Chapter 4 Exponents and Radicals

4.1 Square Roots and Cube Roots

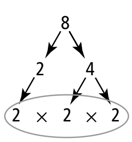
1. **a)** 81 **b)** 225
c) −625 **d)** $\frac{4}{9}$
e) $\frac{-25}{8}$ **f)** $\left(\frac{36}{49}\right)$

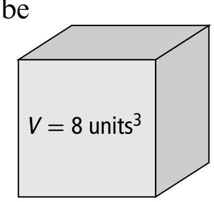
2. **a)** 729 **b)** −27
c) −216 **d)** 8
e) $\frac{-1}{3}$ **f)** $\frac{125}{343}$

3. **a)** 5 **b)** 14
c) 28 **d)** 2
e) $\frac{2}{3}$ **f)** $\frac{4}{7}$
g) $\frac{1}{3}$ **h)** $6x$
i) $\frac{7a}{13b}$

4. **a)** 2 **b)** 3
c) 12 **d)** 20
e) 3 **f)** $\frac{3}{5}$
g) $\frac{2}{7}$ **h)** $5y$
i) $9a$

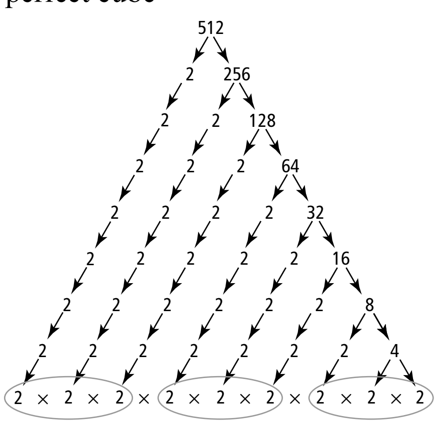
5. **a)** perfect cube

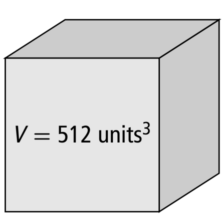




$s = 2$ units

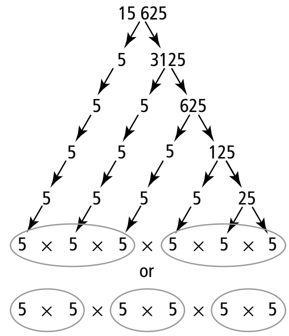
b) perfect cube

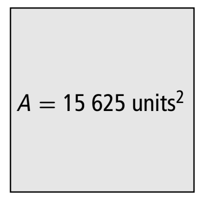




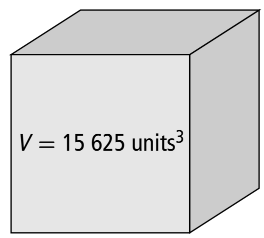
$s = 8$ units

c) both



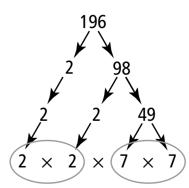


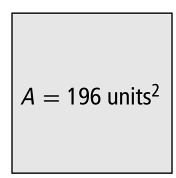
$s = 125$ units



$s = 25$ units

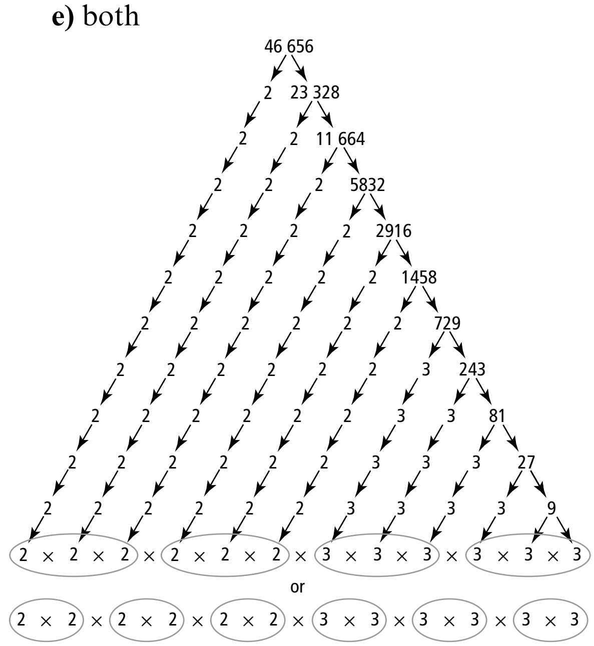
d) perfect square

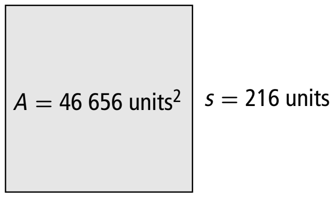


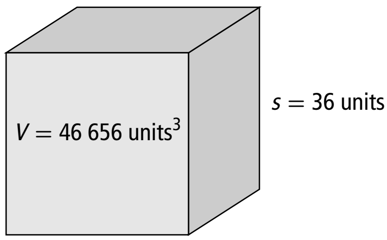


$s = 14$ units

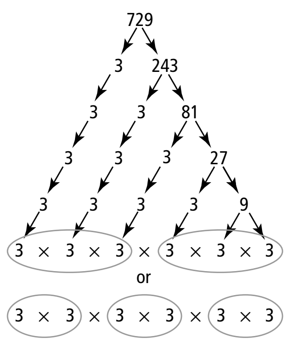
e) both

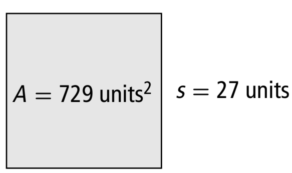


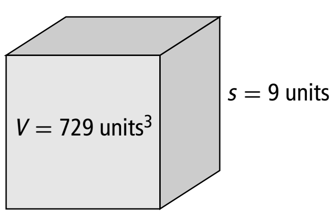




f) both







6. **a)** perfect square **b)** perfect square
c) both **d)** perfect square
e) both **f)** neither

7. **a)** perfect square **b)** perfect cube
c) perfect cube **d)** perfect square
e) perfect square **f)** neither

8. **a)** 17 **b)** 23
c) 14 **d)** 22
e) 31 **f)** 27

9. The storage container will measure 1.4 m by 1.4 m by 1.4 m (or 140 cm by 140 cm by 140 cm).

10. The side length of the patio is 23 ft.

11.
$$V = s^3$$
$$2146.2 = s^3$$
$$\sqrt[3]{2146.2} = s$$
$$12.899\,02\ldots = s$$
The edge length of the cube would be approximately 12.9 cm.

12. 24 ft

13. 12.5 ft

14. 6 m × 6 m × 6 m

15. 1331 mm^3

16. approximately 6.2 m

17. 9 m by 9 m by 9 m

18. 16 cm

19. **a)** $y = 60$ **b)** $y = 192$

20. **a)** $x = 6$ **b)** $x = 23$

21. Volume of the tank in cubic inches:
$$1 \text{ ft}^3 = (12 \text{ in.})(12 \text{ in.})(12 \text{ in.})$$
$$= 1728 \text{ in.}^3$$
$$54 \text{ ft}^3 = (54)(1728)$$
$$= 93\,312$$
The volume of the tank is equal to 93 312 in.^{3}
Volume of one balloon:
$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(6)^3$$
$$V = 288\pi$$
$$V = 904.778\,761\ldots$$
The volume of one balloon is approximately 904.78 in.^{3}
Number of balloons inflated per full tank:
$$\frac{\text{volume of tank}}{\text{volume of balloon}} = \frac{93\,312}{904.778\,761\ldots}$$
$$V = 103.13\ldots$$
A full tank will inflate approximately 103 balloons.

22. 575 cm^2

23. a)

$\sqrt{25}$	5
$\sqrt{2.5}$	1.581…
$\sqrt{0.25}$	0.5
$\sqrt{0.025}$	0.158…
$\sqrt{0.0025}$	0.05
$\sqrt{0.00025}$	0.015…

b)

$\sqrt{81}$	9
$\sqrt{8.1}$	2.846…
$\sqrt{0.81}$	0.9
$\sqrt{0.081}$	0.284…
$\sqrt{0.0081}$	0.09
$\sqrt{0.00081}$	0.028…

c) Answers may vary. Look for the idea that a perfect decimal square exists if it has an even number of zeros before the perfect square number.

24. The expression $\sqrt{-25}$ is not a perfect square because when you multiply two positive or two negative numbers the answer is always positive. The expression $\sqrt[3]{-27}$ is a perfect cube because when you multiply three negative numbers, such as (−3)(−3)(−3), the answer is negative. Therefore, it is possible to have a negative perfect cube.

25. a) When you double the side lengths of a square, the area increases by a factor of 2^2 or 4. Example:

$$A = s^2 = (2s)^2 = 4s^2$$

When you triple the side lengths, the area increases by a factor of 3^2 or 9. Example:

$$A = s^2 = (3s)^2 = 9s^2$$

b) When you double the edge lengths of a cube, the volume increases by a factor of 2^3 or 8. Example:

$$V = s^3 = (2s)^3 = 8s^3$$

When you triple the edge lengths, the volume increases by a factor of 3^3 or 27. Example:

$$V = s^3 = (3s)^3 = 27s^3$$

4.2 Integral Exponents

1. a) $\frac{1}{4^2}$ **b)** $\frac{3}{x^3}$

c) $\frac{1}{(5x)^2}$ or $\frac{1}{25x^2}$ **d)** $\frac{6}{a^3b^2}$

e) $\frac{-5}{a^4}$ **f)** $\frac{-4a^4}{b^5}$

g) $\left(\frac{3}{2}\right)^3$ **h)** $-3x^2y^4$

i) $\frac{6}{a^3b^4}$

2. No. Shelby's answer is incorrect. The correct answer is $\frac{16x^{10}}{y^6}$.

3. a) 0.3644 **b)** −0.125

c) 0.0625 **d)** −1

e) 4096 **f)** 2.8477

4. a) $\frac{a^4}{b^5}$ **b)** $\frac{-2b^2}{a^3}$

c) p^{12} **d)** $3s^{10}$

e) $\frac{x^8}{6^2}$ **f)** t^{16}

g) $\frac{1}{n^4}$ **h)** $\frac{y^6}{x^2}$

5. a) $(6)^{-3}(6) = 6^{-3+1} = 6^{-2} = \frac{1}{6^2}$

b) $\frac{(-2)^{-6}}{(-2)^{-3}} = (-2)^{-6-(-3)} = (-2)^{-3} = \frac{1}{(-2)^3}$

c) $\frac{3^3}{3^{-2}} = 3^{3-(-2)} = 3^5$

d) $\left(\frac{4^0}{4^{-2}}\right)^2 = (4^{0-(-2)})^2 = (4^2)^2 = 4^4$

e) $(6^{-4})^2 = 6^{(-4)(2)} = 6^{-8} = \frac{1}{6^8}$

f) $-(3^4)^{-3} = -(3)^{(4)(-3)}$
$= -(3)^{-12}$
$= \frac{-1}{(3)^{12}}$

g) $[(2^4)(2^{-7})]^{-3} = [(2)^{4+(-7)}]^{-3}$
$= [(2)^{-3}]^{-3}$
$= 2^{(-3)(-3)}$
$= 2^9$

h) $\left(\frac{3^3}{4^3}\right)^{-2} = \frac{(3)^{(3)(-2)}}{(4)^{(3)(-2)}}$
$= \frac{3^{-6}}{4^{-6}}$
$= \frac{4^6}{3^6}$

i) $(4a^{-3})^{-2} = (4)^{-2}a^{(-3)(-2)}$
$= (4)^{-2}a^6$
$= \frac{a^6}{4^2}$

j) $-3[(2^4)(2^{-3})]^{-2} = -3[(2)^{4+(-3)}]^{-2}$
$= -3[(2)^1]^{-2}$
$= -3(2)^{-2}$
$= \frac{-3}{2^2}$

6. **a)** 27 200 cm^2 **b)** 7 130 316 800 cm^2

7. approximately 1638 caribou

8. **a)** 3200 bacteria **b)** 6 710 886 400 bacteria **c)** 50 bacteria

9. $\left[\left((2^{-1})^2\right)^3\right]^{-1} = \left[\left(\left(\frac{1}{2}\right)^2\right)^3\right]^{-1}$
$= \left[\left(\frac{1}{4}\right)^3\right]^{-1}$
$= \left[\frac{1}{64}\right]^{-1}$
$= 64$

Or, some students may evaluate as $2^6 = 64$.

10. No. Kevin is incorrect. Example: Since the bases are not the same, you cannot add the exponents. When simplified, the expression $(2^3)(3^2) = (8)(9) = 72$. The power $6^5 = 7776$.

11. **a)** 25 g **b)** 800 g

12. **a)** $d = \frac{1}{2}gt^2$
$= \frac{1}{2}(9.8)(12.4^2)$
$= (4.9)(153.76)$
$= 753.424$
The penny falls from a height of approximately 753.4 m.

b) $d = \frac{1}{2}gt^2$
$28.5 = \frac{1}{2}(9.8)t^2$
$28.5 = (4.9)t^2$
$\frac{28.5}{4.9} = t^2$
$t^2 = 5.816\,326\,5\ldots$
$t = \sqrt{5.816\,326\,5}$
$t = 2.411\,706\,1\ldots$
It takes approximately 2.4 s for the penny to fall.

c) $d = \frac{1}{2}gt^2$
$248 = \frac{1}{2}(9.8)t^2$
$248 = (4.9)t^2$
$\frac{248}{4.9} = t^2$
$t^2 = 50.612\,244\ldots$
$t = \sqrt{50.612\,244\ldots}$
$t = 7.114\,228\,2\ldots$
It takes approximately 7.1 s for the penny to fall.

13. **a)** approximately 7.6×10^9 or 7.6 billion people
b) approximately 8.3×10^9 or 8.3 billion people

14. **a)** approximately 3.65×10^7 or 36.5 million people
b) approximately 3.75×10^7 or 37.5 million people

15. **a)** $A = 0.01(2)^3 = 0.08$ After 3 years, the payment will be \$0.08.
$A = 0.01(2)^{10} = 10.24$ After 10 years, the payment will be \$10.24.
$A = 0.01(2)^{25} = 335\,544.32$. After 25 years, the payment will be \$335 544 32.

b) Accept any reasonable justification. Examples:
- I would accept the double the money offer because it is worth more over time.
- I would accept the cash prize because it is immediate and I have few financial resources at the present time.

c) Years 0–10 total = \$20.47; years 11–20 total = \$20 951.04; years 21–25 total = \$650 117.12. The total value over 25 years is \$671 088.63.

16. a) 21.5 g **b)** approximately 1.34 g
c) approximately 0.34 g

17. a) $x = -4$ **b)** $x = 6$
c) $x = \frac{2}{3}$ **d)** $x = 3$

18. a) approximately 1.05 g
b) approximately 0.22 g

19. Yes. Example: When you multiply the exponents within each expression, both are equal to 2^{24}.

20.
$$\begin{aligned} 2^x + 2^x + 2^x + 2^x &= 256 \\ 2^x(1+1+1+1) &= 256 \\ 2^x &= \frac{256}{4} \\ 2^x &= 64 \\ 2^x &= 2^6 \\ x &= 6 \end{aligned}$$
or
$$\begin{aligned} 2^x + 2^x + 2^x + 2^x &= 256 \\ 2^x(4) &= 256 \\ 2^x(2^2) &= 256 \\ 2^{x+2} &= 2^8 \\ x + 2 &= 8 \\ x &= 6 \end{aligned}$$

21. For $2^2 + 2^3 + 2^4$, use the order of operations to evaluate each power and then add the resulting values: $4 + 8 + 16 = 28$. For $(2^2)(2^3)(2^4)$, since the powers have a common base, you can multiply by adding the exponents : $2^9 = 512$.

22. Example: calculating student enrollment at schools in the community.

a) You would use a positive exponent to predict enrollment in future years beyond the current year.

b) You would use a negative exponent to calculate student enrollment in years before the current year.

4.3 Rational Exponents

1. a) $a^{\frac{15}{2}}$ **b)** $y^{\frac{5}{6}}$
c) $x^{0.9}$ or $x^{\frac{9}{10}}$ **d)** $a^{0.6}$
e) x^{-4} or $\frac{1}{x^4}$ **f)** 9
g) $\frac{-4x^{\frac{1}{12}}}{3}$ **h)** $-10a^{\frac{11}{10}}$
i) $4a^{1.5}$ or $4a^{\frac{3}{2}}$

2. a) $\frac{1}{a^{\frac{5}{4}}}$ **b)** $\frac{1}{4}$
c) $y^{\frac{2}{3}}$ **d)** $\frac{1}{a^{\frac{7}{8}}}$
e) $a^{1.5}b^3$ or $a^{\frac{3}{2}}b^3$ **f)** $\frac{64x^{\frac{9}{4}}}{125}$
g) $\frac{3y^{\frac{2}{3}}}{2x^{\frac{3}{2}}}$ **h)** $\frac{3x^{\frac{1}{6}}}{5y^{\frac{3}{20}}}$

3. a)
$$\begin{aligned} \left(x^{\frac{2}{3}}\right)^q &= x^{\frac{4}{3}} \\ x^{\frac{2q}{3}} &= x^{\frac{4}{3}} \\ \frac{2q}{3} &= \frac{4}{3} \\ 2q &= 4 \\ q &= 2 \\ \left(x^{\frac{2}{3}}\right)^2 &= x^{\frac{4}{3}} \end{aligned}$$

b)
$$\begin{aligned} \left(x^{\frac{-2}{3}}\right)(x^q) &= x^{\frac{-1}{6}} \\ x^{\frac{-2}{3}+q} &= x^{\frac{-1}{6}} \\ \frac{-2}{3} + q &= \frac{-1}{6} \\ q &= \frac{-1}{6} + \frac{2}{3} \\ q &= \frac{-1}{6} + \frac{4}{6} \\ q &= \frac{3}{6} = \frac{1}{2} \\ \left(x^{\frac{-2}{3}}\right)\left(x^{\frac{1}{2}}\right) &= x^{\frac{-1}{6}} \end{aligned}$$

c) $\dfrac{y^{\frac{2}{3}}}{y^{q}} = y^{\frac{11}{12}}$

$y^{\frac{2}{3} - q} = y^{\frac{11}{12}}$

$\frac{2}{3} - q = \frac{11}{12}$

$-q = \frac{11}{12} - \frac{2}{3}$

$-q = \frac{11}{12} - \frac{8}{12}$

$-q = \frac{3}{12}$

$q = \frac{-3}{12}$

$q = \frac{-1}{4}$

$\dfrac{y^{\frac{2}{3}}}{y^{\frac{-1}{4}}} = y^{\frac{11}{12}}$

d) $(27x^2)^{\frac{1}{3}}(qx^2)^{\frac{-1}{2}} = \dfrac{3}{2x^{\frac{1}{3}}}$

$\left(3x^{\frac{2}{3}}\right)\left(\dfrac{x^{-1}}{q^{\frac{1}{2}}}\right) = \dfrac{3}{2x^{\frac{1}{3}}}$

$\dfrac{3x^{\frac{2}{3}+(-1)}}{q^{\frac{1}{2}}} = \dfrac{3}{2x^{\frac{1}{3}}}$

$\dfrac{3x^{\frac{2}{3}+\left(\frac{-3}{3}\right)}}{q^{\frac{1}{2}}} = \dfrac{3}{2x^{\frac{1}{3}}}$

$\dfrac{3x^{\frac{-1}{3}}}{q^{\frac{1}{2}}} = \dfrac{3}{2x^{\frac{1}{3}}}$

$\dfrac{3}{q^{\frac{1}{2}}x^{\frac{1}{3}}} = \dfrac{3}{2x^{\frac{1}{3}}}$

$q^{\frac{1}{2}} = 2$

$\sqrt{q} = 2$

$q = 4$

$(27x^2)^{\frac{1}{3}}(4x^2)^{\frac{-1}{2}} = \dfrac{3}{2x^{\frac{1}{3}}}$

e) $(5^q)(-3^{-q}) = \dfrac{-125}{27}$

$\left(\dfrac{5^q}{-3^q}\right) = \dfrac{-125}{27}$

$\left(\dfrac{5}{-3}\right)^q = \dfrac{-125}{27}$

$\left(\dfrac{5}{-3}\right)^q = \left(\dfrac{-5^3}{3^3}\right)$

$\left(\dfrac{5}{-3}\right)^q = \left(\dfrac{-5}{3}\right)^3$

$q = 3$

$(5^3)(-3^{-3}) = \dfrac{-125}{27}$

4. **a)** 8 **b)** 9
c) $\frac{1}{32}$ **d)** $\frac{343}{27}$
e) $\dfrac{25x^{\frac{4}{3}}}{4y^2}$ **f)** 5

5. **a)** 0.037 **b)** 512
c) 52.1959 **d)** 11.0553
e) 0.037 **f)** 6.3227

6. **a)** 863 trout **b)** 1409 trout
c) 911 trout **d)** 1264 trout

7. **a)** Error: A common denominator is needed to subtract exponents.

$\dfrac{a^{\frac{2}{3}}}{a^{\frac{1}{4}}} = a^{\frac{2}{3} - \frac{1}{4}}$

$= a^{\frac{8}{12} - \frac{3}{12}}$

$= a^{\frac{5}{12}}$

b) Errors: The negative exponent needs to be converted to a positive exponent. The expression $16^{0.5}$ is equal to 4, not 8.

$\left(16y^{-6}\right)^{-0.5} = (16)^{-0.5}(y^{-6})^{-0.5}$

$= \dfrac{1}{16^{0.5}}y^{(-6)(-0.5)}$

$= \dfrac{1}{4}y^3$

8. **a)** \$1651.05 **b)** \$1624.86

9. **a)** 1.5 represents the growth rate; 1000 represents the starting population
b) approximately 2646 bacteria
c) approximately 614 bacteria

10. **a)** 35.600 million people
b) 33.155 million people

11. **a)** $A = 28(0.5)^{\frac{t}{20}}$

$= 28(0.5)^{\frac{45}{20}}$

$= 5.886\,274\ldots$

After 45 min, approximately 5.89 g remain.

b) $A = 28(0.5)^{\frac{t}{20}}$

$= 28(0.5)^6$

$= 0.4375$

After 2 h, approximately 0.44 g remain.

c) $A = 28(0.5)^{\frac{t}{20}}$

$= 28(0.5)^{\frac{195}{20}}$

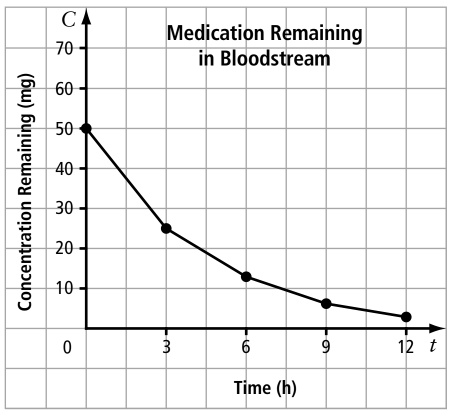
$= 0.032\,58$

After $3\frac{1}{4}$ h, approximately 0.03 g remain.

12. a)

Time (t)	0	3	6	9	12
Concentration (C)	50	25	12.5	6.25	3.125

b)



c) $t = 24$ h **d)** $A = 0.003\,05$ mg

13. 81.92 g

14. a) \$3432.14 **b)** \$759.57

15.
$$4^{\frac{1}{2}} + 4^{\frac{1}{2}} + 4^{\frac{1}{2}} + 4^{\frac{1}{2}} = 4^x$$
$$4^{\frac{1}{2}}(1 + 1 + 1 + 1) = 4^x$$
$$4^{\frac{1}{2}}(4) = 4^x$$
$$4^{\frac{1}{2}+1} = 4^x$$
$$4^{\frac{1}{2}+\frac{2}{2}} = 4^x$$
$$4^{\frac{3}{2}} = 4^x$$
$$x = \frac{3}{2}$$

4.4 Irrational Numbers

1. a) $(\sqrt[3]{5})^2$ **b)** $(\sqrt[4]{8})^3$
c) $(\sqrt[5]{6})^3$ **d)** $\sqrt{81}$
e) $\frac{1}{9^{\frac{5}{3}}} = \left[\left(\frac{1}{9}\right)^{\frac{1}{3}}\right]^5 = \left(\sqrt[3]{\frac{1}{9}}\right)^5$
f) $\sqrt[4]{x^3}$ **g)** $(\sqrt[3]{a})^2$
h) $\left(\sqrt[3]{\frac{x}{y}}\right)^2$

2. a) $3^{\frac{3}{4}}$ **b)** $(5t)^{\frac{4}{3}}$
c) $x^{\frac{2}{3}}$ **d)** $\left(\frac{a^2}{b^3}\right)^{\frac{1}{5}}$ or $\frac{a^{\frac{2}{5}}}{b^{\frac{3}{5}}}$
e) $y^{\frac{5}{6}}$ **f)** $2^{\frac{3}{a}}$

3. a) 0.5 **b)** 4
c) 10.3923 **d)** 1.25
e) 4.5861 **f)** 0.7274

4. a) $4\sqrt{5} = \sqrt{(4^2)}\sqrt{5}$
$= \sqrt{(16)(5)}$
$= \sqrt{80}$
b) $\sqrt{36}$
c) $\sqrt{325}$ **d)** $\sqrt{384.4}$
e) $\sqrt{174.24}$ **f)** $\sqrt{\frac{10}{25}}$ or $\sqrt{\frac{2}{5}}$

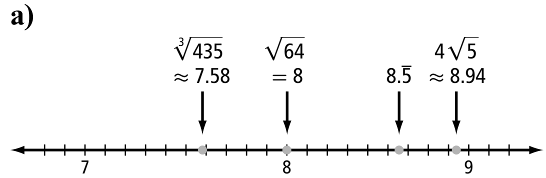
5. a) $\sqrt[3]{135}$ **b)** $\sqrt[3]{1029}$
c) $\sqrt[3]{750}$ **d)** $\sqrt[4]{112}$
e) $\sqrt[3]{\frac{5}{8}}$ **f)** $\sqrt[4]{50.625}$

6. a) $4\sqrt{2}$ **b)** $2\sqrt{11}$
c) $3\sqrt{10}$ **d)** $4\sqrt{5}$
e) $6\sqrt{10}$ **f)** $5\sqrt{19}$

7. a) $2\sqrt[3]{6}$ **b)** $2\sqrt[3]{15}$
c) $3\sqrt[3]{12}$ **d)** $2\sqrt[4]{3}$
e) $3\sqrt[4]{5}$ **f)** $2\sqrt[4]{13}$

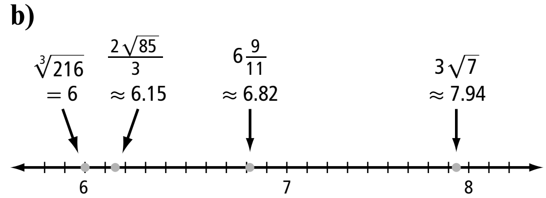
8. a) $0.\overline{7}$, $\frac{3}{4}$, $0.5\sqrt{2}$, $\sqrt{0.49}$; $0.5\sqrt{2}$ is an irrational number.
b) $\sqrt[3]{0.343}$, $\frac{2}{3}$, 0.62, $\sqrt{0.38}$; $\sqrt{0.38}$ is an irrational number.

9. a)



$\sqrt[3]{435}$ and $4\sqrt{5}$ are irrational numbers.

b)



$\frac{2\sqrt{85}}{3}$ and $3\sqrt{7}$ are irrational numbers.

10. approximately 10.093 cm

11. approximately 16.54 cm

12.
$$V = s^3$$
$$(1.3)(10^9) = s^3$$
$$\sqrt[3]{(1.3)(10^9)} = s$$
$$1091 = s$$
The edge length of a cube that contained Earth's estimated total volume of water would be approximately 1091 km.

13. a) approximately 3.16 cm
b) approximately 7.68 cm

14. 2.72 s

15. approximately 86 cm

16. a) solution B: 0.15 M
b) solution A: 0.73 M

17. a) approximately 110 m
b) approximately 38 013.3 m^2

18.
$$SA = 2\pi\left[h\left(\sqrt{\frac{V}{\pi h}}\right) + \left(\frac{V}{\pi h}\right)\right]$$
$$= 2\pi\left[26\left(\sqrt{\frac{26465}{26\pi}}\right) + \left(\frac{26465}{26\pi}\right)\right]$$
$$= 2\pi[26(18.000076) + (324.002736)]$$
$$= 2\pi[468.001976 + 324.002736]$$
$$= 2\pi[792.004712]$$
$$= 4976.31237$$

The surface area of the cylinder is 4976 m^2.

19. a) 2 **b)** 5

c)
$$\sqrt{4 + \sqrt{19 + \sqrt{36}}} = \sqrt{4 + \sqrt{19 + 6}}$$
$$= \sqrt{4 + \sqrt{25}}$$
$$= \sqrt{4 + 5}$$
$$= \sqrt{9}$$
$$= 3$$

d)
$$\sqrt[4]{13 + \sqrt[3]{22 + \sqrt[3]{125}}} = \sqrt[4]{13 + \sqrt[3]{22 + 5}}$$
$$= \sqrt[4]{13 + \sqrt[3]{27}}$$
$$= \sqrt[4]{13 + 3}$$
$$= \sqrt[4]{16}$$
$$= 2$$

20. a)
$$\sqrt[3]{\sqrt{7}} = (\sqrt{7})^{\frac{1}{3}}$$
$$= \left(7^{\frac{1}{2}}\right)^{\frac{1}{3}}$$
$$= 7^{\frac{1}{6}}$$

b)
$$\sqrt[4]{\sqrt[3]{5^2}} = \left(\sqrt[3]{5^2}\right)^{\frac{1}{4}}$$
$$= \left[(5^2)^{\frac{1}{3}}\right]^{\frac{1}{4}}$$
$$= 5^{(2)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)}$$
$$= 5^{\frac{1}{6}}$$

c) $\left(\frac{1}{8}\right)^{\frac{1}{10}}$ **d)** $\left(\frac{2}{5}\right)^{\frac{1}{2}}$

21. The expression $\sqrt[4]{x^3}$ does not have a solution when x is negative. It is not possible to determine the even root of a negative number. Example: The expression $\sqrt[4]{x^3} = \sqrt[4]{(-3)^3} = \sqrt[4]{-27}$ has no solution.

22. The expression $\sqrt[3]{x^4}$ always has a solution because when you raise a negative number to an even exponent, the result is always a positive number and then it is possible to take the cube root of the positive number.

23. a) Example: For all non-perfect squares, the calculator screen shows 9 decimals.
b) Yes. Example: The square root of each non-perfect square is an irrational number since it cannot be expressed as a terminating or a repeating decimal.