9.2 Solving Systems of Linear Equations by Elimination

‹‹ KEY IDEAS ››

- A table can help you organize information in a problem. This can help you to determine the equations in a linear system.
  - You can solve a linear system by elimination.
  $$3x + 2y = 2$$
  $$4x = 12 - 5y$$

  - If necessary, rearrange the equations so that like variables appear in the same position in both equations. The most common form is $ax + by = c$.

    $3x + 2y = 2$ (Equation 1) $\qquad$ $4x = 12 - 5y$ (Equation 2)
    $$4x + 5y = 12 - 5y + 5y$$
    $$4x + 5y = 12$$

  - Determine which variable to eliminate. If necessary, multiply one or both equations by a constant to eliminate the variable by addition or subtraction.

    Eliminate the variable y.

  - The lowest common multiple of 2 and 5 is 10. Multiply Equation 1 by 5 and multiply Equation 2 by 2 so that the coefficients of the terms involving y add to zero.

    $5(3x + 2y) = 5(2)$ $\qquad$ $2(4x + 5y) = 2(12)$
    $15x + 10y = 10$ $\qquad\quad$ $8x + 10y = 24$

    Subtract the second equation from the first equation to eliminate y.
    $$15x + 10y = 10$$
    $$\underline{-(8x + 10y = 24)}$$
    $$7x = -14$$

  - Solve for the remaining variable.
    $$\frac{7x}{7} = \frac{-14}{7}$$
    $$x = -2$$

  - Solve for the second variable by substituting the value for the first variable into one of the original equations.
    $$3x + 2y = 2$$
    $$3(-2) + 2y = 2$$
    $$-6 + 2y = 2$$
    $$-6 + 6 + 2y = 2 + 6$$
    $$2y = 8$$
    $$\frac{2y}{2} = \frac{8}{2}$$
    $$y = 4$$

  - Check your solution by substituting each value into both original equations.

    $3x + 2y = 2$ $\qquad\qquad$ $4x = 12 - 5y$
    $3(-2) + 2(4) = 2$ $\qquad$ $4(-2) = 12 - 5(4)$
    $-6 + 8 = 2$ $\qquad\qquad$ $-8 = 12 - 20$
    $2 = 2$ $\qquad\qquad\qquad$ $-8 = -8$

    The solution is $x = -2$ and $y = 4$.

Example

The Fête au Village is an annual festival in Legal, Alberta, to celebrate the French culture of Legal's pioneers. Two popular events are the tug o' war and the demolition derby. A tug o' war team consists of 8 people. The entry fee is $20 per team. Each demolition derby team has 2 members. The entry fee is $100 per team. If a total of 94 people enter both events and $1660 is collected in fees, how many teams are entered in each event?

Solution

Let W represent the number of teams entered in the tug o' war.
Let D represent the number of teams entered in the demolition derby.
Write an equation that represents the total number of people entered in both events.
$8W + 2D = 94$ (Equation 1)

Write an equation that represents the total entry fees collected.
$20W + 100D = 1660$ (Equation 2)

Solve the system by elimination.
Eliminate variable W.

The lowest common multiple of 8 and 20 is 40. Multiply Equation 1 by 5 and multiply Equation 2 by 2.

$$5(8W + 2D) = 5(94) \qquad 2(20W + 100D) = 2(1660)$$
$$40W + 10D = 470 \qquad\quad 40W + 200D = 3320$$

Subtract Equation 1 from Equation 2.

$$\begin{aligned} 40W + 200D &= 3320 \\ -(40W + 10D &= 470) \\ \hline 190D &= 2850 \end{aligned}$$

Solve for D.
$$\frac{190D}{190} = \frac{2850}{190}$$
$$D = 15$$

Substitute $D = 15$ into Equation 1. Solve for W.
$$\begin{aligned} 8W + 2D &= 94 \\ 8W + 2(15) &= 94 \\ 8W + 30 &= 94 \\ 8W + 30 - 30 &= 94 - 30 \\ 8W &= 64 \\ \frac{8W}{8} &= \frac{64}{8} \\ W &= 8 \end{aligned}$$

Check the answer by substituting $D = 15$ and $W = 8$ into both equations.
$$\begin{aligned} 8W + 2D &= 94 \qquad & 20W + 100D &= 1660 \\ 8(8) + 2(15) &= 94 \qquad & 20(8) + 100(15) &= 1660 \\ 64 + 30 &= 94 \qquad & 160 + 1500 &= 1660 \\ 94 &= 94 \qquad & 1660 &= 1660 \end{aligned}$$

Therefore, there are 8 teams entered for the tug o' war and 15 teams entered for the demolition derby.

A Practise

1. Solve, using elimination.

   a) $2x - y = -5$
   $5x + y = -2$

   b) $4x + 2y = 6$
   $4x - 3y = 1$

   c) $3x + 7y = 17$
   $2x - 3y = -4$

   d) $2x - 3y = 20$
   $2x + 2y = 12$

   e) $7x + 2y = 3$
   $4x - 3y = -48$

2. Rearrange the terms of the equations to the form $ax + by = c$.

   a) $x + 3y = -1$
   $2x + 4y - 12 = 0$

   b) $3y = -2x + 1$
   $4x = 2y + 10$

   c) $3x - 5 = 2y$
   $4y - 1 - 5x = 0$

   d) $x - 3y = -4$
   $4x + 2y - 12 = 0$

   e) $2y = -3x - 9$
   $3y + 2x - 9 = 0$

3. Solve the equations in question 2 by elimination.

4. Solve the following systems of linear equations by the elimination method.

   a) $y = \frac{1}{2}x + 2$
   $x + y = 5$

   ☆**b)** $\frac{1}{2}x - \frac{1}{3}y = 1$
   $x + \frac{1}{4}y = 2$

   c) $\frac{2}{3}x + \frac{1}{5}y = -2$
   $\frac{1}{3}x - \frac{1}{2}y = -7$

5. Solve the following systems of linear equations by the elimination method. For each system, explain the result and determine the solution by another method.

   a) $x + 3y = -1$
   $2x + 6y + 2 = 0$

   b) $3x - 2y = 5$
   $-6x + 4y = 1$

B Apply

Solve problems 6 to 14 by the elimination method. Check your answers.

6. Mrs. Chan's Math class contributed $2 coins and $1 coins to an earthquake relief fund. The number of $1 coins contributed was 8 less than 5 times the number of $2 coins contributed. If the class raised a total of $160, how many coins of each type were collected?

7. A sports club charges an initiation fee and a monthly fee. At the end of 5 months, Christelle had paid a total of $170. Her friend, Keaton, had paid $295 at the end of 10 months. What is the initiation fee and what is the monthly fee?

8. Huyen paid $124 to rent a car for 3 days and drove a total distance of 160 km. When she rented the same car for 5 days and drove 400 km, it cost Huyen $240. What was the rental charge per day and what was the charge per kilometre?

9. Robyn wishes to invest $660 so that the income from an investment paying interest at 10% per annum is equal to the interest from a bond paying 12% annual interest. How much should Robyn invest at each rate?

10. A 500-space parking lot is filled with motorcycles and passenger cars, with only one vehicle in each space. How many motorcycles and cars are there if the total number of tires on the parked vehicles is 1650?

11. General admission tickets to the Calgary Zoo cost a total of $109 for a group of 4 adults and 3 children. Tickets for a group of 2 adults and 5 children cost a total of $93. What is the cost for one adult ticket? What is the admission price for one child?

12. John lives 1.2 miles from his school. Each morning, he walks the entire distance in a time of 24 min. On his return each afternoon, John runs part of the way so that it takes him only 15 min to reach home. If John runs twice as fast as he walks, how far does he run on his way home from school?

13. The Golden Ears Bridge is a toll bridge across the Fraser River in British Columbia. It connects Pitt Meadows and Maple Ridge on the north side to Langley and Surrey on the south side. The regular toll for passenger cars is $3.95. The fee is reduced to $2.80 if a vehicle is equipped with an electronic transponder to pay the toll automatically. One day, 8200 cars crossed the bridge. If the total of the tolls paid was $30 032.50, how many vehicles had a transponder?

☆**14.** Shanice purchased a total of 50 oranges and granola bars as a snack for the girls' soccer team. Oranges cost $2.40 per dozen and granola bars cost $3.25 for a 5-bar box. If Shanice paid a total of $19, how many boxes of granola bars did she buy?

C Extend

15. Cashews and peanuts are mixed together and sold by the pound. You can buy 3 lb of peanuts and 4 lb of cashews for $14.90. You can also buy 5 lb of peanuts and 2 lb of cashews for $12.70. What is the price per pound of each of cashews and peanuts?

☆**16. a)** The sum of the digits of a two-digit number is 14. The number formed by reversing the digits is 36 more than the original number. What is the original number?

  b) The sum of the digits of a two-digit number is 11. If the digits are reversed, the second number is 9 more than the original number. What is the original number?

17. For what values of m and n is (2, 5) the solution of the linear system $mx + y = 19$ and $nx - 2y = -6$?

18. Using the equation $2x + y = 3$, write a second equation to form a linear system that has
  a) an infinite number of solutions
  b) no solution
  c) one solution

19. If (2, −5) and (−5, −2) both satisfy the equation $Ax + By = -29$, what is the value of A?

D Create Connections

20. Consider the system of linear equations $3x + 4y = 1$ and $5x - 3y = -8$.
  a) Solve the system by substitution.
  b) Solve the system by elimination.
  c) Which method do you prefer? Explain.
  d) What do you need to consider when choosing whether to use the substitution method or the elimination method to solve a system of linear equations?