

9.3 Solving Problems Using Systems of Linear Equations

KEY IDEAS

- Systems of linear equations can be solved
 - graphically
 - algebraically by substitution or elimination

Method	Advantages	Disadvantages
Graphical	<ul style="list-style-type: none">• provides a visual that can show how two variables relate• can be done with or without a graphing calculator• can result in an accurate and quick solution when using a graphing calculator	<ul style="list-style-type: none">• can be time-consuming• may not provide an exact solution
Algebraic	<ul style="list-style-type: none">• allows for an exact solution relatively quickly• can be done using more than one method (substitution and elimination)	<ul style="list-style-type: none">• does not provide any visual insight into how the two variables relate• can result in an incorrect answer due to a minor arithmetic error

- It may be better to use a graphical approach to solve linear equations when you wish to see how the two variables relate, such as for cost analysis and speed problems.
- It may be better to use an algebraic approach when
 - you need only the solution (intersection point)
 - it is unclear where to locate the solution on a coordinate plane

Example

An aircraft travels 5432 km from Montreal to Paris in 7 h with a tailwind and returns in 8 h against the wind. Determine the wind speed and the speed of the aircraft in still air. What assumption(s) are you making?

Solution

Assume that the wind speed and the aircraft's speed in still air are constant throughout both trips.

Let W represent the wind speed.

Let A represent the aircraft's speed in still air.

Organize the information about distance, D , aircraft groundspeed, S , and time, T .

	D (km)	S (km/h)	T (h)
With a tailwind	5432	$A + W$	7
Against the wind	5432	$A - W$	8

Write a system of linear equations from the information in the chart.

$$D = ST$$

$$5432 = (A + W)(7)$$

$$5432 = (A - W)(8)$$

Choose a method to solve the system. For example, use the algebraic method involving elimination.

Expand each equation using the distributive property.

$$5432 = 7A + 7W$$

$$5432 = 8A - 8W$$

Choose a variable to eliminate. Eliminate the variable W by multiplying the first equation by 8 and the second equation by 7.

$$8(5432) = 8(7A + 7W)$$

$$7(5432) = 7(8A - 8W)$$

Rewrite the first equation so the like variables in the equations line up. Then, add the equations.

$$\begin{array}{r} 43\,456 = 56A + 56W \\ + 38\,024 = 56A - 56W \\ \hline 81\,480 = 112A \end{array}$$

$$81\,480 = 112A$$

$$\frac{81\,480}{112} = \frac{112A}{112}$$

$$727.5 = A$$

Substitute $A = 727.5$ into the first equation and solve for W .

$$5432 = 7A + 7W$$

$$5432 = 7(727.5) + 7W$$

$$5432 = 5092.5 + 7W$$

$$5432 - 5092.5 = 5092.5 - 5092.5 + 7W$$

$$339.5 = 7W$$

$$\frac{339.5}{7} = \frac{7W}{7}$$

$$48.5 = W$$

Verify the solutions by substituting $W = 48.5$ and $A = 727.5$ into both original equations.

$$5432 = 7(A + W)$$

$$5432 = 8(A - W)$$

$$5432 = 7(727.5 + 48.5)$$

$$5432 = 8(727.5 - 48.5)$$

$$5432 = 7(776)$$

$$5432 = 8(679)$$

$$5432 = 5432$$

$$5432 = 5432$$

Therefore, the wind speed was 48.5 km/h and the aircraft's speed in still air was 727.5 km/h.

A Practise

1. Solve each system of linear equations by an algebraic method. Verify your answer graphically.

a) $x - 2y = 10$
 $3x - y = 0$

b) $3x + 2y = 6$
 $3x - 5y = -15$

c) $5x + 4y = 2$
 $2x - 3y = 10$

d) $3x - 2y = 10$
 $4x + y = -5$

e) $2x + 4y = 12$
 $\frac{x}{5} + 3y = 17$

2. Solve each system of linear equations by a method of your choice. Leave any non-integer answers in fraction form. Explain your choice of method.

★a) $2x - 5y = -18$
 $8x - 13y = -58$

b) $5x = y$
 $-x + 3y = 3$

c) $\frac{1}{3}x + \frac{1}{2}y = -\frac{1}{2}$
 $\frac{1}{5}x - \frac{1}{3}y = \frac{8}{5}$

B Apply

To solve the following problems, write a system of equations and use a method of your choice to solve the system.

3. The perimeter of a rectangular field is 6400 m. Two times the width is 40 m more than the length. Determine the dimensions of the field.
4. Jimal plans to invest \$12 000 in two types of bonds which yield 9% and 11% annually. If he wants to earn a total of \$1200 annually, how much should Jimal invest in each type of bond?
5. A boat travelling against a current took 3 h to travel 36 mi. Travelling with the current, the boat took 2 h for the return trip. Determine the speed of the boat in still water.
6. Liam paid a total of \$27 for 6 tennis balls and 8 golf balls. Jessica paid \$43.50 for 14 tennis balls and 10 golf balls at the same store. What is the price of a tennis ball?
- ★7. During the winter, Mason feeds his horse a daily diet of hay and a mixture of grain. The total weight of the hay and grain is 20 pounds. He recently ordered enough grain at \$2.10/lb and hay at \$0.08/lb to last 60 days and paid a total of \$702.00. How much hay and how much grain does Mason feed his horse per day?
8. Raymark's basketball team scored a total of 93 points in its last game. A total of 49 baskets were made, consisting of free throws worth 1 point each, field goals worth 2 points each, and three-point shots worth 3 points apiece. If the team made 11 free throws, how many field goals and three-point shots did it make?
9. Devon has a summer job planting trees. He is paid \$0.07 per tree when planting in ideal conditions and \$0.30 per tree for planting in rocky or muskeg regions. One day, Devon planted 1750 trees and earned \$180.00. How many trees did he plant in rocky or muskeg areas?
10. During a 20-day period, Tom spent 70 hours mountain biking and swimming. He burned 860 calories per hour when he swam and 730 calories per hour when he rode his bike. If Tom burned a total of 54 350 calories, how many hours did he spend doing each activity?

11. Diobel operates a student painting company with a crew of five painters. They are paid at two different rates—an hourly wage for outdoor work and an hourly wage for indoor work. During one week, Diobel's payroll was \$3060 for 180 hours of indoor work and 20 hours of outdoor work. The following week, her payroll was \$3555 for 15 hours of indoor work and 185 hours of outdoor work. What is a painter's pay per hour for indoor work? for outdoor work?
12. Company A rents cars at a price of \$25 per day and 15 cents per kilometre. Company B charges a daily rental fee of \$30 and 10 cents per kilometre.
 - a) Create a system of linear equations relating the cost of a rental (y dollars) to the distance driven (x kilometres).
 - b) How many kilometres do you have to drive for the total rental charges of Company A and Company B to be equal?
13. Hockey teams are awarded 2 points for winning a game and 1 point for tying a game by the end of regulation time. The Vancouver Giants have 26 points this season. Three times the number of ties is 1 more than the number of wins the Giants have.
 - a) Write a system of linear equations to represent the situation.
 - b) How many wins do the Giants have?
15. Two snowmobile riders start at the same point heading in opposite directions, but leave at different times and travel at different speeds. The first rider leaves at 1:00 p.m. and travels east. The second rider leaves at 3:00 p.m., travelling west at a speed 15 km/h faster than the first rider. At 6:00 p.m., the riders are 365 km apart. What is the speed of each snowmobile rider? What assumption are you making?
16. A 95% sulfuric acid solution needs to be diluted with 60% solution to obtain a 70% solution. If 2700 mL of the new solution is required, how much of the 95% solution is needed?
- ★17. On a multiple choice test with 5 possible answers for each question, students are penalized 0.2 points for each incorrect answer. If the test consisted of 76 questions and a student achieved a score of 58, how many questions did the student answer correctly? What assumption are you making?
18. The lines that enclose a triangle can be represented by graphs of the equations $x = -4$, $y = 3x + 4$, and $y = -2x + 9$. Use a system of linear equations to determine the area of the triangle.

D Create Connections

C Extend

14. A car averages 8.5 L/100 km in city driving and 6.4 L/100 km in highway driving. In a trip that covered a distance of 720 km, the car used 55.7 L of fuel.
 - a) How far was the car driven in the city?
 - b) How much fuel did the car consume in city driving?
19. Create a system of linear equations.
 - a) Solve your system using a method of your choice.
 - b) Explain your method of choice. What did you consider when making your choice?
 - c) Try solving your system by another method. Which method do you prefer? Explain.