7.4 Parallel and Perpendicular Lines

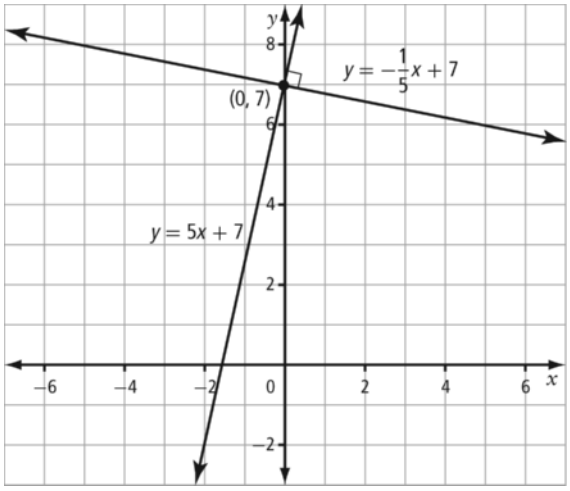
KEY IDEAS

- Parallel lines have the same slope and different intercepts. Vertical lines are parallel to each other, as are horizontal lines, if they have different intercepts.
- Perpendicular lines have slopes that are negative reciprocals of each other. A vertical line with an undefined slope and a horizontal line with a slope of zero are also perpendicular.
- The properties of parallel and perpendicular lines can give information about the slopes. Knowing the slopes can help you develop an equation.

A line perpendicular to $y = 5x + 7$ has the same y-intercept.

The line $y = 5x + 7$ has a slope of 5 and a y-intercept of 7.

The perpendicular line has a slope of $-\frac{1}{5}$ and a y-intercept of 7. So, the equation of the perpendicular line is $y = -\frac{1}{5}x + 7$.



Example

a) Write an equation, in slope-intercept form, for the line that has a y-intercept of -4 and is parallel to the line $5x + 3y - 6 = 0$. Graph the lines using graphing technology.

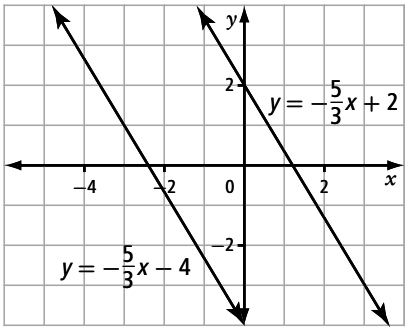
b) Write the equation, in slope-intercept form, for the line that has a y-intercept of -4 and is perpendicular to $5x + 3y - 6 = 0$. Graph the lines using graphing technology.

c) Write the equations for the lines you created in parts a) and b) in general form.

Solution

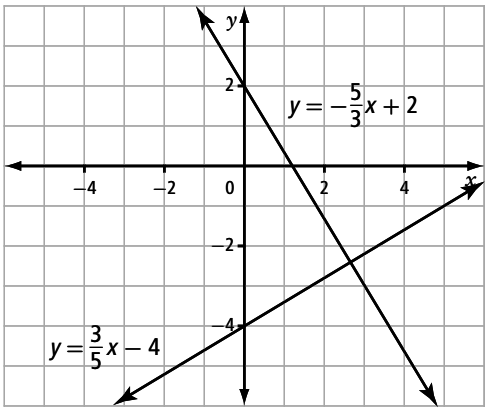
a) Since the new line is parallel to the line $5x + 3y - 6 = 0$, the two lines have the same slope.
Determine the slope of $5x + 3y - 6 = 0$ by rewriting it in slope-intercept form, $y = mx + b$.

$$\begin{aligned} 5x + 3y - 6 &= 0 \\ 5x + 3y - 6 + 6 &= 0 + 6 \\ 5x + 3y &= 6 \\ 5x - 5x + 3y &= -5x + 6 \\ 3y &= -5x + 6 \\ \frac{3y}{3} &= \frac{-5x}{3} + \frac{6}{3} \\ y &= \frac{-5x}{3} + 2 \end{aligned}$$



The slope of the first line is $\frac{-5}{3}$.
Since the second line has the same slope and a y-intercept of -4, the equation for the second line is $y = \frac{-5}{3}x - 4$.

b) A perpendicular line has a slope that is the negative reciprocal of the slope of the first line. Since the slope of the first line is $\frac{-5}{3}$, the slope of the perpendicular line is $\frac{3}{5}$. So, the equation of the perpendicular line running through $(0, -4)$ is $y = \frac{3}{5}x - 4$.



c) Rewrite the equations from part a) and b) in general form.

Parallel line: $y = \frac{-5}{3}x - 4$
Multiply each term by 3.

$$3y = 3\left(\frac{-5}{3}x - 4\right)$$
$$3y = -5x - 12$$

Bring all the terms to one side of the equal sign.

$$3y + 5x + 12 = -5x + 5x - 12 + 12$$
$$5x + 3y + 12 = 0$$

The equation in general form is $5x + 3y + 12 = 0$.

Perpendicular line: $y = \frac{3}{5}x - 4$.
Multiply each term by 5.

$$5y = 5\left(\frac{3}{5}x - 4\right)$$
$$5y = 3x - 20$$

Bring all the terms to one side of the equal sign.

$$5y - 3x + 20 = 3x - 3x - 20 + 20$$
$$-3x + 5y + 20 = 0$$

The equation in general form is $3x - 5y - 20 = 0$.

A Practise

1. Given the slopes of two different lines, determine whether the lines are parallel, perpendicular, or neither.

a) $m_1 = \frac{1}{2}; m_2 = -2$

b) $m_1 = \frac{3}{4}; m_2 = \frac{6}{8}$

c) $m_1 = \frac{-1}{4}; m_2 = 4$

d) $m_1 = -0.5; m_2 = 2$

e) $m_1 = 1; m_2 = -1$

f) $m_1 = \frac{1}{4}; m_2 = 0.25$

2. For each given line, state the slope of a line that is parallel and the slope of a line that is perpendicular.

a) $y = -3x - 4$

b) $y = x$

c) $4x + y - 4 = 0$

d) $8y - 7 = 0$

e) $5x - 2y + 3 = 0$

3. The following are slopes of parallel lines. Determine the value of n.

a) $2, \frac{n}{2}$ **b)** $-3, \frac{6}{n}$

c) $\frac{2}{n}, \frac{4}{5}$ **d)** $\frac{-n}{3}, \frac{-2}{4}$

4. The following are slopes of perpendicular lines. Determine the value of r.

a) $3, \frac{r}{6}$ **b)** $\frac{r}{9}, \frac{-3}{5}$

c) $\frac{9}{2}, \frac{4}{r}$ **d)** $\frac{-1}{2}, \frac{r}{4}$

5. Write the general form of the equation of a line that is parallel to the given line and passes through the given point.

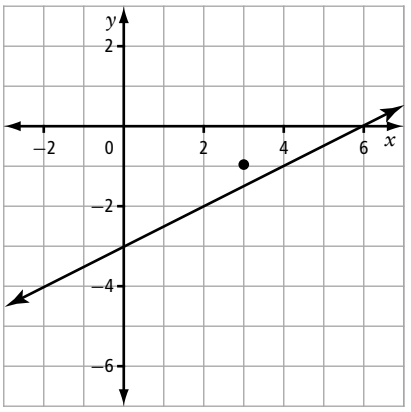
a) $5x + y + 8 = 0; (2, -3)$

b) $y = \frac{-1}{3}x + 2; (-4, 0)$

c) $y + 2 = -(x + 1); (-2, 6)$

6. Write the general form for the equation of a line that is perpendicular to the given line and passes through the given point.

a)



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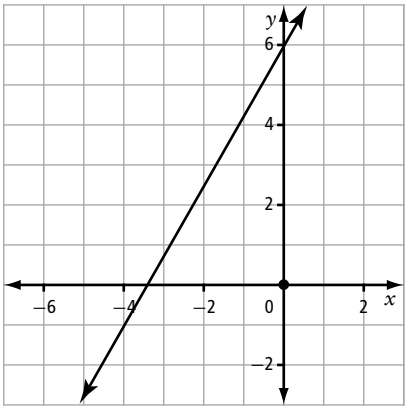

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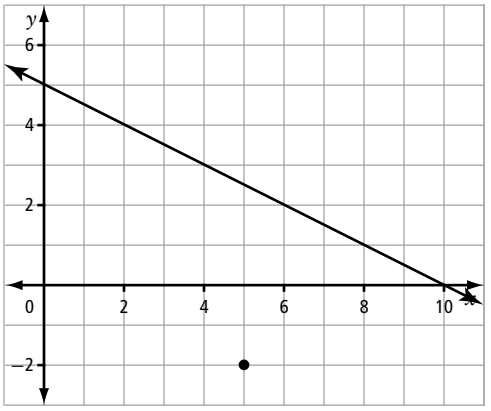


B Apply

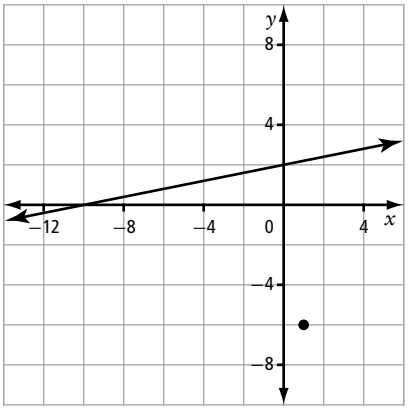
7. Consider lines $x - 2y + 10 = 0$ and $y = \frac{1}{2}x + 5$.

a) What are the slopes of the lines?

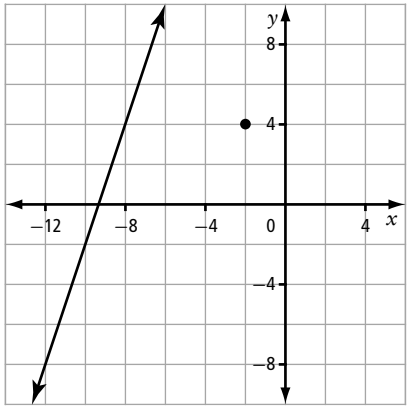
b) Are the lines parallel? Explain your answer.

8. Write the general form for the equation of a line passing through the given point and running parallel to the line.

a)



b)



9. Write the equation of a line that is perpendicular to the y-axis and has a y-intercept of 15.

10. Given three points, determine whether or not the points are vertices of a right triangle. Explain.

a) M(1, 1), N(−2, 5), C(3, −2)

b) D(2, 4), F(−2, 2), G(5, −2)

☆**11.** Consider the linear equation $4x + y - 11 = 0$.

a) Write an equation of a line parallel to the given line. How many such equations can be written?

b) Write an equation of a line perpendicular to the given line. How many such equations can be written?

12. Write the equation of the line perpendicular to $x - 12y + 15 = 0$ and having the same y-intercept as $7x + 4y - 12 = 0$.

13. Sketch the graph of a line parallel to the line $3x + 6y - 7 = 0$ and passing through the origin.

C Extend

14. The line passing through A(−2, 3) and B(0, 4) is perpendicular to the line passing through C(k, 4) and D(1, −6). What is the value of k.

15. What is the value of k if the lines $x - 2y + 6 = 0$ and $kx + 8y + 1 = 0$ are parallel?

16. For what value of k are the lines $3kx - 7y - 10 = 0$ and $2x + y - 7 = 0$ perpendicular?

☆**17.** Determine the value(s) of k for which the lines $kx - 2y - 1 = 0$ and $8x - ky + 3 = 0$ are

a) parallel

b) perpendicular

18. Write the equation of a line that passes through the point of intersection of the lines $5x + y - 11 = 0$ and $2x + 3y - 7 = 0$ and is

a) parallel to the x-axis

b) perpendicular to the line $2x - y + 4 = 0$

D Making Connections

19. Write the equations of lines that form the sides of a square or a rectangle so that no sides are vertical or horizontal lines.