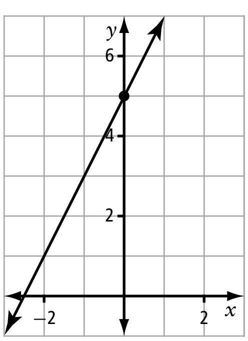
Chapter 7 Linear Equations and Graphs

7.1 Slope-Intercept Form

1. **a)** $m = \frac{1}{2}$, y-intercept: −2
b) $m = -4$, y-intercept: 3
c) $m = 1$, y-intercept: 0
d) $m = 0.75$, y-intercept: 3.5

2. **a)** $x - x + y = 7 - x$
$y = -x + 7$
$m = -1$, y-intercept: 7
b) $y - 4x + 4x = 12 + 4x$
$y = 4x + 12$
$m = 4$, y-intercept: 12
c) $5x - 5x + 2y = -5x + 10$
$2y = -5x + 10$
$\frac{2y}{2} = \frac{-5x + 10}{2}$
$y = \frac{-5}{2}x + 5$
$m = \frac{-5}{2}$, y-intercept: 5
d) $x - 3y + 3y - 12 = 0 + 3y$
$x - 12 = 3y$
$\frac{x - 12}{3} = \frac{3y}{3}$
$\frac{1}{3}x - 4 = y$
$y = \frac{1}{3}x - 4$
$m = \frac{1}{3}$, y-intercept: −4

3. **a)** $y = 4x - 1$ **b)** $y = \frac{-1}{2}x + 7$
c) $y = \frac{2}{3}x - 2$ **d)** $y = 0.5x$
e) $y = -5x + 1$ **f)** $y = x + \frac{4}{5}$

4. **a)**


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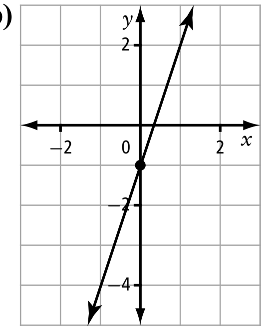

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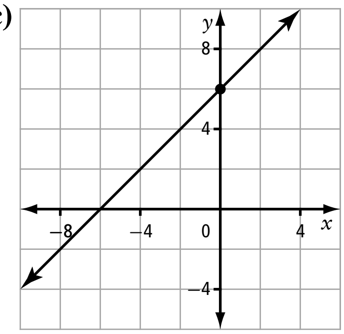

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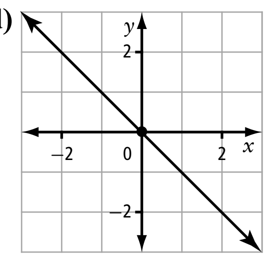

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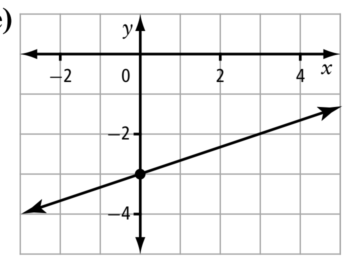

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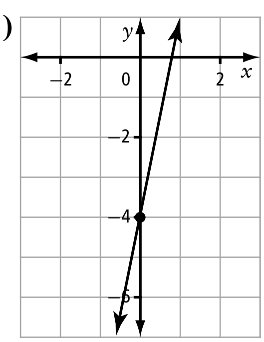


5. a) $m = 1, b = 1, y = x + 1$
b) $m = -1, b = 4, y = -x + 4$
c) $m = \frac{2}{3}, b = 0, y = \frac{2}{3}x$
d) $m = -4, b = 2, y = -4x + 2$
e) $m = 0.6, b = -2, y = 0.6x - 2$
f) $m = \frac{-6}{5}, b = 6, y = \frac{-6}{5}x + 6$

6. a) Replace x with 12 and y with 8 in the equation $y = \frac{1}{2}x + b$.

$$y = \frac{1}{2}x + b$$
$$8 = \frac{1}{2}(12) + b$$

Solve for b.

$$8 = 6 + b$$
$$8 - 6 = 6 - 6 + b$$
$$2 = b$$

b) Replace x with −3 and y with $\frac{1}{2}$ in the equation $y = \frac{1}{2}x + b$.

$$y = \frac{1}{2}x + b$$
$$\frac{1}{2} = \frac{1}{2}(-3) + b$$

Solve for b.

$$\frac{1}{2} = \frac{-3}{2} + b$$
$$\frac{1}{2} + \frac{3}{2} = \frac{-3}{2} + \frac{3}{2} + b$$
$$\frac{4}{2} = b$$
$$2 = b$$

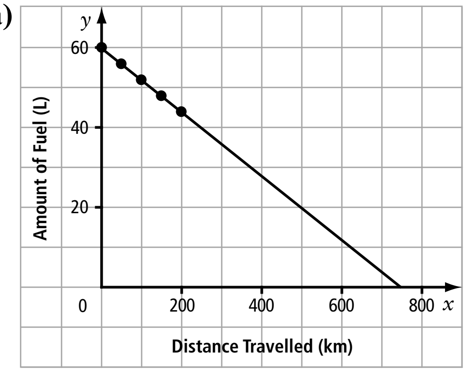
7. a) $m = 2$ b) $m = -4$

8. a) $y = 2x - 250$
b) $m = 2$; the price per raffle ticket
c) $b = -250$; the cost of the pair of hockey tickets
d) 275 tickets

9. a) $y = 75x - 600$
b) \$255 loss; \$525 profit; \$1275 profit
c) 8 competitors

10. a) \$25
b) $y = 15x + 25$
c) $b = 25$, which represents the fixed charge
d) discrete because rental is charged per whole hour

11. a)



b) Substitute two points on the line, for example, (0, 60) and (50, 56), into the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{56 - 60}{50 - 0}$$
$$m = \frac{-4}{50}$$
$$m = \frac{-2}{25}$$

The line intersects the y-axis at 60, so $b = 60$.

c) Replace m with $\frac{-2}{25}$ and b with 60 in the slope-intercept form: $y = \frac{-2}{25}x + 60$.

d) The amount of fuel in the car's tank before driving any distance.

e) The tank is empty when $y = 0$. Replace y with 0 and solve for x:

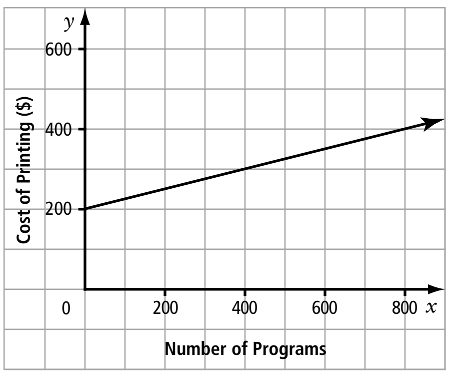
$$y = \frac{-2}{25}x + 60$$
$$0 = \frac{-2}{25}x + 60$$
$$0 - 60 = \frac{-2}{25}x + 60 - 60$$
$$-60 = \frac{-2}{25}x$$
$$\left(\frac{-25}{2}\right)(-60) = \left(\frac{-25}{2}\right)\left(\frac{-2}{25}x\right)$$
$$750 = x$$

The tank will be empty after 750 km.

12. a)

Number of Programs	Cost of Printing ($)
0	200.00
50	212.50
100	225.00
150	237.50
200	250.00
250	262.50

b)



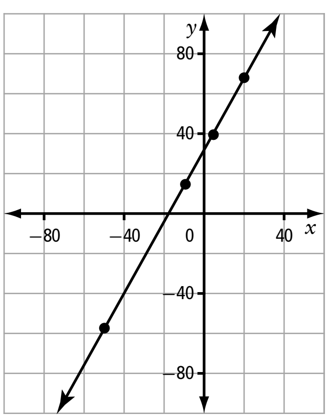
c) $m = \frac{1}{4}$; the cost of printing each program

d) $b = 200$; the fixed cost

e) $y = \frac{1}{4}x + 200$

f) 600 programs

13. a)



b) Use two points from the table to find the slope: (20, 68) and (−10, 14).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{14 - 68}{-10 - 20}$

$m = \frac{-54}{-30}$

$m = \frac{9}{5}$

c) $b = 32$, which represents the temperature in Fahrenheit when the temperature is 0 °C.

d) Replace m with $\frac{9}{5}$ and b with 32 in the slope-intercept form: $y = \frac{9}{5}x + 32$.

e) In the equation $y = \frac{9}{5}x + 32$, replace y with x and x with y. Then, solve for y.

$$x = \frac{9}{5}y + 32$$

$$x - 32 = \frac{9}{5}y + 32 - 32$$

$$x - 32 = \frac{9}{5}y$$

$$\frac{5}{9}(x - 32) = \left(\frac{5}{9}\right)\left(\frac{9}{5}y\right)$$

$$\frac{5}{9}x - \frac{160}{9} = y$$

f) Use the $y = \frac{9}{5}x + 32$ form of the equation, replacing x with −40.

$y = \frac{9}{5}x + 32$

$y = \frac{9}{5}(-40) + 32$

$y = -72 + 32$

$y = -40$

−40 °C = −40 °F

Use the $y = \frac{5}{9}x - \frac{160}{9}$ form of the equation, replacing x with 100.

$y = \frac{5}{9}(100) - \frac{160}{9}$

$y = 37.8$

100 °F = 37.8 °C

Use the $y = \frac{9}{5}x + 32$ form of the equation, replacing x with 0.

$y = \frac{9}{5}x + 32$

$y = \frac{9}{5}(0) + 32$

$y = 32$

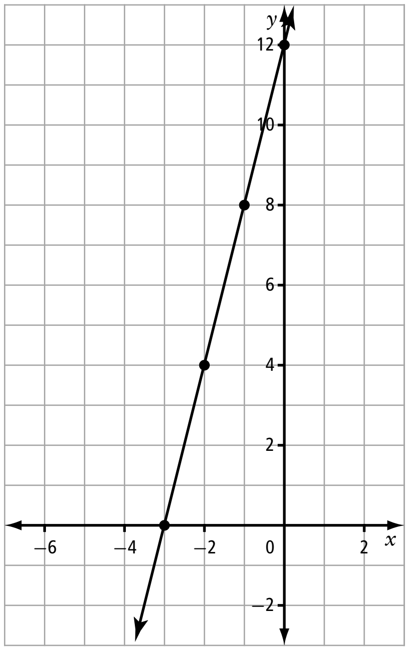
0 °C = 32 °F

14. 80 people

15. a) Example:

x	*y*
0	12
−3	0
−2	4
−1	8

b) The slope is 4 and the y-intercept is (0, 12).

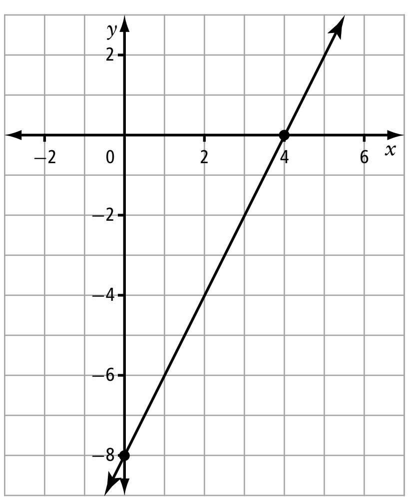


c) Example: I prefer using the slope and y-intercept. There is less computation involved when writing the equation in slope-intercept form. It is simple to graph the line using the slope and the y-intercept.

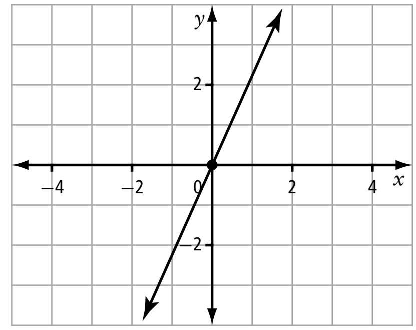
7.2 General Form

1. **a)** $x - 3y + 15 = 0$ **b)** $2x + 7y = 0$
c) $8y - 1 = 0$ **d)** $2x + 10y - 12 = 0$

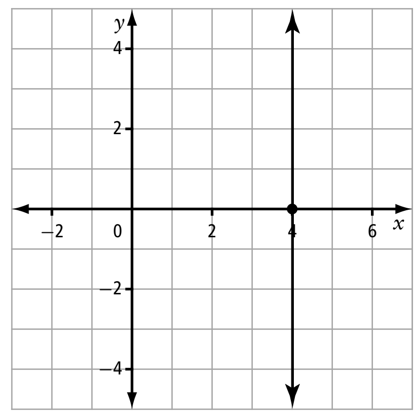
2. **a)** (4, 0), (0, −8)



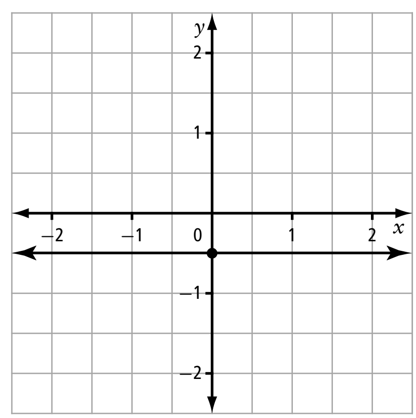
b) (0, 0)



c) (4, 0), no y-intercept



d) no x-intercept, $(0, \frac{-1}{2})$



3. **a)** domain: $x \in R$; range: $y \in R$
To find the slope, use the points (0, 4) and (3, 0).
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{4 - 0}{0 - 3}$
$m = \frac{-4}{3}$
The x-intercept is (3, 0) and the y-intercept is (0, 4).

In the slope-intercept form, replace m with $\frac{-4}{3}$ and b with 4:
$y = \frac{-4}{3}x + 4$.
Multiply both sides of the equation by 3:
$3y = 3\left(\frac{-4}{3}x + 4\right)$
$3y = -4x + 12$
Bring all terms to one side of the equation:
$3y = -4x + 12$
$3y + 4x = -4x + 4x + 12$
$4x + 3y = 12$
$4x + 3y - 12 = 12 - 12$
$4x + 3y - 12 = 0$

b) domain: $x \in R$; range: $y = -3$
Since this is a horizontal line, the slope is 0.
The y-intercept is (0, −3).
The equation of the line in general form is $y + 3 = 0$.

4. Examples:
a) $y + 5 = 0$ **b)** $x + 0 = 0$
c) $x - 8 = 0$ **d)** $3x - 7y = 0$
e) $x + y - 3 = 0$

5. **a)** $B = -1$ **b)** $A = 4$
c) $C = 0$

6. **a)** Let $14x$ represent the number of calories burned swimming for x minutes. Let $12y$ represent the number of calories burned biking for y minutes. The equation to represent the total number of calories burned is $14x + 12y = 4200$.

b) To find the x-intercept, replace y with 0.
$14x + 12y = 4200$
$14x + 12(0) = 4200$
$14x = 4200$
$x = 300$

The x-intercept represents how many minutes he would need to spend swimming if he burned 4200 calories by only swimming.

To find the y-intercept, replace x with 0.
$14x + 12y = 4200$
$14(0) + 12y = 4200$
$12y = 4200$
$y = 350$

The y-intercept represents how many minutes he would need to spend biking if he did not spend any time swimming.

c) The domain is $0 \le x \le 300$. There can be no values less than 0 or greater than 300. The range is $0 \le y \le 350$. There can be no values less than 0 or greater than 350.

d) Replace y with 120 and solve for x.
$14x + 12y = 4200$
$14x + 12(120) = 4200$
$14x + 1440 = 4200$
$14x + 1440 - 1440 = 4200 - 1440$
$14x = 2760$
$x = 197$
He would need to swim 197 minutes, or 3 hours and 17 minutes.

7. **a)** $5x + 2y - 2250 = 0$
b) $m = \frac{-5}{2}$; (450, 0) and (0, 1125);
domain: $0 \le x \le 450$,
range: $0 \le y \le 1125$
c) 400 minutes, or 6 hours and 40 minutes

8. 108 square units

9. Examples:
a) (−4, 1) and (6, 4), $3x - 10y + 22 = 0$
b) (2, 8) and (7, 8), $y - 8 = 0$
c) (−2, −1) and (6, −4), $3x + 8y + 14 = 0$
d) (−3.5, 6) and (−3.5, −2), $2x + 7 = 0$

7.3 Slope-Point Form

1. a) $m = 4, (3, -7)$
   b) $m = \frac{1}{3}, (-5, 5)$
   c) $m = -2, (6, 0)$
   d) $m = 1, (3, -1)$
2. a) $y = \frac{2}{3}x + \frac{11}{3}$; $2x - 3y + 11 = 0$
   b) $y = -2x - 2$; $2x + y + 2 = 0$
   c) $y = \frac{3}{4}x - 3$; $3x - 4y - 12 = 0$
   d) $y = 3x + 19$; $3x - y + 19 = 0$
3. a) For slope-point form, replace m with $\frac{4}{3}$ and (x_1, y_1) with $(-1, -5)$:
   $y - y_1 = m(x - x_1)$
   $(y + 5) = \frac{4}{3}(x + 1)$
   For slope-intercept form, rewrite $(y + 5) = \frac{4}{3}(x + 1)$ in the form $y = mx + b$:
   $$\begin{aligned}(y + 5) &= \frac{4}{3}(x + 1)\\ y + 5 &= \frac{4}{3}x + \frac{4}{3}\\ y + 5 - 5 &= \frac{4}{3}x + \frac{4}{3} - 5\\ y &= \frac{4}{3}x - \frac{11}{3}\end{aligned}$$
   For general form, rewrite $y = \frac{4}{3}x - \frac{11}{3}$ in the form $Ax + By + C = 0$:
   $$\begin{aligned}3y &= 3\left(\frac{4}{3}x - \frac{11}{3}\right)\\ 3y &= 4x - 11\\ 3y - 3y &= 4x - 3y - 11\\ 0 &= 4x - 3y - 11\end{aligned}$$
   b) Slope-point form: $(y + 3) = 1\left(x + \frac{1}{2}\right)$
   Slope-intercept form: $y = x - \frac{5}{2}$
   General form: $0 = 2x - 2y - 5$
   c) Slope-point form: $(y - 4) = -1.5(x - 1)$
   Slope-intercept form: $y = -1.5x + 5.5$
   General form: $0 = 15x + 10y - 55$
   d) Slope-point form:
   First find the slope of the line through the points $(-5, -8)$ and $(-7, -9)$.
   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
   $$m = \frac{-9 - (-8)}{-7 - (-5)}$$
   $$m = \frac{1}{2}$$
   Then, replace m with $\frac{1}{2}$ and (x_1, y_1) with $(-5, -8)$ in the slope-point form.
   $y - y_1 = m(x - x_1)$
   $(y + 8) = \frac{1}{2}(x + 5)$
   Slope-intercept form:
   $y = \frac{1}{2}x - \frac{11}{2}$
   General form:
   $$\begin{aligned}y &= \frac{1}{2}x - \frac{11}{2}\\ 2y &= 2\left(\frac{1}{2}x - \frac{11}{2}\right)\\ 2y &= x - 11\\ 2y - 2y &= x - 2y - 11\\ 0 &= x - 2y - 11\end{aligned}$$
   e) Slope-point form: $(y + 2) = \frac{1}{2}(x + 1)$
   Slope-intercept form: $y = \frac{1}{2}x - \frac{3}{2}$
   General form: $0 = x - 2y - 3$
4. Examples:
   a) $y - 2 = \frac{1}{2}(x - 6)$
   b) $y - 2 = -1(x - 2)$
   c) $y + 5 = \frac{-4}{3}(x - 2)$
5. a) $y - 1 = 0(x + 3)$; $y - 1 = 0$
   b) $y - 8 = 2(x + 1)$; $2x - y + 10 = 0$
   c) The slope of the line $5x + 2y - 10$ is $-\frac{5}{2}$. So, the second line must have this same slope. Since it passes through the point $(-1, 4)$, the equation of the second line in slope-point form is $y - 4 = \frac{-5}{2}(x + 1)$. To convert this to general form, move all terms to the left side of the equation:
   $$\begin{aligned}y - 4 &= \frac{-5}{2}(x + 1)\\ y - 4 &= \frac{-5}{2}x - \frac{5}{2}\\ (2)(y - 4) &= (2)\left(\frac{-5}{2}x - \frac{5}{2}\right)\\ 2y - 8 &= -5x - 5\\ 2y - 8 + 5 &= -5x - 5 + 5\\ 2y - 3 &= -5x\\ 5x + 2y - 3 &= -5x + 5x\\ 5x + 2y - 3 &= 0\end{aligned}$$
   d) $y + 6 = \frac{-5}{2}(x - 2)$; $5x + 2y + 2 = 0$
   e) $y - 3 = \frac{3}{5}(x - 0)$; $3x - 5y + 15 = 0$
   f) $y - 0 = \frac{-3}{2}(x - 0)$; $3x + 2y = 0$

6. Write the equation of the line with x-intercept (10, 0) and y-intercept (0, −5).
Find the slope.

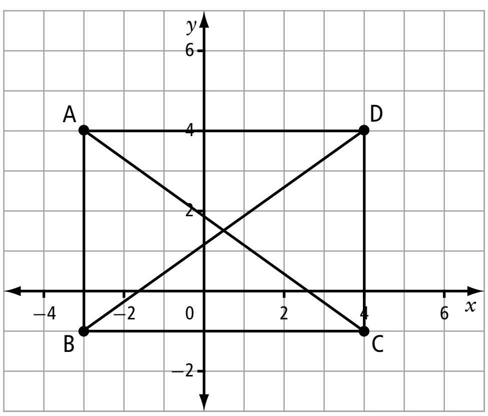
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-5 - 0}{0 - 10}$$
$$m = \frac{-5}{-10}$$
$$m = \frac{1}{2}$$

Substitute $m = \frac{1}{2}$ and $b = -5$ into the slope-intercept form: $y = \frac{1}{2}x - 5$.
Replace x with −2 and y with −6.

$$y = \frac{1}{2}x - 5$$
$$-6 = \frac{1}{2}(-2) - 5$$
$$-6 = -1 - 5$$
$$-6 = -6$$

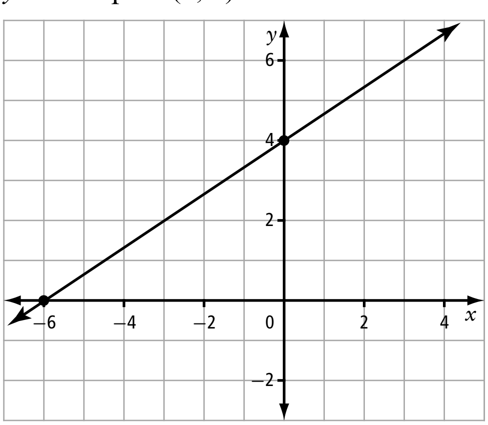
Since replacing x with −2 and y with −6 in the equation of the line results in a true statement, the point (−2, −6) is on the line.

7.



AC: $5x + 7y - 13 = 0$
BD: $5x - 7y + 8 = 0$

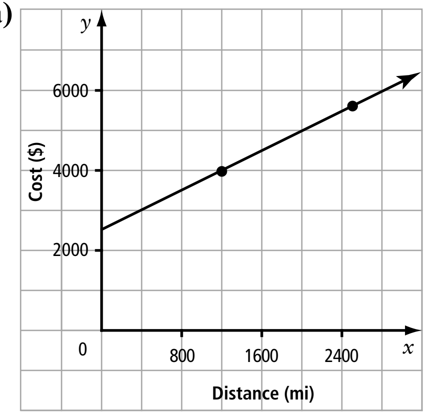
8. The x-intercept is (−6, 0) and the y-intercept is (0, 4).



9. $k = 1$

10. a) Lines 3 and 4 **b)** Line 5
c) Lines 1 and 2

11. a)



b) $m = \frac{5}{4}$, which represents the cost of operating a snowmobile per mile
c) $b = 2500$; the fixed cost of operating a snowmobile
d) $5x - 4y + 10\,000 = 0$
e) \$3625

12. a) $5x - 2y - 52 = 0$
b) 2.5 cm/h; at 1400 hours it was 21 cm tall
c) rate of burn per hour
d) the height at 1400 hours

13. Rewrite each equation in slope-intercept form. Enter the equations into a graphing calculator or graphing program and read the intersection points, which are the vertices.

Line 1:

$$2x + 3y - 18 = 0$$
$$2x - 2x + 3y - 18 = 0 - 2x$$
$$3y - 18 = -2x$$
$$3y - 18 + 18 = -2x + 18$$
$$3y = -2x + 18$$
$$\frac{3y}{3} = \frac{-2x}{3} + \frac{18}{3}$$
$$y = \frac{-2x}{3} + 6$$

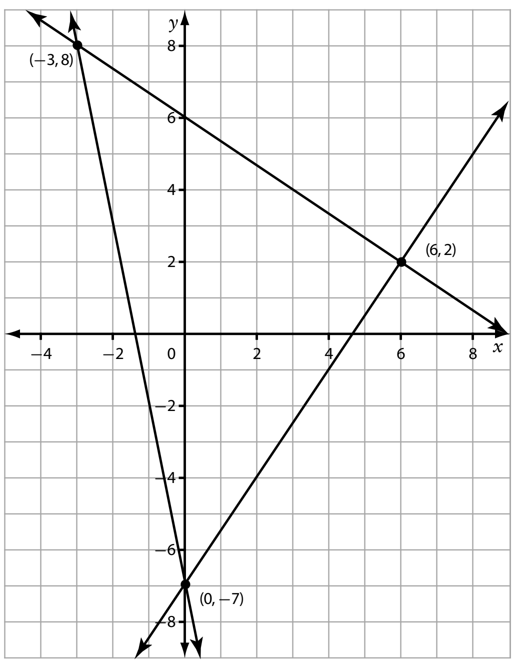
Line 2:

$$5x + y + 7 = 0$$
$$5x - 5x + y + 7 = 0 - 5x$$
$$y + 7 = -5x$$
$$y + 7 - 7 = -5x - 7$$
$$y = -5x - 7$$

Line 3:

$$3x - 2y - 14 = 0$$
$$3x - 3x - 2y - 14 = 0 - 3x$$
$$-2y - 14 = -3x$$
$$-2y - 14 + 14 = -3x + 14$$
$$-2y = -3x + 14$$
$$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{14}{-2}$$
$$y = \frac{3x}{2} - 7$$

Enter the equations into a graphing calculator or graphing program.



The vertices of the triangle are (−3, 8), (0, −7), and (6, 2).

14. a) $3x - 4y + 24 = 0$

b) The x-intercept is (−8, 0) and the y-intercept is (0, 6). The denominator of the x-term in the original equation is the x-intercept. The denominator of the y-term in the original equation is the y-intercept.

c) Example: Predict that the x-intercept is (3, 0) and the y-intercept is (0, −5). Verify by replacing y with 0 and solving for x to find the x-intercept.

$$\frac{x}{3} - \frac{y}{5} = 1$$
$$\frac{x}{3} - \frac{0}{5} = 1$$
$$\frac{x}{3} = 1$$
$$x = 3$$

Therefore, the x-intercept is (3, 0).

Verify by replacing x with 0 and solving for y to find the y-intercept.

$$\frac{x}{3} - \frac{y}{5} = 1$$
$$\frac{0}{3} - \frac{y}{5} = 1$$
$$0 - \frac{y}{5} = 1$$
$$\frac{-y}{5} = 1$$
$$y = -5$$

Therefore, the y-intercept is (0, −5).

The above shows that the prediction was correct.

15. a) Example: $y - 1.4 = -\frac{31}{90}(x - 2010)$
b) in the year 2014

7.4 Parallel and Perpendicular Lines

1. a) perpendicular **b)** parallel
c) perpendicular **d)** perpendicular
e) perpendicular **f)** parallel

2. a) parallel: −3; perpendicular: $\frac{1}{3}$
b) parallel: 1; perpendicular: −1
c) parallel: −4; perpendicular: $\frac{1}{4}$
d) parallel: 0; perpendicular: undefined
e) parallel: $\frac{5}{2}$; perpendicular: $\frac{-2}{5}$

3. a) $n = 4$ **b)** $n = -2$
c) $n = 2.5$ **d)** $n = \frac{3}{2}$

4. a) $r = -2$ **b)** $r = 15$
c) $r = -18$ **d)** $r = 8$

5. a) $5x + y - 7 = 0$ **b)** $x + 3y + 4 = 0$
c) $x + y - 4 = 0$

6. a) $2x + y - 5 = 0$ **b)** $4x + 7y = 0$
c) $2x - y - 12 = 0$

7. a) $\frac{1}{2}, \frac{1}{2}$
b) no, the equations represent the same line

8. a) $x - 5y - 31 = 0$ **b)** $3x - y + 10 = 0$

9. $y - 15 = 0$

10. a) The slope of side MN is $-\frac{4}{3}$, the slope of side NC is $\frac{-7}{5}$, and the slope of side MC is $\frac{-3}{2}$. Since no two slopes have a product of −1, these points do not represent the vertices of a right triangle.

b) The slope of DF is $\frac{1}{2}$, the slope of FG is $\frac{-4}{7}$, and the slope of DG is −2. Since the product of the slopes of sides DF and DG is −1, the points represent the vertices of a right triangle.

11. Examples:

a) Find the slope of $4x + y - 11 = 0$ by writing the equation in slope-intercept form, $y = mx + b$:

$$\begin{aligned} 4x + y - 11 &= 0 \\ 4x - 4x + y - 11 &= 0 - 4x \\ y - 11 &= -4x \\ y - 11 + 11 &= -4x + 11 \\ y &= -4x + 11 \end{aligned}$$

The slope is −4.

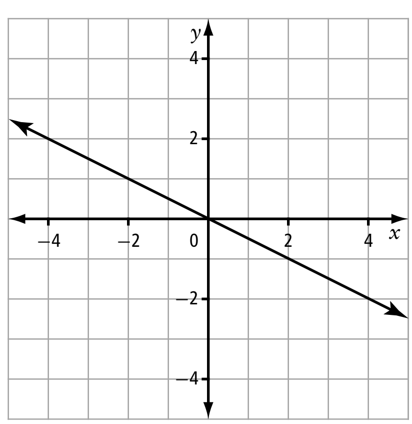
Any line with the same slope but a different y-intercept will be parallel to the given line. Example, $y = -4x + 16$. An infinite number of equations can be written in the form $y = mx + b$, where $m = -4$ and $b \neq 11$.

b) A line perpendicular to $4x + y - 11 = 0$ has a slope that is the negative reciprocal of −4, or $\frac{1}{4}$. Example, $y = \frac{1}{4}x - 6$.

Any line with a slope of $\frac{1}{4}$ will be perpendicular to the given line, regardless of the y-intercept. There is an infinite number of these lines.

12. $12x + y - 3 = 0$

13. The line must have a slope of $\frac{-1}{2}$ and a y-intercept of 0.



14. $k = -4$

15. $k = -4$

16. $k = \frac{7}{6}$

17. a) Write the equation in slope-intercept form:

$$\begin{aligned} kx - 2y - 1 &= 0 \\ kx - kx - 2y - 1 &= 0 - kx \\ -2y - 1 &= -kx \\ -2y - 1 + 1 &= -kx + 1 \\ -2y &= -kx + 1 \\ \frac{-2y}{-2} &= \frac{-kx}{-2} + \frac{1}{-2} \\ y &= \frac{kx}{2} - \frac{1}{2} \end{aligned}$$

Therefore, the slope of the first line is $\frac{k}{2}$.

$$\begin{aligned} 8x - ky + 3 &= 0 \\ 8x - 8x - ky + 3 &= 0 - 8x \\ -ky + 3 &= -8x \\ -ky + 3 - 3 &= -8x - 3 \\ -ky &= -8x - 3 \\ \frac{-ky}{-k} &= \frac{-8x}{-k} - \frac{3}{-k} \\ y &= \frac{8x}{k} + \frac{3}{k} \end{aligned}$$

The slope of the second line is $\frac{8}{k}$.

Since the lines are parallel, the slopes must be equal. Set the two slopes equal to each other and solve for k.

$\frac{k}{2} = \frac{8}{k}$

$k^2 = 16$

$k = \sqrt{16}$

$k = \pm 4$

The values of k are 4 and −4.

b) Since the lines are perpendicular, the slope must have a product of −1.

$\left(\frac{k}{2}\right)\left(\frac{8}{k}\right) = -1$

$\frac{8k}{2k} = -1$

$4 \neq 1$

Therefore, there are no values of k that will work.

18. a) $y - 1 = 0$

b) $x + 2y - 4 = 0$

19. Example:

$7x + 9y - 82 = 0$, $7x + 9y + 48 = 0$,
$9x - 7y + 6 = 0$, $9x - 7y + 136 = 0$