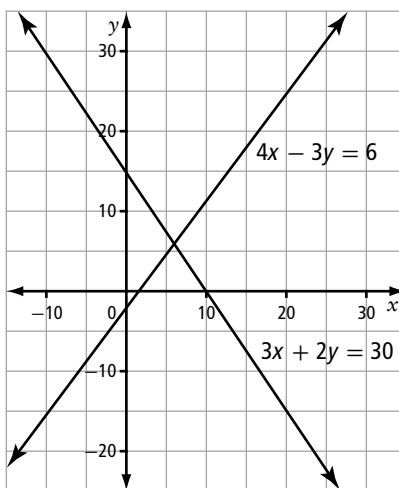# Chapter 8 Solving Systems of Linear Equations Graphically

8.1 Systems of Linear Equations and Graphs

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⟪ KEY IDEAS ⟫

- Systems of linear equations can be modelled numerically, graphically, or algebraically.

x	y
3	7
5	11
7	15
9	19
11	23

$4x - 3y = 6$

$3x + 2y = 30$

$6a + 2b = 0$
$2a + 5b = -13$

- The solution to a linear system is a pair of values that occurs in each table of values, an intersection point of the lines, or an ordered pair that satisfies each equation.

- One way to solve a system of linear equations is to graph the lines and identify the point of intersection on the graph.

- A solution to a system of linear equations can be verified using several methods:
  – Substitute the value for each variable and evaluate the equations.
  – Create a graph and identify the point of intersection.
  – Create tables of values and identify the pair of values that occurs in each table.

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Example
Consider the system of linear equations $y = 3x + 2$ and $2x - y = -4$.
 a) Rewrite the equations in slope-intercept form.
 b) Use technology to graph the equations together and identify the point of intersection.

Solution
 a) Rearrange each equation into slope-intercept form by isolating y. Identify the y-intercept and slope to draw the graph.
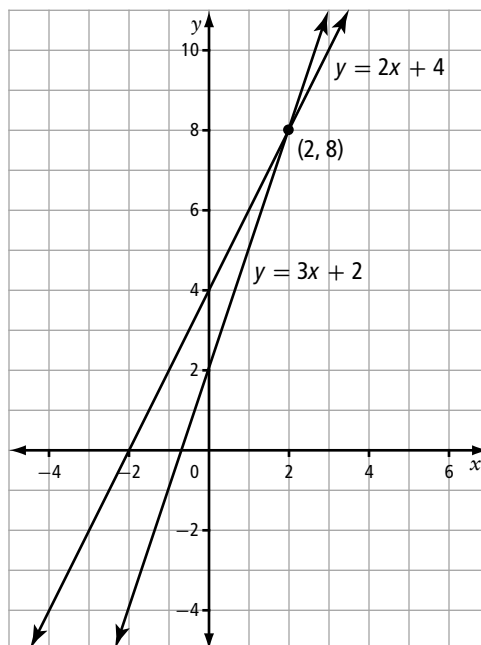
$y = 3x + 2$ is in slope-intercept form already.

$$2x - y = -4$$
$$2x - y + y = -4 + y$$
$$2x = -4 + y$$
$$2x + 4 = -4 + y + 4$$
$$2x + 4 = y$$
$$y = 2x + 4$$

The y-intercept is 2. The slope is 3.

The y-intercept is 4. The slope is 2.

- Using the Y= screen of your calculator, enter $y = 3x + 2$ as Y1 and $y = 2x + 4$ as Y2. Set the viewing window as X [3, 5, 1] and Y [−2, 12, 2]. The graph of the two lines will appear in the form shown. Use the 2nd/Calc/Intersect functions to identify the ordered pair at the point of intersection of the two lines. The point of intersection (2, 8) is the solution to the linear system.

A Practise

1. Rewrite the equations in slope-intercept form. Then, use technology to graph each pair of equations and determine the point of intersection.

a) $x + y = 6$
$2x - 3y = 2$

b) $x - 4y = -2$
$y = -x - 5$

c) $3x + 4y = 0$
$2x - 2y + 14 = 0$

2. Determine graphically whether each given point is a solution to the system of linear equations.

a) $y = 4x - 9$
$y = -2x + 3$
$(2, -1)$

b) $x + y = 7$
$3x - 2y = -3$
$(2, 5)$

3. For each pair of tables of values, determine two equations in slope-intercept form. Then, use technology to graph the equations and identify the point of intersection.

a)

x	0	−1	2	3
y	5	7	1	−1

x	0	−2	1	4
y	2	3	1.5	0

b)

Time (s)	0	2	5	10
Distance (m)	0	7	17.5	35

Time (s)	0	2	4	10
Distance (m)	10	13	16	25

☆4. Solve each system of linear equations by creating a table of values and graphing with pencil and graph paper. Then, verify you solution for each case.

a) $y = 8x - 3$
$y = 2x + 3$

b) $x - y = -2$
$4x + y = 12$

c) $y = \frac{1}{2}x - 6$
$3x - y = -4$

B Apply

5. Alan has $10 and saves $0.50 each day. Vanessa has $5 and saves $1 each day.

a) Create a system of linear equations to model the amount of money, M, in dollars, that each of Alan and Vanessa has in terms of days, d.

b) Use a graph to determine when Alan and Vanessa will have the same amount of money. How much money will each of them have on that day?

☆6. Two large tanks of oil are being drained. The first tank contains 125 m³ of oil and is being emptied at a rate of 2.5 m³ per minute. The second tank contains 80 m³ of oil and is being drained at a rate of 1 m³ per minute.

a) Create a system of linear equations to model the amount of oil, A, remaining in each tank in terms of time, t.

b) Graph the equations together to identify the point of intersection.

c) What does the point of intersection mean in the context of the problem?

d) Use your graph to determine which tank will be empty first.

7. A theatre production sold tickets at a price of $15 for adults and $10 for children. The total revenue from the sale of 69 tickets was $900.

a) Write a system of linear equations to model revenue from ticket sales, where a is the number of adults' tickets sold and c is the number of children's tickets sold.

b) Rewrite the equations in slope-intercept form.

c) Graph the equations together and determine the point of intersection.

d) Explain the meaning of the point of intersection in the context of the problem.

C Extend

8. 200 L of oil in a cylindrical tank are being transferred to a second container that is empty. The oil is being pumped at a rate of 8 L per minute.

a) Write an equation to model the volume, V, in litres, of oil in each tank in terms of time, t, in minutes.

b) Graph the two equations together and determine the point of intersection.

c) What is true about the volumes of oil in both tanks at this point?

d) How long will it take to empty the first tank?

e) Does the shape of the second container affect your answers?

9. The graph represents a system of linear equations.
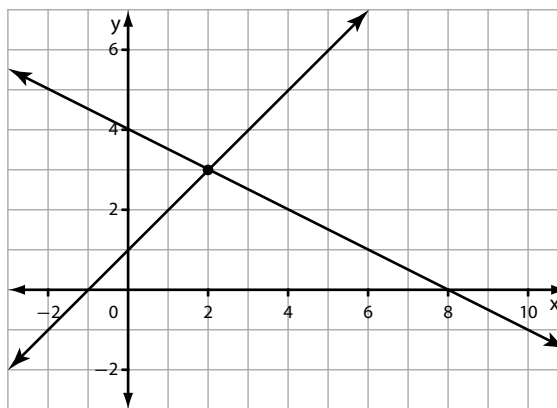
For each line, determine the

a) x-intercept

b) y-intercept

c) slope

d) point of intersection with the other line

e) equation

10. Mr. Darwal told his students that his daughter was getting married. When the students expressed surprise that he had a daughter old enough to be engaged, Mr. Darwal said: "Our ages have a sum of 62 and half of my age is one year more than my daughter's age."

a) Create a system of linear equations for the relationship between the two ages.

b) Solve the system graphically to determine the ages of Mr. Darwal and his daughter.

11. When an airplane is flying with a tailwind, the plane's speed is equal to its airspeed plus the speed of the wind. When a plane is flying into a headwind, its speed is equal to its airspeed minus the speed of the wind. Suppose that the airspeed of an airplane is 180 km/h and that the wind speed is 30 km/h.

a) Create a system of linear equations to model the distance, d, that the plane travels in terms of time, t, when the aircraft is flying with the wind and when it is flying against the wind.

b) What is the point of intersection of the two lines representing these equations?

c) How much farther can the airplane travel in 4 h with a 30-km/h tailwind than with a 30-km/h headwind?

☆**12.** At what point will a line that passes through (1, 1) and (4, 7) intersect a second line that passes through (1, 6) and (3, 0)? Solve algebraically.

13. To convert temperatures expressed in degrees Celsius, C, to temperatures expressed in degrees Fahrenheit, F, you can use the equation $F = \frac{9}{5}C + 32$. To convert temperatures from the Fahrenheit scale to the Celsius scale, you can use the equation $C = \frac{5}{9}(F - 32)$. At what point are temperatures on the two scales equal?

14. Ferdinand lives in the mountains and rides his bicycle to and from school in the valley. Over level ground, Ferdinand rides at an average speed of 20 km/h. He travels at twice that speed coming down the mountain, and pedals back up the trail at half that speed. It takes Ferdinand a total of 1 hour to complete a ride to and from school.

a) Write a system of linear equations to model Ferdinand's ride in each direction in terms of distance travelled, d, in kilometres and time, t, in hours.

b) Use graphing technology to solve your system. What does the point of intersection represent?

c) Assuming that Ferdinand rides at a constant speed in each direction, how long does it take for him to travel from home to school? from school to home?

d) How far from school does Ferdinand live?

D Create Connections

15. Is it possible for two lines to have no point of intersection? Explain.

16. You are starting a website creation company. Your start-up costs are $200. For each website you create you must pay a fee of $25. If you charge customers $50 each to create a website for them, how many customers will it take for you to break even?

☆**17.** For the parallelogram shown, determine the values of x and y.