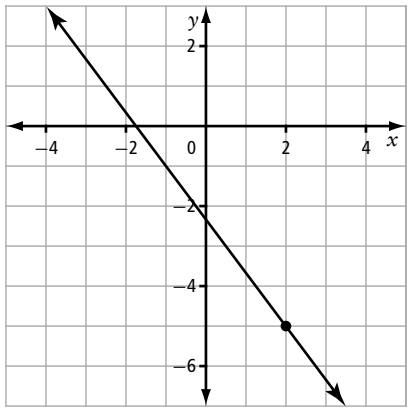


B Apply

5. Write the equation of each line in slope-point form. Then, convert each equation to general form.

a) slope of 0 and passing through $(-3, 1)$

b) same slope as $y = 2x + 2$ and passing through $(-1, 8)$

☆**c)** same slope as $5x + 2y - 10 = 0$ and passing through $(-1, 4)$

d) same y-intercept as $3x - y - 1 = 0$ and passing through $(2, -6)$

e) x-intercept of -5 and y-intercept of 3

f) same slope as $3x + 2y + 6 = 0$, with an x-intercept of 0

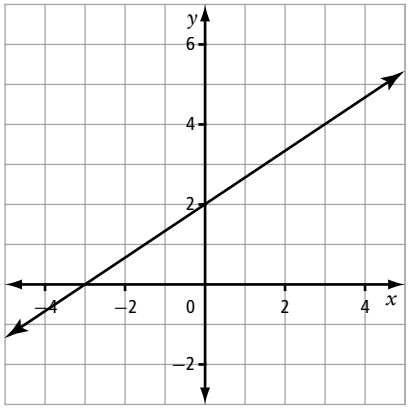
☆**6.** Show that the point $(-2, -6)$ lies on the line that has an x-intercept of 10 and a y-intercept of -5.

7. A rectangle has vertices A$(-3, 4)$, B$(-3, -1)$, C$(4, -1)$, and D$(4, 4)$. Plot the points on a grid and draw the rectangle. Then, draw the two diagonals and write an equation in general form for each.

8. Use graphing technology to identify the x-intercept and y-intercept of the line $2x - 3y + 12 = 0$. Use algebra to verify your answer.

9. Consider the linear equation $8x + ky - 6 = 0$. If the line passes through the point $(1, -2)$, what is the value of k?

10. Compare the following five lines to the line graphed below.
Line 1: $x + y + 3 = 0$
Line 2: $2x + 3y + 6 = 0$
Line 3: $2x - 3y + 18 = 0$
Line 4: $4x - 6y - 9 = 0$
Line 5: $2x - y + 2 = 0$



a) Which line(s) have the same slope as the graphed line?

b) Which line(s) have the same y-intercept as the graphed line?

c) Which line(s) have the same x-intercept as the graphed line?

11. The annual cost of operating a snowmobile depends on the distance driven plus a fixed cost, which includes maintenance, depreciation, and trail fees. The cost is $4000 for 1200 miles driven and $5625 for 2500 miles driven.

a) Sketch a line showing the relationship between distance and cost.

b) Calculate the slope of the line. What does the slope represent?

c) Determine the y-intercept. What does it represent?

d) Write an equation in general form for the cost of operating a snowmobile.

e) Use your equation to determine the cost of operating the snowmobile for 900 miles.

C Extend

12. A candle is lit at 1400 hours. At 1600 hours, it is 16 cm tall. At 2030 hours, it is 4.75 cm tall.

a) Write a linear equation, in general form, with the points representing (hours, height).

b) Use the equation to determine the rate at which the candle burns per hour and its height at 1400 hours.

c) What does the slope of the line represent?

d) What does the ***y***-intercept represent?

13. The following lines pass through the sides of a triangle:

$2x + 3y - 18 = 0$
$5x + y + 7 = 0$
$3x - 2y - 14 = 0$

Determine the vertices of the triangle.

14. Consider the linear equation $\frac{x}{-8} + \frac{y}{6} = 1$.

a) Write the equation in general form.

b) What are the x- and y-intercepts? How do they relate to the original form of the equation?

c) Predict the x- and y-intercepts of the equation $\frac{x}{3} - \frac{y}{5} = 1$. Verify your answer.

D Create Connections

15. In 2001, the pollution in a local lake was measured at 4.5 parts per million. In 2010, the level had decreased to 1.4 parts per million.

a) Write an equation in slope-point form, showing the relationship between time, x, and the pollution rate, y.

b) If the decrease in pollution continues at the same rate, in what year should the pollution level be 0 parts per million?