Chapter 6 Linear Relations and Functions

6.1 Graphs of Relations

1. Examples:

a) a straight line starting at the origin and rising gently to the right

b) a straight line starting at a point on the distance axis (y-axis) above the origin and descending rapidly to the right to the time axis (x-axis)

c) a curved line starting at the origin and rising to the right quickly at first and then becoming less and less steep, but always moving further from the time axis

d) a straight horizontal line quite high above the time axis

e) an inverted V, starting at the origin, with the first segment steeper than the second

f) a straight line starting at the origin and rising to the right, and then becoming a horizontal line leading to the right side of the graph

2. Examples:

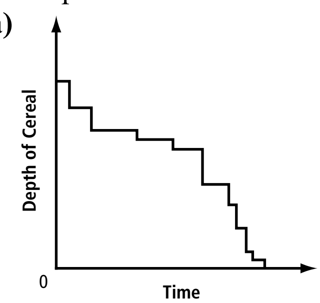
a) Graph A: the lines are straight (suggesting constant rate of change), have the same steepness, and are increasing; Graph B: the lines are straight and have the same steepness; Graph C: the lines are straight, start at the same point on the vertical axis, and are increasing; Graph D: the lines start at the same point on the vertical axis and are increasing

b) Graph A: each line starts at a different point on the vertical axis; Graph B: each line starts at a different point on the vertical axis and one line is increasing, while the other is decreasing; Graph C: the lines have different steepness; Graph D: one line is straight (suggesting constant rate of change) while the other is curved (suggesting a decreasing rate of change)

c) Graph A: The cost to rent DVDs versus the number of DVDs rented. The upper line includes a membership fee and the lower one has no extra fee. The costs of individual DVD rentals are the same for both lines.; Graph B: The increasing line could represent the number of pages read in a book versus time. The decreasing line, then, is the number of pages left to read.; Graph C: The steeper line could represent the cost of removing computer viruses versus the time taken to do the job. The less steep line could represent the same scenario, but at a lower hourly rate.; Graph D: The straight line could represent the height of a jet after takeoff versus time. The curved line could be the height of a smaller, less powerful, private plane.

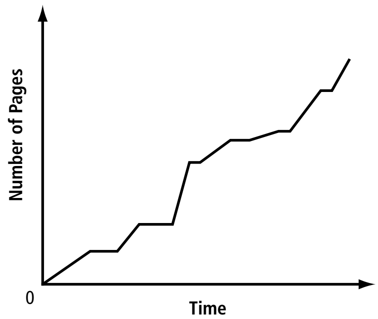
3. Examples:

a)



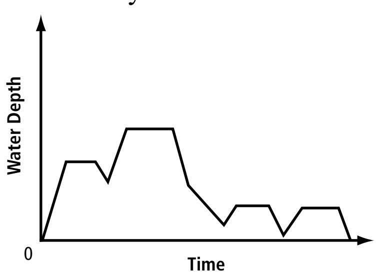
The height of each "step" on the graph illustrates the amount that the cereal goes down with each spoonful. The width of each step is the time between mouthfuls. This graph suggests that the eater starts of at a leisurely pace, perhaps while reading the paper. However, the size of spoonful increases and the time between mouthfuls decreases as if the eater has suddenly realized that the school bus is coming. The point at the x-axis is where the cereal is gone.

b)



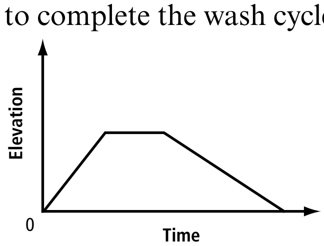
The number of pages read over time changes with the pace of reading. The steeper the slope, the faster the person is reading. At one point, where the slope is very steep the person may even be skimming the text. Where the slope is almost flat, the person is reading very slowly and carefully. The horizontal lines are places are periods where the reader may not even be reading.

c)



This graph shows that the water rises when the washing machine is turned on. The water reaches a particular height, where the clothes soak. The level of the water level dips as the more of the clothes slide into the water, absorbing some of it. At the end of the soaking cycle, more water is added to the tub and the water begins to agitate for a period of time. Then, all the water is drained and fresh water is added to rinse the clothes. This rinse cycle is repeated and then the water drains out to complete the wash cycle.

d)



This graph shows a quick elevation change as the airplane takes off. The plane then levels out. Notice that this is a short flight, so the plane is at cruising altitude for not much longer than it took to get to that elevation. The plane then begins a slower descent to its destination.

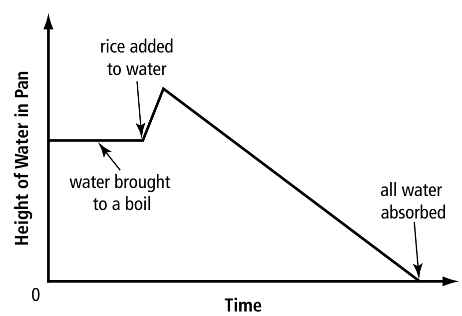
4. Examples:
Using line segments: (constant change theme)
- graphing water used each time a toilet is flushed
- buying grapes at a fixed price per kilogram

Using curves: (changing rate occurring)
- driving in traffic that is speeding up and slowing down
- elevation of a road through a mountain park

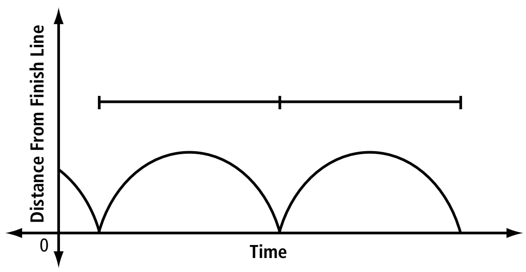
5. **a)** line segments and curves follow a similar path

b) The person graphing may have realized that the relation had more constant increases and decreases, which are best represented by line segments.

6.

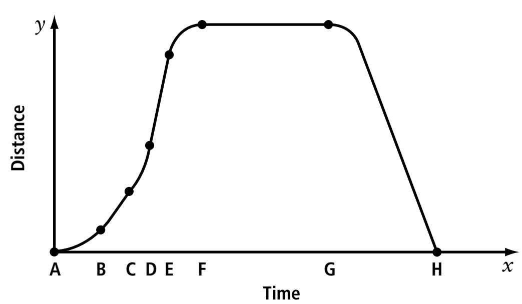


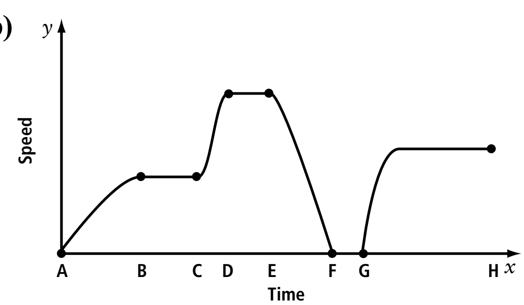
The height of the water rises sharply as the rice is added. This rise is not exactly vertical because it takes some time, even though it very little, to add the rice. As the rice absorbs the water and some evaporates, the water level decreases steadily. When the line reaches the x-axis, all the water has been absorbed and there is none left in the pan.

7. 

Example: The symmetry occurs because the skater moves away from and then returns to the finish line repeatedly and keeps the same pace. If the skater's pace varied, the distance between the points where the graph meets the x-axis would change.

8. **a)** Example: Section B to C should not be horizontal; it should be increasing, but not as steeply as in D to E. Section F to G should not be decreasing; it should be horizontal. Section G to H should not be decreasing to the left. If this were the case, the car would be in two different places at the same time between times A and G. Rather, it should decrease to the right with steepness between that in B to C and D to E.



b) 

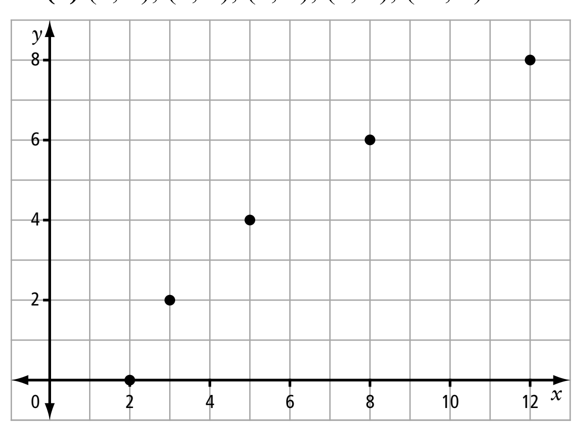
9. **a)** Example: The darker line is the revenue and the lighter line is cost. We know this because there are 0 revenues when 0 belts have been made and sold. Also, the dark line exceeds the lighter line after 500 belts. The distance between the lines is the profit. If the darker line represented cost, the company would lose money as it sold more belts, which does not make sense.
b) \$4000.00
c) 500
d) \$9000.00
e) \$4000.00
f) \$4000.00
g) \$18.00

10. Example: −40 °C = −40 °F; one degree F is smaller than one degree C; 0 °C = 32 °F (water freezes); 100 °C = 212 °F (water boils); as the temperature in C rises, so does the temperature in F; the F scale is positive from about −18° C to 0° C and beyond; doubling the temperature in C is less than doubling the corresponding temperatures in F.

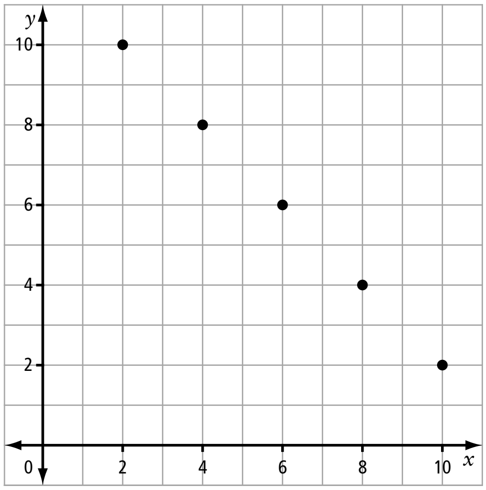
11. Example: Consumer income, competition between stores, scarceness of resource, input or cost of production prices.

6.2 Linear Relations

1. **(a)** (2, 0), (3, 2), (5, 4), (8, 6), (12, 8)



(b) (10, 2), (8, 4), (6, 6), (4, 8), (2, 10)



2. a) $C = 23p$

p	C
1	23
2	46
3	69
4	92

This relation is discrete because you cannot buy a part of a ticket.

b) $ab = 24$

a	b
2	12
2.5	9.6
3	8
3.5	6.857142857

This relation is continuous because you can choose any positive a-value and divide this number into 24 to find a b-value. If you only considered whole numbers, this relation would be discrete.

3. a) Non-linear; The number is always multiplied by one more than itself. $y = (x)(x + 1)$

b) Linear; The number is doubled, then increased by one. $y = 2x + 1$

c) Linear; The number is always subtracted from 3. $y = (3 - x)$

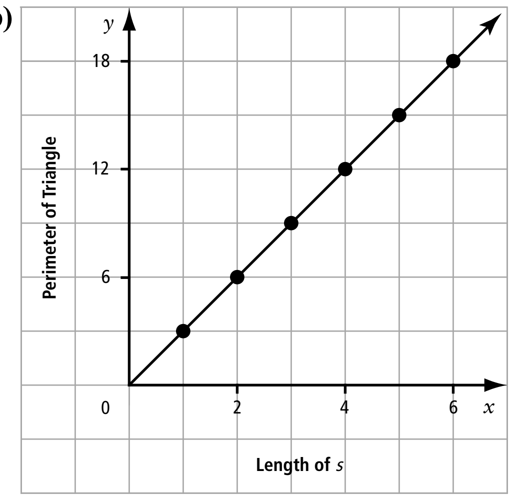
d) Linear; The number is always multiplied by negative one: $y = -x$.

4. a)

r	A
1	3.14
2	12.57
3	28.27
4	50.27

This is non-linear as there is no constant change in the values of A. The radius is independent; the area is dependent.

b)



This is linear as the perimeter is constantly three times the length of the side. The side length is independent, and perimeter is dependent.

c)

Polygon with n Sides	3	4	5	6
Number of Diagonals	0	2	5	9

This is non-linear as there is no constant change in the values of the numbers of diagonals. The number of sides in a polygon is the independent variable; the number of diagonals is the dependent variable.

5. a) linear **b)** linear
c) non-linear **d)** non-linear

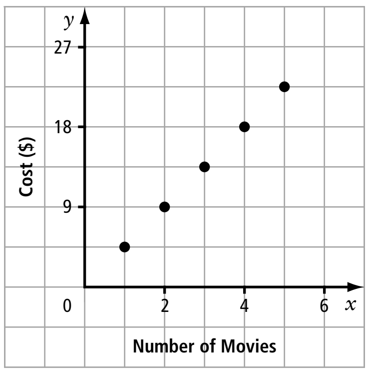
6. a) independent variable: number of movies; dependent variable: cost

b) The cost for renting one new release movie is \$4.50. The cost for renting two is \$9.00. The cost for renting three is \$13.50. The cost of renting four is \$18.00. The cost for renting five is \$22.50.

c) $C = \$4.50(m)$
d) (1, 4.50), (2, 9.00), (3, 13.50), (4, 18.00), (5, 22.50)
e)

Movies Rented	1	2	3	4	5
Cost ($)	4.50	9.00	13.50	18.00	22.50

f)



Plotting for zero does not make sense because people know that if they do not rent anything, the cost is $0.00.

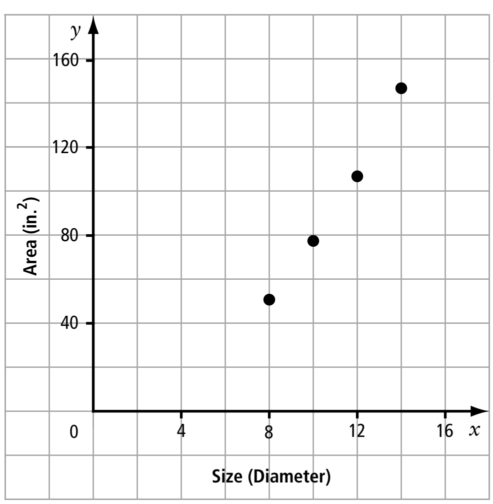
g) Example: The table of values would be easiest for customers to understand.

7. a)

Size (Diameter)	Area of Pizza
8 in.	50.27 in.^{2}
10 in.	78.54 in.^{2}
12 in.	113.1 in.^{2}
14 in.	153.94 in.^{2}

b) diameter

c)



d) non-linear
e) discrete

8. a) 4 in., 5 in., 6 in., 7 in.
b) 50 in.^{2}, 79 in.^{2}, 113 in.^{2}, 154 in.^{2}
c) $0.20
d) $15.80, $22.60, $30.80
e) Example: No. The price for the large pizzas is too high and may not compete well with the competition. Yes. He is charging the same rate for all the pizzas.
f) linear; discrete
g) non-linear

9. a) This would be a linear relation given a fixed or constant speed.

b)

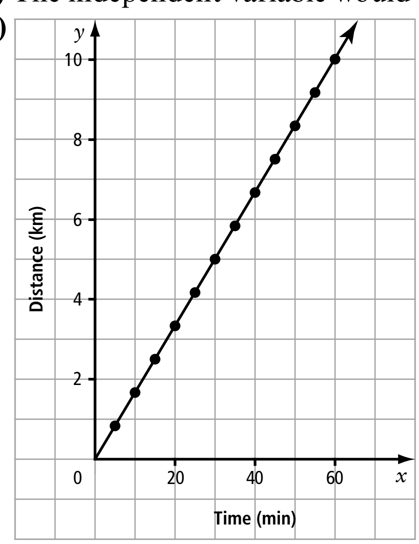
Time (h)	1	2	3	4	5
Distance (km)	10	20	30	40	50

Distance (km)	Time (min)
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48

The second table gives more realistic and detailed information to a reader. Distances of 40 or 50 km are unlikely for most joggers.

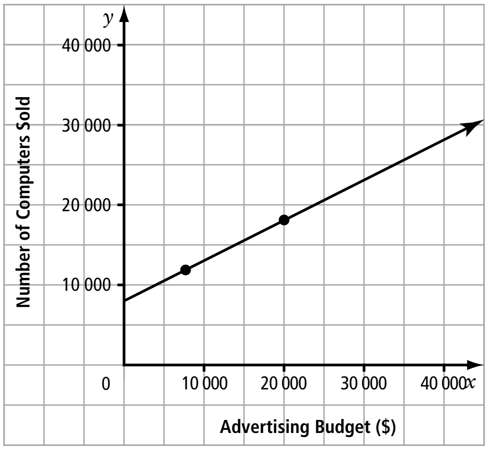
c) The independent variable would be time.

d)



e) The graph would be continuous because at any distance there would be a specific time.

10. a)



b) 28000
c) \$26000
d) 8000

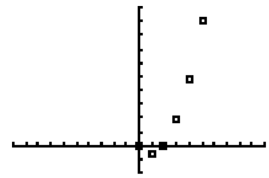
11. a) Answers will vary.
b) Answers will vary.
c) Either variable can be independent or dependent because you are measuring in centimetres for both.
d) linear
e) discrete, but in a population it would be continuous from the shortest arm/foot length to the longest

12. a) bags of flour, jugs of milk, single cans of soup, loaves of bread, packages of cookies
b) independent: number of items; dependent: cost
c) discrete.
d) Example: discrete; jars of pickles (even if bought by the case); continuous: roasts, which are priced per pound or kilogram
e) Items for which the price drops by a fraction if bought in multiples, such as 2 for a price, or 3 for a price.

13. $y = 3x\,(x - 2)$:

x	0	1	2	3	4	5
y	0	−3	0	9	24	45

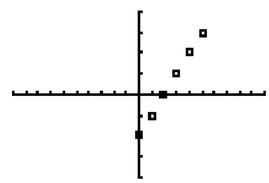
This equation is non-linear because the y-value does not change consistently with consistent change in the x-value.
The points on the graph are in the shape of a parabola so this is a non-linear function.



$y = 5(x - 2)$:

x	0	1	2	3	4	5
y	−10	−5	0	5	10	15

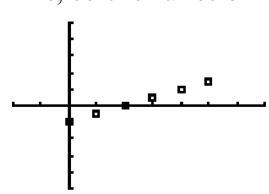
Each increase of 1 in the x-value results in an increase of 5 in the y-value. Since the y-value changes consistently with a consistent change in the x-value, this is a linear equation.
The points on the graph form a straight line, so the function is linear.



$y = \frac{(x - 2)}{2}$:

x	0	1	2	3	4	5
y	−1	−0.5	0	0.5	1	1.5

This equation is linear because the y-value changes consistently by 0.5 whenever the x-value increases by one.
The points on the graph form a straight line, so the function is linear.

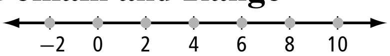


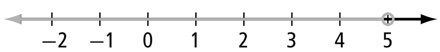
14. a) linear if the independent variable is of degree 1 and multiplied by a constant number
b) linear if both the independent and dependent values change at a constant rate
c) linear if both the independent and dependent values change at a constant rate
d) linear if all the points, whether discrete or continuous, form a line

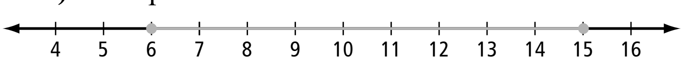
e) Find at least 3 pairs of answers and see if there are regular changes between them. If so, it is linear.

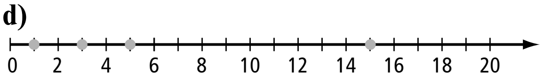
15. Example:
Look back at whether you had freedom to pick any numbers or values you wanted to for the independent variable. If you did, and you could choose numbers like fractions and decimals that work, it is continuous. If you are only allowed certain numbers, or only integers, then a discrete graph is best.

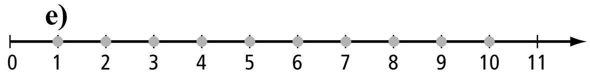
6.3 Domain and Range

1. a)


b)


c) Example:


d)


e)


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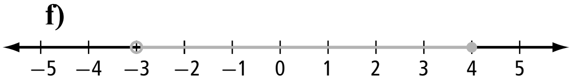


2. a) D = {0, 1, 2, 3, 4};
R = {\$0.00, \$0.38, \$0.76, \$1.14, \$1.52}
b) D = {penny, nickel, dime, quarter, half dollar, loonie, twoonie}
R = {\$0.01, \$0.05, \$0.10, \$0.25, \$0.50, \$1.00, \$2.00}
c) D = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
R = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
d) D = {1, 2, 3, 4, 5, 6}
R = {\$35.00, \$70.00, \$105.00, \$140.00, \$175.00, \$210.00}
e) D = {0, 1, 2, 3, 4, 5, 6}
R = {\$0.00, \$1.50, \$3.00, \$4.50, \$6.00, \$7.50, 9.00}

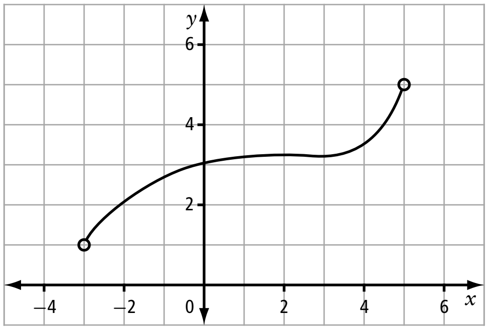
3. a) D = $\{x \mid -4 < x \leq 5\}$ or $(-4, 5]$
R = $\{y \mid -2 \leq y < 2\}$ or $[-2, 2)$
b) D = $\{x \mid 0 \leq x \leq 4\}$ or $[0, 4]$
R = $\{y \mid 0 \leq y \leq 4\}$ or $[0, 4]$
c) D = $\{x \mid -3 \leq x \leq 6\}$ or $[-3, 6]$
R = $\{y \mid -4 \leq y \leq 5\}$ or $[-4, 5]$
d) D = $\{x \mid -6 \leq x \leq 0\}$ or $[-6, 0]$
R = $\{y \mid -8 \leq y \leq 0\}$ or $[-8, 0]$
e) D = $\{x \mid -3 \leq x \leq 5\}$ or $[-3, 5]$
R = $\{y \mid -3 \leq y \leq 7\}$ or $[-3, 7]$
f) D = $\{x \mid -6 \leq x \leq 1\}$ or $[-6, 1]$
R = $\{y \mid -4 \leq y \leq 7\}$ or $[-4, 7]$

4. a) The domain is the number of litres of fuel that might be purchased. Since the capacity of the fuel tank is 40 L, the domain is 0, if no fuel is purchased, to 40 L, if the tank is empty and then filled to capacity. In set notation, D = $\{x \mid 0 \leq x \leq 40\}$
b) The range is the possible cost incurred by purchasing gas and getting a car wash. The assumption is that the car wash will be purchased, regardless if any gasoline is purchased. So the lowest end of the range is \$8.00. If the tank is empty and is filled with 40 L of gas, C = 0.92(40) + 8.00, or 44.80. So, the range is \$8.00 to \$44.80, or R = $\{y \mid 8 \leq y \leq 44.80\}$
c) The cost is dependent on the number of litres of gasoline that is purchased, so n is the independent and C is the dependent variable.

5. a) The lowest internal temperature in the table is for medium-rare beef, veal, and lamb at 63° C. The highest internal temperature is for whole poultry at 85° C. So, the range is from 63 to 85, or R = $\{T \mid 63 \leq T \leq 85\}$
b) It would be better to make a list of each temperature expectation. It would make a chef check carefully which temperature should be chosen.
c) Each temperature depends on the type of food you are cooking. Some are very specific and a range of values will not work. This is especially true for poultry, which must have the highest temperature value of 85.

6. Example:

a)



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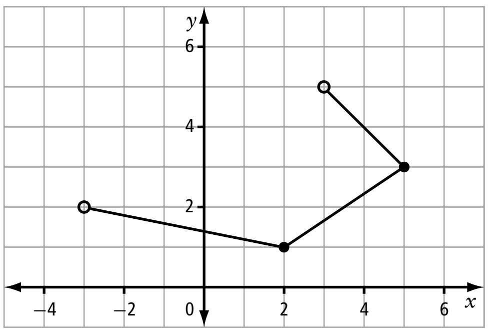
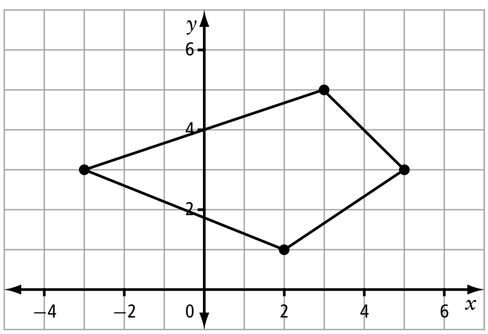
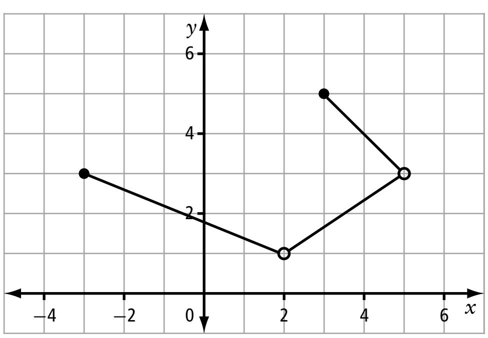


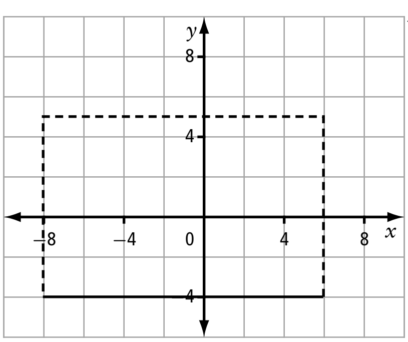
c)



d)



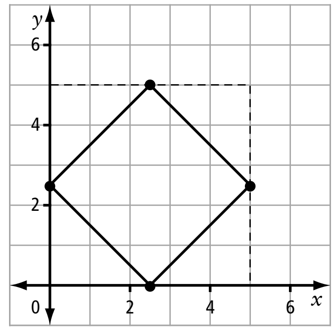
7.



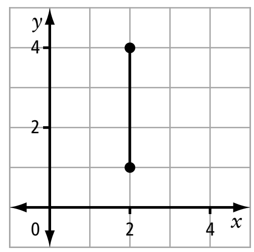
w

8. a ray

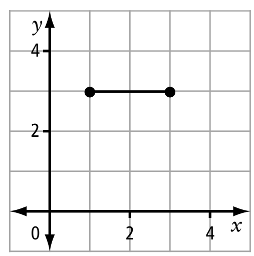
9. a) Example:



b) Example:



c) Example:



10. Example: A domain must be present for a relation to exist. It must have a minimum of one domain element matched with one range element

11. Example: A balloon being blown up: when empty it has no air in it, but it can get bigger and bigger and then pop when it gets to 20 cm

12. Example: Domains: player names, position on the team, team jersey number. Ranges: salary, game minutes, points scored

6.4 Functions

1. **a)** This is not a function because there are two ordered pairs with an x-value of 2, but with different y-values: 4 and 1. If you drew the vertical line, $x = 2$, the line would pass through two y-values proving that this is not a function.
   b) This is a function because each of the x-values is different and has only one corresponding y-value. You could draw a vertical line through each x-value, and it would intersect with only one y-value.
   c) This is not a function because each of the x-values, 1, 4, and 9, has two corresponding y-values. A vertical line drawn through any of these x-values would intersect two y-values.
   d) This is a function because each person has only one shoe size. If you were to plot these points on a graph, a vertical line could be drawn from any of the names and there would only be one corresponding shoe size (y-value).
   e) This is not a function because Anika has 3 different siblings.
   f) This is function because each of the x-values has one and only one corresponding y-value. In other words, you can draw a vertical line through any point on the graph, and it will not pass through any other point.
   g) This is not a function because if you draw a vertical line through any of these points, the line will pass through at least one other point.

2. $A(n) = 500(1 + 0.08)^n$

3. $W = 26p + 1200$

4. **a)** $z(-3) = 16$ **b)** $z(2) = 1$
   c) $a = 0$

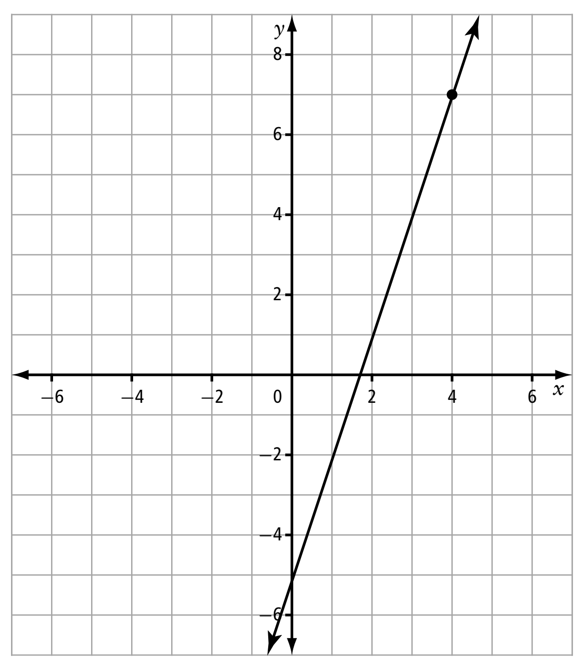
5. **a)** $t(1) = 5$ **b)** $t(20) = 81$
   c) $n = 10$

6. **a)** The price, P, is dependent on the number of months you are a member, m. So, m is independent variable and it represents the number of months you are a member of the club.
   b) The cost of being a member for a year is equal to 12 months of monthly dues plus the $55 initiation fee: $P(12) = 35(12) + 55$, or $475.00.
   c) To find after how many months you will have spent $1000, set P equal to 1000 and then solve for m:
   $$1000 = 35m + 55$$
   $$1000 - 55 = 35m + 55 - 55$$
   $$945 = 35m$$
   $$\frac{945}{35} = \frac{35m}{35}$$
   $$27 = m$$
   d) The competitor sells its membership by the week, so you must solve the equation for 52 weeks, which is equal to one year: $P(52) = 10(52) + 100$, or $620.00. This option is more expensive than belonging to FITFIT.

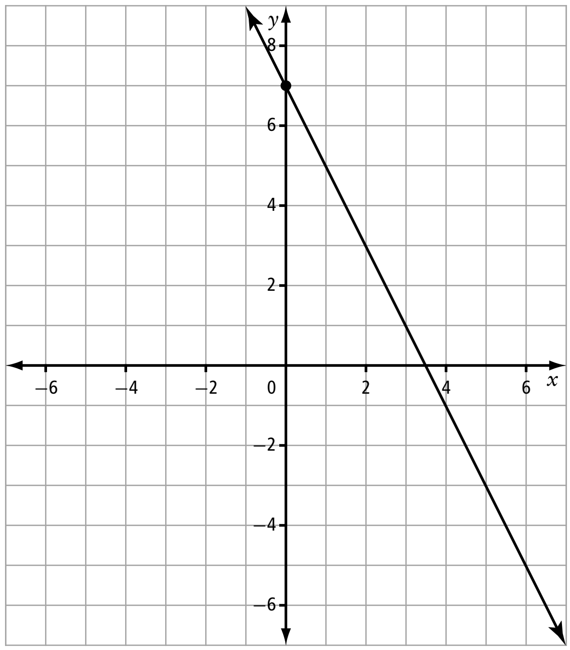
7. **a)** $f(5) = 9$ **b)** $f(5) = 1$
   c) $f(5) = 5$ **d)** $f(5) = -\frac{1}{2}$

8. **a)** $x = -3$ **b)** $x = 13$
   c) $x = -35$ **d)** $x = 63$

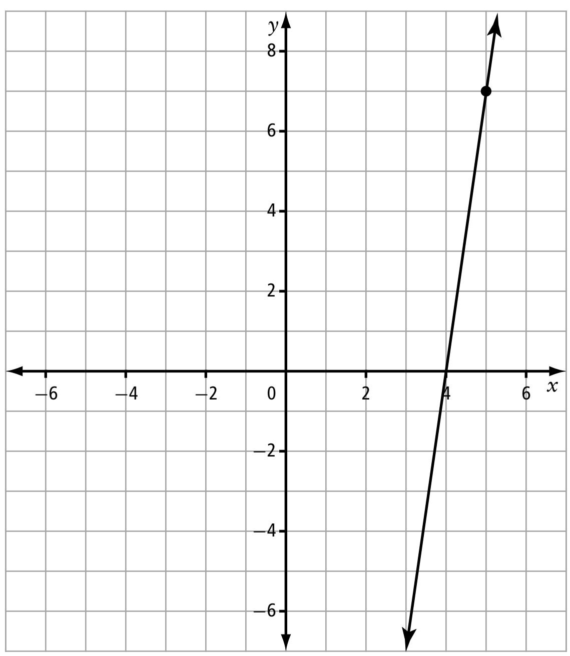
9. **a)** $x = 4$



b) $x = 0$



c) $x = 5$



10. a) $x = 6$ produces the prime number 7
b) $x = 9$ produces the multiple of 8 that is 16
c) $x = 38$ produces the number 103, which is larger than 100
d) $x = 3$ produces −2, the largest negative number in the function

11. $T(m) = (A)m + (B)$ would be $T(m) = (-4)m + (90)$, where $A = -4$ and $B = 90$

12. a) V0B 2P0 is the postal code for all of Yahk, BC. T0H 2P0 is the postal code for all of Meander River, AB. These are not functions. R2J 0A1 refers to only one address: 520 Lagimodiere Blvd, Winnipeg, MB (the Royal Canadian Mint). This postal code is a function. (Notice that this postal code returns two results when you search Canada Post's online database: one response in English and one in French. Therefore, you might also answer that this postal code is not a function within the context of the database because there are two records associated with it.)
b) Postal codes are usually specific to one street name and either even or odd street numbers. If you live in a rural setting or are the ONLY even or odd numbered house on a street, you may be unique. If your postal code is used only for you, then it is a function. If you share your postal code with other addresses, it is not a function.

6.5 Slope

1.

Positive Slope	Negative Slope	Zero Slope
AH GH FE BE	AB HE GF ED BC	HB DC

2. Slopes are: AB $= -\frac{1}{3}$; BC $= -\frac{2}{1}$; CD $= 0$; DE $= \frac{2}{1}$; EF $= -\frac{1}{3}$; FG $= 2$; GA $= -\frac{7}{2}$;

3. a) $m = +\frac{1}{2}$, less steep than a 45° line
b) $m = +\frac{8}{5}$, steeper than a 45° line
c) $m = -\frac{9}{4}$, steeper than a 45° line
d) $m = -\frac{1}{3}$, less steep than a 45° line

4. a) $m = -\frac{1}{2}$ **b)** $m = 0$
c) $m = \frac{3}{7}$ **d)** $m = \frac{5}{2}$

5.

Given Point A(x, y)	Slope	Next Point to the Right of A
(3, 5)	$-\frac{1}{2}$	(5, 4)
(3, 5)	$\frac{2}{3}$	(6, 7)
(−3, 7)	$\frac{3}{7}$	(4, 10)
(2, −5)	$-\frac{4}{1}$	(3, −9)
(0,−4)	$\frac{5}{4}$	(4, 1)

6. 10.08 m or approximately 10 m

7. $7.50: the amount of money she gives her mother each week

8. a) The slope is equal to $\frac{\text{vertical change}}{\text{horizontal change}}$, or $\frac{\text{rise}}{\text{run}}$. The slope on this road is 32% grade. So, $0.32 = \frac{130}{d}$, where d is the horizontal distance. Solving for d,

$$0.32 = \frac{130}{d}$$
$$d(0.32) = d\left(\frac{130}{d}\right)$$
$$d(0.32) = 130$$
$$\frac{d(0.32)}{(0.32)} = \frac{130}{0.32}$$
$$d = 406.25$$

The horizontal distance is approximately 406 m.

b) Expressed as rise over run, the slope of this road is $\frac{130}{406}$.

c) The actual road surface is approximately 426 m long.

9. 1.25 cm of hair growth per month

10. 3 cm

11. increase of 1925 people per year; in 2021, an increase of 28 878, or a population to 231 218

12. a) For both trips, the employee paid for 3 days of use. The only variable that changes to account for the change in cost is the number of kilometres driven. On the second trip, the cost of the car was $63.75 more and the number of additional kilometres driven was 255 more km. So, $255(d) = 63.75$, where d is the charge per kilometre:

$$255(d) = 63.75$$
$$\frac{255(d)}{255} = \frac{63.75}{255}$$
$$d = 0.25$$

The company charges $0.25/km.

b) The daily cost vehicle, C, is the number of kilometres × 0.25 plus the daily charge. In this case, the employee paid for 3 days for both trips. Since the daily charge does not change for each trip, so we can solve the equation $C = 0.25(d) + 3(r)$, where d is the number of kilometres driven and r is the daily rate. Solving for Trip A:

$$301.25 = 0.25(425) + 3(r)$$
$$301.25 = 106.25 + 3r$$
$$301.25 - 106.25 = 106.25 - 106.25 + 3(r)$$
$$195 = 3(r)$$
$$\frac{195}{3} = \frac{3(r)}{3}$$
$$65 = r$$

The daily fee for the car is $65.00.

c) If the company simply paid employees $0.50/km to use their own car, Trip A would have cost $212.50, while Trip B would cost $340.00. This means that paying the employee for the use of their car would be cheaper.

d) The cost for renting a car is the same as for using the employees car when the kilometres travelled is 780 km. The cost in both scenarios is $390.00. However, since the rental company charges less per kilometre, if the trip is more than 780 km long in a 3-day period, it becomes cheaper to rent.

13. Example: If the slope definition were run over rise, when the line segments got steeper, the ratios would be getting smaller and smaller, rather than larger. Having the slope formula as rise over run makes these ratios increase in value as the actual line segments get steeper.

14. Example: The denominator is increasing quicker than the numerator, so the slope is getting increasingly smaller. However, since the numerator will never equal 0, no matter how small the slope gets, it will never be 0.