

8.2 Modelling and Solving Linear Systems

KEY IDEAS

- When modelling word problems, assign variables that are meaningful to the context of the problem.
- To assist in visualizing or organizing a word problem, you can use a diagram or a table of values, or both.
- If a situation involves quantities that change at constant rates, you can represent it using a system of linear equations.
- If you know the initial values and rates, you can write the equations directly in slope-intercept form because the initial value is the y -intercept and the rate of change is the slope. Otherwise, you can determine the rate of change using start and end values.
- You can interpret information from graphs of linear systems.

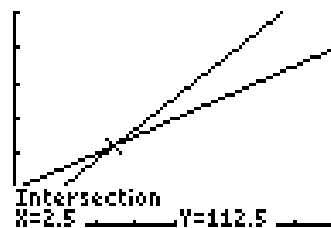
Example

Sean is comparing the costs that two computer repair companies charge for home visits. Company A charges a rate of \$45 per hour. Company B charges a flat rate of \$50, plus \$25 per hour for labour.

- Create a system of linear equations to model the rates that both companies charge.
- Solve the system of linear equations graphically. Explain what the solution represents.

Solution

- Let h represent the number of hours for a home visit and let C represent the cost, in dollars. For Company A, the equation to express its rate is $C = 45 \times h$. For Company B, its rate may be modelled by the equation $C = 50 + 25 \times h$. These two equations form a linear system.
- Use technology to graph the equations.



The point of intersection is $(2.5, 112.5)$. This indicates that the charges of the two companies are equal, at \$112.50, when 2.5 hours of labour are needed for a computer repair.

A Practise

- Model each situation using a system of linear equations.
 - Shandra is three times as old as Cory. In four years she will be twice as old as Cory will be.
 - One vehicle has 5 L of fuel in its tank and is being filled at a rate of 0.9 L/s. A second vehicle has 3 L of fuel in its tank and is being filled at a rate of 1.2 L/s.

- ★2. The sum of two numbers is 168. Their difference (subtracting the second number from the first) is 18.

- a) The table shown has x - and y -values adding to 168. Calculate the difference between x and y and put these values in the table. If you want the difference to be 18, what inferences can you make about the size of x and the size of y ?

x	y	$x - y$
30	138	-118
50	118	
70	98	
80	88	
90	78	
100	68	

- b) Create a system of equations to model the relationships between the two numbers.
- c) Rewrite the equations in slope-intercept form.
- d) Graph the equations to determine the intersection point.
- e) Does the intersection point confirm the inferences you made using the table in part a)?
3. Josee invests a total of \$15 000 in two different investments. The first amount is put into a long-term account that pays interest at a rate of 6.5% per year. The second amount is put into a short-term account earning interest at a rate of 5% per year. Josee's investments earn a total of \$885 in interest in one year.
- a) Write equations to represent the total amount invested and the total interest earned.
- b) Rewrite the equations in slope-intercept form.
- c) Use technology to graph the two equations and determine the point of intersection. Explain how this point relates to the investments.

B Apply

- ★4. A chemist wants to make 5 L of bromine solution with 32% concentration.

The chemist has two available bromine solutions of concentrations 40% and 25%. She needs to determine the amount of each solution to mix to obtain the final amount in the desired concentration.

- a) Write an equation to express the final amount of solution.
- b) Write a second equation using the concentrations and amounts of the available solutions.
- c) Rewrite the equations in slope-intercept form, if necessary, and graph them using technology.
- d) Use the point of intersection to determine the amount of each bromine solution that the chemist needs to use.

5. A rectangle with a perimeter of 72 m has a length that is three times its width.

- a) Create a table of values for each equation and determine five ordered pairs to satisfy each equation.

$$l = 3w$$

w						
l						

$$2l + 2w = 72$$

w						
l						

- b) Plot the ordered pairs on graph paper with axes labelled w and l . Connect each set into a line, and estimate the intersection point of the lines.
- c) Check your estimate using technology.
- d) Explain the meaning of the point of intersection.

6. A food company wants to produce 0.5-kg bags of a mixture of cashews and peanuts to sell for \$5.00. The company pays \$12/kg for cashews and \$3/kg for peanuts. The company needs to determine the amount of each to put into the bags to at least break even.
- Create an equation for the amount of cashews, c , and peanuts, p , in each bag. Create a second equation for the total cost of the different nuts and the mixture.
 - Rewrite the equations in the form $y = mx + b$ and graph them on a graphing calculator.
 - Solve the system of linear equations.
7. Aircraft are landing and taking off on parallel runways at a busy airport. On its approach, one aircraft descends from an altitude of 1200 m to an altitude of 500 m in 35 s. During the same time, a departing aircraft climbs from an altitude of 200 m to an altitude of 1250 m.
- Write a system of linear equations to model the altitudes of the aircraft.
 - When are the aircraft at the same altitude? What is that altitude?
9. Janna and Jordan are planning a birthday party and are comparing prices from two restaurants. Both restaurants have a flat rate for renting a banquet hall for a maximum of 100 guests, as well as a set meal price per guest.
- The prices that the restaurants charge can be modelled by the equations $C = 175 + 20n$ and $C = 100 + 22.5n$. Explain what each equation means.
 - Graph the equations together and determine the point of intersection.
 - For how many guests is the first restaurant the less expensive choice? For how many guests is the second restaurant the less expensive choice? Express each answer as a range of values.
- ★10. The tables of values model the relationship between time and distance for two drivers who are travelling from Calgary in the same direction but who leave at different times.

Driver A

Time (h)	0	1	2	5	6
Distance (km)	0	75	150	375	450

Driver B

Time (h)	0	1	2	3	6
Distance (km)	0	0	95	190	475

C Extend

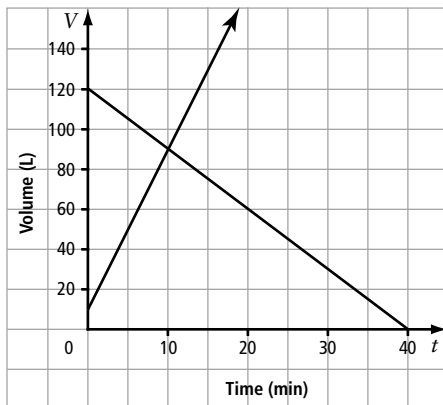
8. The highest point in Alberta is Mount Columbia. The highest point in Saskatchewan is Cypress Hills. Their elevations are related by the system of linear equations $a - s = 2279$ and $a + s = 5215$, where a is the height, in metres, of Mount Columbia above sea level, and s is the height, in metres, of Cypress Hills above sea level.
- Explain the meaning of each equation.
 - Rewrite each equation to isolate the variable s .
 - Graph the equations together and determine the heights of Mount Columbia and Cypress Hills.
- Which driver left one hour later than the other?
 - Which driver is travelling at a faster rate?
 - What happens when 6 h have elapsed?
 - Plot the points in the tables of values on graph paper, using time and distance axes, and estimate the number of hours it takes for the drivers to travel the same distance.

D Create Connections

11. A boat travels 50 km along a river in 2.5 h when it is moving downstream with the current at a constant speed. When moving upstream against the current, it takes 4 h for the boat to cover the same distance.

- Write equations to model the travel of the boat in each direction, using s for the constant speed of the boat and c for the constant speed of the current, each in kilometres per hour.
- Rewrite the equations and graph them together to determine the point of intersection.
- What does the point of intersection indicate about the boat's travel on the river?

- ★12. The graph shows the change in volume of water in two tanks over time. V represents volume, in litres, and t represents time, in minutes.

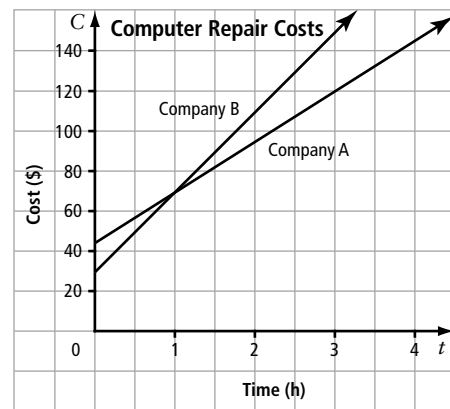


- How much water is in each tank at $t = 0$?
- What is different about how the volume of water is changing in the two tanks?
- When do the tanks have the same volume of water and how much water is that?
- At what rate is the volume of water changing for each tank?

- Determine a system of linear equations to express the relationship between volume and time that matches the graphs.

13. The graph shows the charges, C , in dollars, of two appliance repair companies, A and B, in terms of hours of labour, t .

- What is the point of intersection of the lines on the graph?
- Which company charges less if a repair takes several hours?
- Create a system of linear equations to match the graphs shown.



14. Create a problem or situation that could be modelled by each linear system.

- $$C = 100 + 18.5t$$

$$C = 75 + 20t$$
- $$a + b = 900$$

$$0.05a + 0.045b = 74$$
- $$y = x + 8$$

$$5x + 9y = 100$$

15. Create a problem involving a system of linear equations that has a solution of $(3, 24)$.