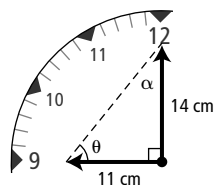


Chapter 3 Right Triangle Trigonometry

3.1 The Tangent Ratio

- c
 - a
 - b
- $\tan A = \frac{a}{b}$
 - $\tan B = \frac{b}{a}$
- 0.83
 - 1.26
 - 18
- 5.31 m
- $\tan X = \frac{\text{opposite}}{\text{adjacent}}$
 $\tan X = \frac{1.85}{1.6}$
 $\tan X = 1.15625$
 Use the inverse function on a calculator to apply the tangent ratio in reverse:
 $\tan X = 1.15625$
 $X = \tan^{-1}(1.15625)$
 $X = 49.1446\dots^\circ$
 $\angle X$ is 49.1° , to the nearest tenth of a degree.
 - $\tan Y = \frac{\text{opposite}}{\text{adjacent}}$
 $\tan Y = \frac{1.6}{1.85}$
 $\tan Y = 0.86486$
 Use the inverse function on a calculator to apply the tangent ratio in reverse:
 $\tan Y = 0.86486$
 $Y = \tan^{-1}(0.86486)$
 $Y = 40.8552\dots^\circ$
 $\angle Y$ is 40.9° , to the nearest tenth of a degree.
- $\angle A = 52^\circ$; $\angle B = 38^\circ$; $c = 4.06$ m; Side c , which is opposite the right angle, is the hypotenuse. Because the lengths of the other two sides (a and b) are known, the length of side c can be determined by substituting the values for a and b in the equation $a^2 + b^2 = c^2$ and solving for c .
- 293 cm
- 12.9 m
 - 14.8°

- 3.59 m
- 200 cm, or 2 m
- Organize the information and sketch a diagram to illustrate the problem.



- Form the tangent ratio for $\angle \alpha$ from the diagram.

$$\tan \alpha = \frac{11}{14}$$

$$\alpha = \tan^{-1}\left(\frac{11}{14}\right)$$

$$\alpha = 38.2^\circ$$

The angle formed between the line and the minute hand is 38.2° .

- Form the tangent ratio for $\angle \theta$.

$$\tan \theta = \frac{14}{11}$$

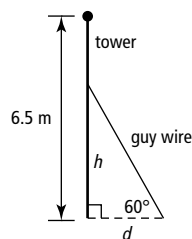
$$\theta = \tan^{-1}\left(\frac{14}{11}\right)$$

$$\theta = 51.8^\circ$$

The angle between the line and the hour hand is 51.8° .

- 359 m
- The plane travelled 13 186 m.
- 6.1 m
 - 7.6 m
- 24.3°

- Organize the information and sketch a diagram to illustrate the problem.



First, find the value of h , the height in metres up the pole where the wires should be attached.

$$6.5 \text{ m} \times \frac{2}{3} = 4.33 \text{ m}$$

Next, use a tangent ratio to find the distance, d , in metres, that each wire should extend from the base of the tower.

$$\begin{aligned}\tan 60^\circ &= \frac{4.33}{d} \\ d &= \frac{4.33}{\tan 60^\circ} \\ d &= 2.5 \text{ m}\end{aligned}$$

Since Ramon needs 2.5 m *on each side* of the tower, the available width of 4.2 m is not enough.

- b)** Since the available width is 4.2 m, the maximum value of d is $\frac{4.2 \text{ m}}{2}$, or 2.1 m. To find the corresponding value of h when d equals 2.1 m, use a tangent ratio:

$$\begin{aligned}\tan 60^\circ &= \frac{h}{2.1} \\ (2.1) \tan 60^\circ &= h \\ 3.64 \text{ m} &= h\end{aligned}$$

Since h represents only two thirds of the tower's height, the maximum possible tower height is

$$3.64 \times \frac{3}{2} = 5.46 \text{ m}$$

- 17. a)** $a = 2b$ or $a = \frac{b}{2}$; $b = 2a$ or $b = \frac{a}{2}$
b) $a = \frac{b}{2}$ or $a = 2b$; $b = \frac{a}{2}$ or $b = 2a$
c) The values or ratios in parts a) and b) are the same. This is because the values are simply switched from numerator to denominator, or vice versa.

3.2 The Sine and Cosine Ratios

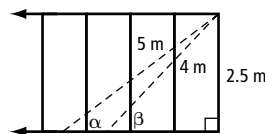
- 1. a)** $\sin A = \frac{a}{c}$ **b)** $\cos A = \frac{b}{c}$
c) $\sin B = \frac{b}{c}$ **d)** $\cos B = \frac{a}{c}$
- 2. a)** $\cos A = \frac{12}{17}$ **b)** $\sin A = \frac{10}{15}$
c) $\sin B = \frac{1.9}{2.4}$ **d)** $\cos B = \frac{2.6}{3.9}$
e) $a = \frac{75}{6} = \frac{25}{2}$ **f)** $a = \frac{9}{3} = 3$
- 3. a)** $\angle R = 48.4^\circ$ **b)** $\angle S = 41.6^\circ$
- 4. a)** 54.3° **b)** 56.7°
c) 12.6° **d)** 87.2°
- 5. a)** $x = 1.85 \text{ cm}$ **b)** $x = 4.39 \text{ m}$
- 6. a)** $z = 72.48 \text{ mm}$ **b)** $z = 9.70 \text{ m}$

- 7.** The angle is 61° and the backing piece must have a height of 155 mm.

- 8. a)** 6.3 ft **b)** 54.9°

- 9.** 34 m

- 10.** First, label the given diagram.



Then, form appropriate ratios:

$$\sin \alpha = \frac{2.5 \text{ m}}{5 \text{ m}} \quad \sin \beta = \frac{2.5 \text{ m}}{4 \text{ m}}$$

Solve for the angles:

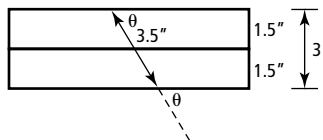
$$\alpha = \sin^{-1}\left(\frac{2.5}{5}\right) \quad \beta = \sin^{-1}\left(\frac{2.5}{4}\right)$$

$$\alpha = 30^\circ \quad \beta = 39^\circ$$

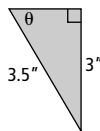
The 5-m rod forms an angle of 30° with the floor. The 4-m rod forms an angle of 39° with the floor.

- 11.** The boat ramp must be 4.8 m in length.

- 12.** First, modify and label the given diagram.



Then, create a simplified diagram.



Select the appropriate trigonometric ratio and solve for θ :

$$\sin \theta = \frac{3}{3.5}$$

$$\theta = \sin^{-1}\left(\frac{3}{3.5}\right)$$

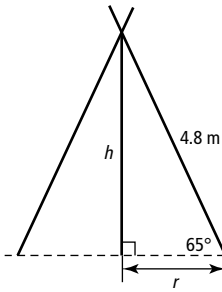
$$\theta = 59^\circ$$

The screws should be driven in at an angle of 59° or less to prevent the points from sticking through.

- 13. a)** The water gun can spray the middle 7.2 m of the opposite side.
b) 2.8 m (1.4 m at each end)
c) 7.8 m

- 14.** $CD = 14.7 \text{ cm}$

15. Consider the teepee as a cone.



Use a ratio to find the radius of the base.

$$\cos 65^\circ = \frac{r}{4.8 \text{ m}}$$

$$(4.8 \text{ m}) \times \cos 65^\circ = r$$

$$2.03 \text{ m} = r$$

Since the radius is one half of the diameter, multiply the value by 2:

$$\text{diameter} = 2 \times \text{radius}$$

$$\text{diameter} = 2 \times 2.03 \text{ m}$$

$$\text{diameter} = 4.06 \text{ m, or } 4.1 \text{ m, to the nearest tenth of a metre}$$

The diameter of the teepee is 4.1 m.

16. a)

θ	$\tan \theta$	$\sin \theta$	$\cos \theta$
15°	0.2679	0.2588	0.9659
30°	0.5774	0.5	0.8660
45°	1	0.7071	0.7071
60°	1.7321	0.8660	0.5
75°	3.7321	0.9659	0.2588

- b) As the values of the tangent and sine increase, the value of the cosine decreases.
c) The sine and cosine generate the same values, but in the opposite order.

3.3 Solving Right Triangles

1. a) $\angle B = 23^\circ$; $a = 5.2$; $c = 5.6$
b) $\angle B = 42^\circ$; $b = 12.9$; $c = 19.2$
c) $\angle Z = 49.0^\circ$; $\angle Y = 41.0^\circ$; $x = 2.3$
d) Use the Pythagorean theorem to calculate the length, x , of side YZ:

$$(YZ)^2 = (XY)^2 + (XZ)^2$$

$$x^2 = (1.75)^2 + (1.52)^2$$

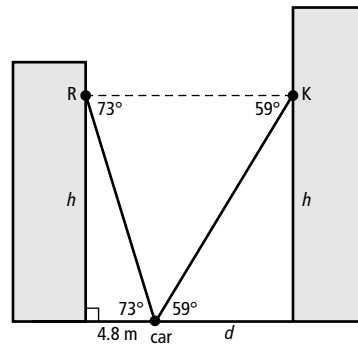
$$x^2 = 3.0625 + 2.3104$$

$$x^2 = 5.3729$$

$$x = \sqrt{5.3729}$$

$$x = 2.317\dots, \text{ or } 2.3 \text{ units, to the nearest tenth of a unit}$$

2. a) $\angle A$, $\angle C$
b) $\angle B$, $\angle D$
c) $\angle A = \angle B$, $\angle C = \angle D$
3. a) Both are correct. The name of the angle changes when the point of view changes.
b) The angle of elevation from one point to a second point is equal to the angle of depression from the second point to the first point.
4. a) $\angle C = 48^\circ$; $\angle B = 52^\circ$; $\angle D = 42^\circ$;
 $b = 18.3$; $c = 23.2$; $x = 20.3$
b) $\angle X = 27^\circ$; $\angle Z = 61^\circ$; $x = 2.6$; $y = 5.4$;
 $z = 5.3$
5. a) 42.1 cm b) 54.9°
6. Sketch and label a diagram with the given information to illustrate the problem.



Note that the angles of depression to the car equal the angles of elevation from the car.

Use an appropriate ratio to find h first:

$$\tan 73^\circ = \frac{h}{4.8 \text{ m}}$$

$$(4.8 \text{ m}) \times \tan 73^\circ = h$$

$$15.7 \text{ m} = h$$

This is the same h as for Kenneth's window.

Now we find d :

$$\tan 59^\circ = \frac{15.7 \text{ m}}{d}$$

$$d = \frac{15.7 \text{ m}}{\tan 59^\circ}$$

$$d = 9.43 \text{ m.}$$

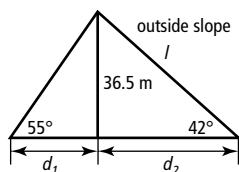
Add the two horizontal distances:

$$4.8 \text{ m} + 9.43 \text{ m} = 14.23 \text{ m, or } 14.2 \text{ m, to the nearest tenth of a metre}$$

The distance between the two windows is 14.2 m.

7. 8.6 m

8. a) 202 m
b) 714 m
c) 436 m
9. a) 98 m b) 50°
10. a) No, the diagonal length across the bottom of the box is only 118.5 cm, which is 1.5 cm less than the length of the cane.
b) Yes, the diagonal distance across opposite corners is 121.8 cm, which is greater than the length of the cane. Therefore, the lid can be closed.
11. Sketch a modified diagram and label it with the given information to illustrate the problem.



- a) Width of dike = $d_1 + d_2$
Find d_1 : Find d_2 :
 $\tan 55^\circ = \frac{36.5 \text{ m}}{d_1}$ $\tan 42^\circ = \frac{36.5 \text{ m}}{d_2}$
 $d_1 = \frac{36.5 \text{ m}}{\tan 55^\circ}$ $d_2 = \frac{36.5 \text{ m}}{\tan 42^\circ}$
 $d_1 = 25.56 \text{ m}$ $d_2 = 40.54 \text{ m}$

Width = $25.56 \text{ m} + 40.54 \text{ m} = 66.1 \text{ m}$
The width of the dike is 66.1 m.

- b) The length of the mesh is the hypotenuse of the outside triangle:

$$\sin 42^\circ = \frac{36.5 \text{ m}}{\ell}$$

$$\ell = \frac{36.5 \text{ m}}{\sin 42^\circ}$$

$$\ell = 54.5 \text{ m}$$

The mesh needs to be 54.5 m long.

- c) Let the height be 39 m. Then, solve as in part a):

$$d_1 = \frac{39 \text{ m}}{\tan 55^\circ}$$

$$d_1 = 27.31 \text{ m}$$

$$d_2 = \frac{39 \text{ m}}{\tan 42^\circ}$$

$$d_2 = 43.31 \text{ m}$$

Width = $27.31 \text{ m} + 43.31 \text{ m} = 70.6 \text{ m}$
The width at the base must be 70.6 m.

- d) Let the height equal 31 m. Then, solve as in part a):

$$d_1 = \frac{31 \text{ m}}{\tan 55^\circ}$$

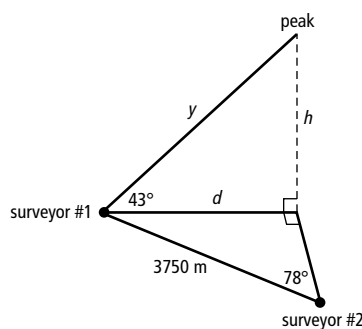
$$d_1 = 21.71 \text{ m}$$

$$d_2 = \frac{31 \text{ m}}{\tan 42^\circ}$$

$$d_2 = 34.43 \text{ m}$$

Width = $21.71 \text{ m} + 34.43 \text{ m} = 56.14 \text{ m}$
The student-engineer is not correct.
The base of the dike needs to be 56.1 m wide, not 54.8 m.

12. Sketch a simplified diagram to illustrate the problem, converting all distances to metres.



- a) Find the distance d , in metres, from surveyor #1 to a point directly under the peak:

$$\sin 78^\circ = \frac{d}{3750 \text{ m}}$$

$$(3750 \text{ m}) \times \sin 78^\circ = d$$

$$3668 \text{ m} = d$$

Now, use this distance in the other triangle to find h :

$$\tan 43^\circ = \frac{h}{3668 \text{ m}}$$

$$(3668 \text{ m}) \times \tan 43^\circ = h$$

$$3420 \text{ m} = h$$

The mountain is 3420 m high.

- b) Find the hypotenuse, y , of the second triangle:

$$\cos 43^\circ = \frac{3668 \text{ m}}{y}$$

$$y = \frac{3668 \text{ m}}{\cos 43^\circ}$$

$$y = 5015 \text{ m}$$

The cable needs to be 5015 m long.

- c) Use trigonometric ratios as in part a) to determine the distance, in metres, of each of d and h . Then, use the Pythagorean theorem to determine the distance y , in metres:

$$y^2 = d^2 + h^2$$

$$y^2 = (3668)^2 + (3420)^2$$

$$y^2 = 13\,454\,224 + 11\,696\,400$$

$$y^2 = 25\,150\,624$$

$$y = \sqrt{25\,150\,624}$$

$$y = 5015.03\dots, \text{ or } 5015 \text{ m, to the nearest metre}$$