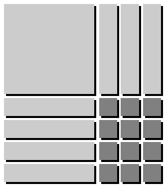
Chapter 5 Polynomials

5.1 Multiplying Polynomials

1. **a)** $3x^2 - 5x + 2$; $(3x - 2)(x - 1)$
b) $2x^2 + x - 6$; $(2x - 3)(x + 2)$

2. **a)**


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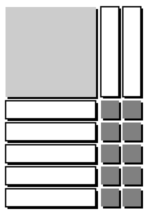

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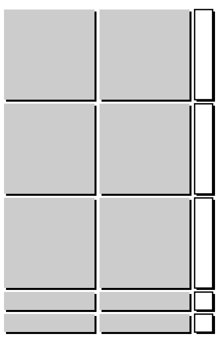

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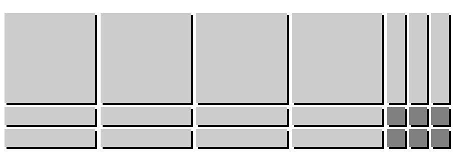

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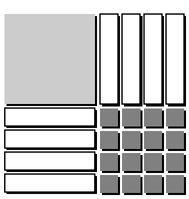
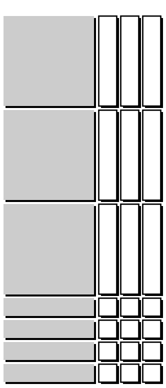


f)


3. **a)** $2x^2 - 4x - 16$
b) $t^2 + 9t + 20$
c) $6w^2 - 23w - 18$
d) $z^2 - 4$
e) $a^2 + 2ab + b^2$
f) $30e^2 + 25e - 5$

4. **a)** E **b)** H
c) A **d)** G
e) B **f)** C
g) D **h)** F

5. **a)** A **b)** D
c) C **d)** B
e) B **f)** A

6. **a)** $2d^3 + 11d^2 + 13d - 6$
b) $4s^3 - 41s^2 + 41s + 5$
c) $5k^3 - k^2 + 7k$
d) $3c^3 + 18c^2 + 45c + 42$
e) $10y^4 + 8y^3 - 32y^2 + 6y$
f) $(r^2 - 5r - 3)(3r^2 - 4r - 5)$
$= r^2(3r^2 - 4r - 5) - 5r(3r^2 - 4r - 5)$
$-3(3r^2 - 4r - 5)$
$= 3r^4 - 4r^3 - 5r^2 - 15r^3 + 20r^2 + 25r$
$- 9r^2 + 12r + 15$
$= 3r^4 - 4r^3 - 15r^3 - 5r^2 - 9r^2 + 20r^2$
$+ 25r + 12r + 15$
$= 3r^4 - 19r^3 + 6r^2 + 37r + 15$

7. **a)** $120y^3 - 68y^2 - 144y - 36$
b) $8a^2 - 25a + 47$
c) $12d^2 - 32de - 15e^2$
d) $9n^2 - 4n + 58$
e) $6w^4 - 13w^3 - 15w^2 - 26w - 24$
f) $2(4t + 5s)(2t - 3s) - (5t - s)$
$= 2[4t(2t - 3s) + 5s(2t - 3s)] - 5t + s$
$= 2[8t^2 - 12st + 10st - 15s^2] - 5t + s$
$= 2[8t^2 - 2st - 15s^2] - 5t + s$
$= 16t^2 - 4st - 30s^2 - 5t + s$
$= 16t^2 - 5t - 4st + s - 30s^2$

8. **a)** $8a^2 + 5a + 1$
b) $4b^2 + 6b + 21$
c) $5x^2 - 5xy + 5y^2$
d) $23a^2 - 68ac - 28c^2$
e) $2x^4 - x^3 - 4x^2 + 17x - 12$
f) $12b^2 - 18bd - 5d^2$

9. **a)** Step 2;
$28t^2 - 33t - 7$
b) Step 3;
$2xy^2 + x^2y + xy - 3x$

10. a) The dimensions of the deck and the pool are $(x + 4)$ by $(x + 4)$.
The area of the deck and pool is $(x + 4) \times (x + 4) = (x + 4)^2$.
$$(x + 4)^2 = x(x + 4) + 4(x + 4) = x^2 + 4x + 4x + 16 = x^2 + 8x + 16.$$
b) The area of the pool is 49 m^2. If $x^2 = 49$, then $x = \sqrt{49} = 7$.
The area of pool and deck is $x^2 + 8x + 16$.
Using the value for x, the total area is $(7)^2 + 8(7) + 16 = 49 + 56 + 16 = 121$.
Thus, the area of the deck and the pool is 121 m^2.

11. a) $A(5x + 6)(2x + 4) = 10x^2 + 32x + 24$
b) 920 in.^{2}

12. a) $A = x^2 + 8x + 12$
b) The area of the diamond is one half the area of the rectangle.

13. a) $(5x - 2)(2x + 1)$
b) $(x - 1)(x + 3)$
c) $(5x - 2)(2x + 1) - (x - 1)(x + 3)$, $9x^2 - x + 1$

14. a) length: $30 - 2x$; width: $20 - 2x$; height: x
b) $x(30 - 2x)(20 - 2x)$
c) $600x - 100x^2 + 4x^3$

15. a) 4 cm × 5 cm × 6 cm = 120 cm^3
b) $V = n(n + 1)(n + 2)$
c) One way:
10 cm × 11 cm × 12 cm = 1320 cm^3
Another way:
$V = n(n + 1)(n + 2)$
$V = n^3 + 3n^2 + 2n$
$V = (10)^3 + 3(10)^2 + 2(10)$
$V = 1000 + 300 + 20$
$V = 1320$ cm^3

16. a) The square of the middle number is 4 greater than the product of the first and third numbers.
b) $(x - 2)$ and $(x + 2)$
c) $(x - 2)(x + 2) = x(x + 2) - 2(x + 2)$
$(x - 2)(x + 2) = x^2 + 2x - 2x - 4$
$(x - 2)(x + 2) = x^2 - 4$
$x^2 = (x - 2)(x + 2) + 4$

17. a)

Table A	
Numbers	Total
6, 7	42
7, 8	56
8, 9	72
9, 10	90
10, 11	110

Table B				
Numbers			Total	
5	25	15	2	42
6	36	18	2	56
7	49	21	2	72
8	64	24	2	90
9	81	27	2	110

b) $(n + 1)(n + 2) = n^2 + 3n + 2$

18. a) $(n + 3)(n + 2) - n(n + 1)$
b) $4n + 6$
c) $12 - 2 = 10, 4(1) + 6 = 10; 20 - 6 = 14, 4(2) + 6 = 14; 30 - 12 = 18, 4(3) + 6 = 18; 42 - 20 = 22; 4(4) + 6 = 22$

5.2 Common Factors

1. a) 10: 1, 2, 5, 10; 15: 1, 3, 5, 15; GCF: 5
b) 24: 1, 2, 3, 4, 6, 8, 12, 24; 36: 1, 2, 3, 4, 6, 9, 12, 18, 36; GCF: 12
c) 16: 1, 2, 4, 8, 16; 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48; GCF: 16
d) 40: 1, 2, 4, 5, 8, 10, 20, 40; 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60; GCF: 20
e) 18: 1, 2, 3, 6, 9, 18; 45: 1, 3, 5, 9, 15, 45; GCF: 9
f) 14: 1, 2, 7, 14; 24: 1, 2, 3, 4, 6, 8, 12, 24; GCF: 2

2. a) $6x^2 = (2)(3)(x)(x)$;
$12x = (2)(2)(3)(x)$
b) $20c^2d^3 = (2)(2)(5)(c)(c)(d)(d)(d)$;
$30cd^2 = (2)(3)(5)(c)(d)(d)$
c) $4b^2c^3 = (2)(2)(b)(b)(c)(c)(c)$;
$6bc^2 = (2)(3)(b)(c)(c)$
d) $18xy^2z = (2)(3)(3)(x)(y)(y)(z)$;
$24x^2y^3z^2 = (2)(2)(2)(3)(x)(x)(y)(y)(y)(z)(z)$
e) $5m^3n = (5)(m)(m)(m)(n)$;
$20mn^2 = (2)(2)(5)(m)(n)(n)$

3. a) 2, 3, x; GCF: $(2)(3)(x) = 6x$
b) 2, 5, c, d; GCF: $(2)(5)(c)(d)(d) = 10cd^2$
c) 2, b, c; GCF: $(2)(b)(c)(c) = 2bc^2$
d) 2, 3, x, y, z;
GCF: $(2)(3)(x)(y)(y)(z) = 6xy^2z$
e) 5, m, n; GCF: $(5)(m)(n) = 5mn$

4. a) 7 **b)** $-5n$
c) 1 **d)** $4fg^2$
e) $-15de$ **f)** $9j^2k$

5. a) 80 **b)** 120
c) $18x$ **d)** $12t^3$
e) $2ab$ **f)** $504c^3d^2e^3$

6. a) $6(s + 5)$ **b)** $4(t + 7)$
c) $5(a - 1)$ **d)** $4r(4r - 3)$
e) $7x(y + 2y - 7z)$ **f)** $3(c^3 - 3c^2 - 9d^2)$

7. a) $3w - 1$ **b)** $2a^2$
c) $x^2y - 5x$ **d)** $g + 2$
e) $5xy$ **f)** $2r$

8. a) $(x - 6)$ **b)** $(a + 3)$
c) $(d - 9)$ **d)** $ab(b + 2)$
e) $x(x + 2)$ **f)** $2m(n - 1)$

9. a) $(s + 5)(s - 2)$ **b)** $(r - 7)(r - 4)$
c) $(g + 6)(g + 9)$ **d)** $(p + 3)(p + 4)$
e) $(b - 3)(b - 7)$ **f)** $(r - 3)(r + 2s)$

10. a) To find the largest number of centrepieces, identify the GCF of 36, 48, and 60.
The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.
The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.
The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.
The GCF is 12, so that factor represents the largest number of centrepieces.
To determine how many of each type of flower per centrepiece, divide the number of each flower by 12:
$\frac{36}{12} = 3, \frac{48}{12} = 4, \frac{60}{12} = 5$
Each centrepiece will contain three roses, four daffodils, and five tulips.
b) The cost of each centrepiece will be the unit cost of each flower multiplied by the number of each flower added together:
$3 \times \$2.50 + 4 \times \$1.70 + 5 \times \$1.50 = \$21.80$
Therefore, the cost of each centrepiece is \$21.80.

11. a) no; $6(2x - 1)$ **b)** no; $-10w(w + 1)$
c) yes **d)** no; $(x + 3)(x + 2y)$
e) yes **f)** yes

12. Examples:
a) $4x^2 + 8x$
b) $6r^2s^2 + 9rs$
c) $10m^2n^2 + 15m^3n^3$
d) $a^3b^3 + 2a^2b^2 + 3ab$
e) $4c^4d^4 + 6c^3d^3 + 8c^2d^2$
f) $2e^4 + 6e^3 + 8e^2 + 4e$
g) $ac + 4a - bc - 4b$

13. a) 4 by $(x - 1)$; $4(x - 1) = 4x - 4$
b) $(x - 2)$ by $(x + 3)$;
$(x - 2)(x + 3) = x^2 + 3x - 2x - 6$
$= x^2 + x - 6$
c) $(2x - 3)$ by $(x + 2)$;
$(2x - 3)(x + 2) = 2x^2 + 4x - 3x - 6$
$= 4x^2 + x - 6$

14. a) The length is $t - 3 - 3 = t - 6$ and the width is $s - 3 - 3 = s - 6$.
b) Substituting 10 cm for t, and 8 cm for s, the dimensions are $(10 - 6)$ by $(8 - 6)$ or 4 by 2. So, the area is 4 cm × 2 cm = 8 cm^2. Multiplying the binomials to find the area first, and then substituting, is another way to find the area:
$(t - 6)(s - 6) = ts - 6t - 6s + 36$
$= (10)(8) - (6)(10) - (6)(8) + 36$
$= 80 - 60 - 48 + 36$
$= 8$ cm^2

15. a) 6" by 3"
b) 3" by 3" (22 servings)

16. 30 and 90, 45 and 60. The only other number with a GCF of 15 less than 30 is 15. The other number has to be greater than 100, because 90 is already close to 100.

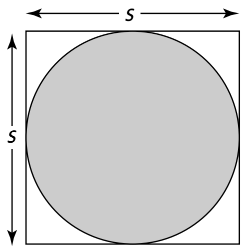
17. 5

18. a) $A = \pi r^2 + \pi(r + 3)^2 + \pi(r + 6)^2$;
Other algebraic expressions are possible, depending on which radius is assigned the value of r; examples:
$A = \pi r^2 + \pi(r - 3)^2 + \pi(r + 3)^2$ or
$A = \pi r^2 + \pi(r - 3)^2 + \pi(r - 6)^2$
b) $A = 3\pi(r^2 + 6r + 15)$

19. $8x^5 - 16x^3 + 24x$

20. 1948 and 2922

21. a)



b) $A = \pi\left(\frac{s}{2}\right)^2 = \frac{\pi s^2}{4}$

c) $A = s^2 - \frac{\pi s^2}{4}$;

$A = s^2\left(1 - \frac{\pi}{4}\right)$

22. a) 3 s

b) $15t - 5t^2 = 5t(3 - t)$. The product equals 0 when $t = 0$ or when $t = 3$. Because the ball is at its initial height when the product is 0, it can be seen that the ball will be at the same height when $t = 3$. Thus, factoring simplifies the process used to calculate the answer in part a).

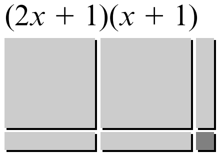
23. a) $5x$

b) The width is 10 cm, the length is 15 cm and the volume is 900 cm^3.

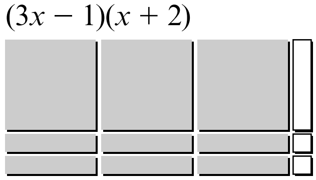
5.3 Factoring Trinomials

1. **a)** $x^2 + 5x + 6$; $x + 2$ by $x + 3$
b) $x^2 - 9$; $x - 3$ by $x + 3$
c) $x^2 - 3x - 4$; $x - 4$ by $x + 1$
d) $x^2 - x - 6$; $x - 3$ by $x + 2$

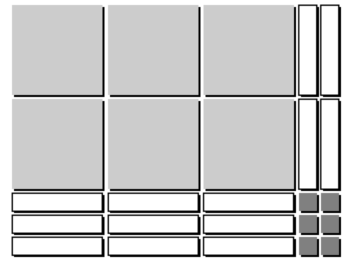
2. **a)** $(2x + 1)(x + 1)$



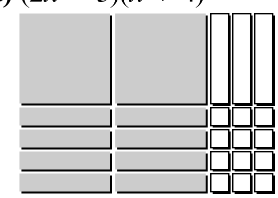
b) $(3x - 1)(x + 2)$



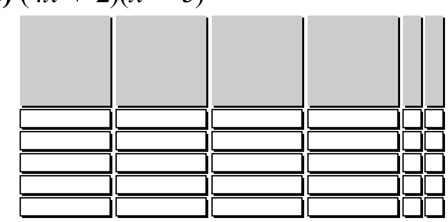
c) $(3x - 2)(2x - 3)$



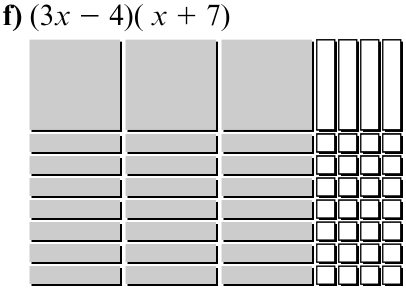
d) $(2x - 3)(x + 4)$



e) $(4x + 2)(x - 5)$



f) $(3x - 4)(x + 7)$



3. **a)** 2, 6 **b)** not possible
c) −4, 1 **d)** −8, −3
e) −2, 21 **f)** not possible

4. **a)** $(y + 6)(y + 2)$ **b)** $(x + 3)(x + 7)$
c) $(a - 10)(a - 9)$ **d)** not possible
e) $(m - 7n)(m + 6n)$ **f)** $(b + 2)(b + 17)$

5. **a)** $(g - 4)(g - 6)$ **b)** $(n - 2)(n - 13)$
c) $(c - 8)(c - 7)$ **d)** $(s - 2t)(s - 5t)$
e) not possible **f)** $(3v - 2)(v + 1)$

6. **a)** $(2r + 7)(r + 2)$ **b)** $(2l + 3)(l + 4)$
c) $(3w + 6)(w + 1)$ **d)** not possible
e) $(y + 3z)(y + 2z)$ **f)** $(3a + 4)(4a + 1)$

7. **a)** $(2f - 3)(f + 5)$ **b)** $(r - 10)(r + 11)$
c) not possible **d)** $(5m - n)(2m - 3n)$
e) not possible **f)** $(3g - 2f)(3g - f)$
g) $2(l + 3)(3l + 7)$ **h)** $(5a - 7)(a - 9)$

8. Examples:
a) 5, 7, −5, −7
b) 2, 14, −2, −14
c) 9, 11, 19, −9, −11, −19
d) 5, 13, −5, −13

9. Examples:
a) −5, 5, −1, 1
b) −9, 9, −6, 6
c) −57, 57, −30, 30, −18, 18, −15, 15
d) −19, 19, −1, 1, −8, 8

10. Examples:
a) There are four sets of two integers with the product of −10: −1 and 10, 1 and −10, −2 and 5, and 2 and −5. Possible values of p are the sums of each set: −1 + 10, 1 + −10, −2 + 5, and 2 + −5. So, the possible values of p are 9, −9, 3, and −3.
b) −4, 4
c) 9, 15
d) 12, 36

11. **a)** 12 **b)** 28
c) 7 **d)** −12
e) 21 **f)** 6

12. **a)** width = $2x - 6$; length = $3x + 8$
b) 18 yards by 44 yards

13. **a)** $A = x^2 + 11x + 24 = (x + 8)(x + 3)$;
width: $x + 3 = (12) + 3 = 15$ cm;
length: $x + 8 = (12) + 8 = 20$ cm
b) $A = 8x^2 + 6x - 2 =)(2x + 2)(4x - 1)$;
width: $2x + 2 = 2(12) + 2 = 26$ cm;
length: $4x - 1 = 4(12) - 1$
$= 48 - 1$
$= 47$ cm
c) $A = x^2 + 3x - 10 = (x - 2)(x + 5)$;
width: $x - 2 = (12) - 2 = 10$ cm;
length: $x + 5 = (12) + 5 = 17$ cm

14. **a)** $(-6t - 3)(t - 5)$
b) 27 ft

15. **a)** $h = (x + 6)$, $b = (x + 7)$;
$h = 24$ cm, $b = 25$ cm
b) $h = (2x + 3)$, $b = (3x - 1)$;
$h = 39$ cm, $b = 53$ cm

16. 7 by 1 and 6 by 4

17. 14

18. **a)** square
b) 16 and 36 are squares.
c) $(4s - 6)(4s - 6) = (4s - 6)^2$

19. **a)** square
b) $(x + 3)(x + 3)$
c) The area of the second figure is four times the area of the original square, meaning that the side dimension is doubled: $2(x + 3) = 2x + 6$.

20. Example: In trinomials such as $n^2 - 20n - 44$, one needs to find two numbers whose sum is −20 and whose product is −44. The important thing to notice to make a connection between the two types of trinomials is that the product −44 comes from multiplying the coefficient for n, which is 1, by the final term in the trinomial. For trinomials such as $6n^2 + 13n - 5$, one must ask what two numbers have a product of −30 (because $6 \times -5 = -30$) and a sum of 13. These two numbers are used to break up the middle term. Then, factoring by grouping completes the process.

21. **a)** 5, 6
b) $(x + m)(x + n)$
$= x^2 + nx + mx + mn$
$= x^2 + (n + m)x + mn$
c) 30, 13
d) $(ax + m)(x + n)$
$= ax^2 + anx + mx + mn$
$= ax^2 + (an + m)x + mn$

22. **a)** x by $2x - 5$ by $3x + 1$
b) dimensions: 5 cm by 5 cm by 16 cm; volume: 400 cm^3

5.4 Factoring Special Trinomials

1. **a)** $(x - 2)^2$ **b)** $(x + 3)^2$
c) $(x + 2)(x - 2)$ **d)** $(3x - 2)^2$

2. **a)** $x^2 - 25$ **b)** $9r^2 - 16$
c) $5w^2 - 180$ **d)** $4b^2 - 49c^2$
e) $16x^2 - 36y^2$ **f)** $2x^2y - 18y$

3. **a)** $y^2 + 10y + 25$
b) $9d^2 + 12d + 4$
c) $16m^2 - 40mp + 25p^2$
d) $2e^2 - 24ef + 72f^2$
e) $12z^2 - 48z + 48$
f) $4x^2 - 12xy + 9y^2$

4. **a)** $n^2 - 10n + 25 = (n - 5)^2$
b) $r^2 - s^2 = (r + s)(r - s)$
c) $9c^2 - 16d^2 = (3c - 4d)(3c + 4d)$
d) $4s^2 + 24s + 36 = (2s + 6)^2$
e) $4x^2 + 8x + 4 = (2x + 2)^2$
f) $(4x - 2)^2 = 16x^2 - 16x + 4$

5. **a)** $(a + 10)(a - 10)$
b) $(t + 7)(t - 7)$
c) not possible
d) $(8 + h)(8 - h)$
e) $2(5g + 6h)(5g - 6h)$
f) $3(3p^2 - 5r^2)$
g) not possible
h) $2(6g + 4h)(6g - 4h)$

6. **a)** $(y + 6)^2$
b) $(x - 3)^2$
c) $2(z + 3)^2$
d) not possible
e) $-4(b^2 + 12b - 36)$
f) $(3s + 8)^2$
g) $(5n - 11)^2$
h) not possible

7. **a)** $16(d + 2e)(d - 2e)$
b) $3(3m + 4)(3m - 4)$
c) $-2(k + 6)^2$
d) $3c(c^2 + 17c + 49)$
e) $25(2a + b)(2a - b)$
f) $st(s - 9)^2$
g) $(9g^2 + 4)(3g + 2)(3g - 2)$
h) $3l(2m + n)^2$

8. **a)** $(2a - b)(2a + b)$
b) $(3x + 1)^2$
c) $9(24 - y^2)$
d) $d^2 - 4e^2 = (d + 2e)(d - 2e)$
e) $(7 - h)^2$

9. **a)** $-2, 2$ **b)** $-24, 24$
c) $-60, 60$ **d)** $-48, 48$

10. **a)** $3x^2 + 24x + 48 = 3(x^2 + 8x + 16)$
$= 3(x + 4)^2$
b) $3(x + 4) = 3x + 12$; The dimensions are $x + 4$ by $3x + 12$.
c) Since $x = 5$, the width is $(5) + 4 = 9$ cm and the length is $3(5) + 12 = 27$ cm.
d) Area $= 9 \times 27 = 243$ cm^2; check: $3(5)^2 + 24(5) + 48 = 3(25) + 120 + 48 = 243$

11. **a)** $16^2 - 4^2 = (16 + 4)(16 - 4) = (20)(12)$
$= 240$
b) $7^2 - 27^2 = (7 + 27)(7 - 27)$
$= (34)(-20) = -680$
c) $45^2 - 15^2 = (45 + 15)(45 - 15)$
$= (60)(30) = 1800$
d) $113^2 - 13^2 = (113 + 13)(113 - 13)$
$= (126)(100) = 12\,600$

12. **a)** Area $= \pi(r + 5)^2 - \pi(r + 3)^2 + \pi r^2$
b) Area $= \pi(r^2 + 4r + 16)$
c) Area $= 28\pi = 28(3.14) = 87.9$ cm^2

13. $2r(r - 1)^2(r + 1)^2$; solution:
$2r^5 - 4r^3 + 2r = 2r(r^4 - 2r^2 + 1)$
$= 2r(r^2 - 1)^2$
$= 2r(r^2 - 1)(r^2 - 1)$
$= 2r(r + 1)(r - 1)(r + 1)(r - 1)$
$= 2r(r - 1)^2(r + 1)^2$

14. $(x + 2y)$, $(x - 2y)$, and $(xy - 4)$

15. **a)** 391, 775
b) $(x - 3)(x + 3) = x(x - 3) + 3(x - 3)$
$(x - 3)(x + 3) = x^2 - 3x + 3x - 9$
$(x - 3)(x + 3) = x^2 - 9$
$x^2 = (x - 3)(x + 3) + 9$

16. $9b^2 - 12b$

17. **a)** $A = \pi r^2$
b) $A = \pi(r + 3)^2 = \pi(r^2 + 6r + 9)$
$= \pi r^2 + 6\pi r + 9\pi$
c) Area of walkway = (area of walkway + area of garden) − area of garden
Area of walkway $= (\pi r^2 + 6\pi r + 9\pi) - \pi r^2 = 6\pi r + 9\pi = 3\pi(2r + 3)$
d) $3\pi[2(8) + 3] = 3\pi(19) = 179.1$ m

18. **a)** 144, 143, 169, 168, 196, 195, $15^2 = 225$, $14 \times 16 = 224$
b) The product of the factors that are 1 less and 1 more than the squared number is 1 less than the product of the squared number, the difference of squares equation.
c) $(n - 1)(n + 1) = n^2 - 1$

19. **a)** 600 cm^2
b) The difference between $a^2 + b^2$ and $(a + b)^2$ is $2ab$. So, the difference between $15^2 + 20^2$ and $(15 + 20)^2$ is $2(15 \times 20)$ which equals 600.