Chapter 9 Solving Systems of Linear Equations Algebraically

9.1 Solving Systems of Linear Equations by Substitution

KEY IDEAS

- You can solve systems of linear equations algebraically using substitution.
  - Isolate a single variable in one of the two equations.
  - Where possible, choose a variable with a coefficient of 1 or −1.
  - Solve the linear system.

    $5x + y = 11$ (Equation 1)

    $2x + 3y = 7$ (Equation 2)
  - Isolate the variable y in Equation 1 since its coefficient is 1.

$$\begin{aligned} 5x + y &= 11 \\ 5x + y - 5x &= 11 - 5x \\ y &= 11 - 5x \end{aligned}$$

  - Substitute the expression for y in Equation 2.

$$\begin{aligned} 2x + 3y &= 7 \\ 2x + 3(11 - 5x) &= 7 \\ 2x + 33 - 15x &= 7 \\ 33 - 13x &= 7 \\ -13x &= 7 - 33 \\ -13x &= -26 \\ \frac{-13x}{-13} &= \frac{-26}{-13} \\ x &= 2 \end{aligned}$$

  - Substitute the solution for the first variable into one of the original equations. Solve for the remaining variable.

$$\begin{aligned} 5x + y &= 11 \\ 5(2) + y &= 11 \\ 10 + y &= 11 \\ 10 + y - 10 &= 11 - 10 \\ y &= 1 \end{aligned}$$

    The solution to the system is $x = 2$ and $y = 1$.
  - Check your answer by substituting into both original equations.

$$\begin{aligned} 5x + y &= 11 & 2x + 3y &= y \\ 5(2) + 1 &= 11 & 2(2) + 3(1) &= y \\ 10 + 1 &= 11 & 4 + 3 &= 7 \\ 11 &= 11 & 7 &= 7 \end{aligned}$$

    Therefore, $x = 2$ and $y = 1$ is the correct solution.

Example

Ryan bought 3 tickets and 6 hot dogs at an Edmonton Rush lacrosse game. The total cost of his purchases was $153. His friend Carrie bought 1 ticket and 3 hot dogs and spent $54. What was the price of a single ticket? What was the price of a hot dog?

Solution

Let T represent the price of a ticket.
Let D represent the price of a hot dog.
Write an equation that represents Ryan's purchases.
$3T + 6D = 153$ (Equation 1)
Write an equation that represents Carrie's purchases.
$1T + 3D = 54$ (Equation 2)
Isolate a variable in one of the equations. Isolate T in Equation 2.

$$\begin{aligned} 1T + 3D &= 54 \\ T + 3D - 3D &= 54 - 3D \\ T &= 54 - 3\mathrm{D} \end{aligned}$$

Substitute this expression for T into Equation 1. Solve for D.

$$\begin{aligned} 3T + 6D &= 153 \\ 3(54 - 3D) + 6D &= 153 \\ 162 - 9D + 6D &= 153 \\ 162 - 3D &= 153 \\ 162 - 162 - 3D &= 153 - 162 \\ -3D &= -9 \\ \frac{-3D}{-3} &= \frac{-9}{-3} \\ D &= 3 \end{aligned}$$

Therefore, the cost of a hot dog is $3. Substitute the cost of a hot dog into one of the equations. Solve for T.

$$\begin{aligned} T + 3D &= 54 \\ T + 3(3) &= 54 \\ T + 9 &= 54 \\ T + 9 - 9 &= 54 - 9 \\ T &= 45 \end{aligned}$$

Therefore, the cost of a ticket is $45.
Verify by substituting 45 for T and 3 for D into each equation.

$$\begin{aligned} 3T + 6D &= 153 \\ 3(45) + 6(3) &= 153 \\ 135 + 18 &= 153 \\ 153 &= 153 \end{aligned} \qquad \begin{aligned} T + 3D &= 54 \\ 45 + 3(3) &= 54 \\ 45 + 9 &= 54 \\ 54 &= 54 \end{aligned}$$

Therefore, $T = 45$ and $D = 3$ is the correct solution.
The cost of a ticket is $45 and the cost of a hot dog is $3.

A Practise

1. Solve the following linear systems of equations by substitution.

a) $x = y + 2$
$x + y = 25$

b) $y = 4x$
$x - y = 33$

c) $y = 18 - 2x$
$3x + y = 17$

d) $x - 2y = 8$
$3x + y = 3$

e) $2x + 5y = 5$
$8x - y = 41$

f) $\frac{1}{2}x - y = -2$
$x + y = 5$

★2. Solve the following system of linear equations by isolating x. Then, solve by isolating y. Which method do you prefer? Explain why.

$2x - y = -5$
$5x + y = -2$

★3. Determine algebraically if the point (2, 4) is the solution to the system $3x - y = 2$ and $x + y = 5$. Explain your answer.

B Apply

4. Seventy-five hundred fans attended a rock concert. Ticket prices were $60 for adults and $35 for students. If the total revenue from ticket sales was $300 000, how many students attended the concert?

5. Meredith invested $4000 in the stock market. She used some of the money to buy stock worth $2.50 a share and invested the rest in a stock worth $4.50 a share. If Meredith purchased a total of 1280 shares, how many shares of each type did she buy?

★6. Joel's hockey team collected a total of 900 aluminum cans and plastic bottles for recycling. The team received 10¢ for each can and 25¢ for each bottle. If the team received a total of $145.20, how many cans did they bring in?

7. Canada won 26 medals at the 2010 Winter Olympic Games, including 7 silver medals. The number of gold medals was 4 more than twice the number of bronze medals. How many gold medals did Canada win?

8. A satellite radio station plays 108 new wave and hip-hop songs in a 6-hour time slot. If the station plays 5 times as many new wave songs as hip-hop songs, how many songs of each type does the station play in 1 hour? What assumption(s) did you make?

9. The sum of Jane's age and Tim's age is 40. Four years from now, Jane's age will be 6 years less than twice Tim's age at that time. How old are Jane and Tim now?

10. A bag contains a total of 71 marbles, 14 of which are red. Each remaining marble is either black or white. If the number of black marbles is 3 less than 3 times the number of white marbles, how many marbles are black?

11. Your neighbourhood music store is having a sale. You spend a total of $81.50 to buy 5 music videos and 4 compact discs. Your friend spends a total of $42.25 to buy 3 videos and 1 compact disc. All music videos cost the same amount, and all compact discs cost the same amount.

a) What is the price for one video?

b) What is the price for one CD?

12. Teams in the girls' school basketball league are awarded 2 points for a win and 1 point for a loss. At the end of the season, Team A has 36 points and Team B has 24 points. Deborah, the league statistician, notices that the record of Team A is the reverse of the record of Team B: the number of Team A's wins equals the number of Team B's losses, while the number of Team A's losses is the same as the number of Team B's wins. What is the win-loss record of each team?

13. A student is given the following problem.

> A vending machine in the school cafeteria accepts only \$1, \$2, and 25¢ coins. When it was emptied, the machine contained 125 coins, 45 of which were loonies. If the total value of the coins was \$184, how many coins of each type were in the machine?

The student wrote the following system of linear equations to solve the problem.

Let T represent the number of \$2 coins.
Let Q represent the number of 25¢ coins.
$T + Q + 45 = 125$
$27 + 0.25Q = 184$

Will this linear system provide the correct solution? If so, solve it. If not, write a correct system and solve it.

14. In which step is the first error made in the partial solution of the linear system $3x + y = 2$ and $2x + 5y = 23$?

Step 1: $2x + 5(-3x + 2) = 23$

Step 2: $2x - 15x + 2 = 23$

Step 3: $-13x = 21$

Step 4: $x = \frac{13}{21}$

Correct the error and determine the solution.

C Extend

☆**15.** Mandy drove from Prince George to Kamloops, a distance of approximately 400 km. It took her 4.5 hours to complete the trip. For part of the trip, she drove at an average speed of 80 km/h. For the remaining part, she travelled at an average speed of 100 km/h. For what distance did she drive at the higher speed?

16. $3x + 2y = 2$ and $4x + 5y = 12$ form a linear system in which part of the solution is $x = -2$. Determine the y-value of the solution.

17. Consider the system of linear equations $5x - 4y = 0$ and $x + 3y = 15$.

a) Without solving the system, determine whether there is one solution, an infinite number of solutions, or no solution. Explain your thinking.

b) If there is one solution, in which quadrant will the lines intersect? Show how you determined your answer.

18. Use a substitution method to solve this system of linear equations involving three variables.

$2x - y + 2z = -7$
$4x - 3y + 4z = -33$
$-x + y + z = -3$

D Create Connections

19. Consider the system of linear equations $x - 2y = 10$ and $3x - y = 0$.

a) Solve the system by graphing.

b) Solve the system by using substitution.

c) How are the methods similar? How are they different?

d) Which method do you prefer for solving this system of equations? Explain.