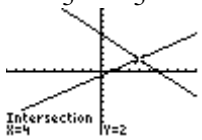
Chapter 8 Solving Systems of Linear Equations Graphically

8.1 Systems of Linear Equations and Graphs

1. a) $y = -x + 6$

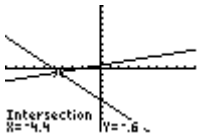
$y = \frac{2}{3}x - \frac{2}{3}$



The point of intersection is (4, 2).

b) $y = \frac{1}{4}x + \frac{1}{2}$

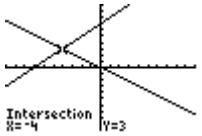
$y = -x - 5$



The point of intersection is $\left(-\frac{22}{5}, -\frac{3}{5}\right)$.

c) $y = -\frac{3}{4}x$

$y = x + 7$

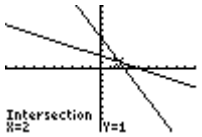


The point of intersection is $(-4, 3)$.

2. a) yes **b)** no

3. a) $y = -2x + 5$

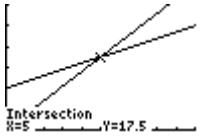
$y = -\frac{1}{2}x + 2$



The point of intersection is (2, 1).

b) $d = 3.5t$

$d = 1.5t + 10$



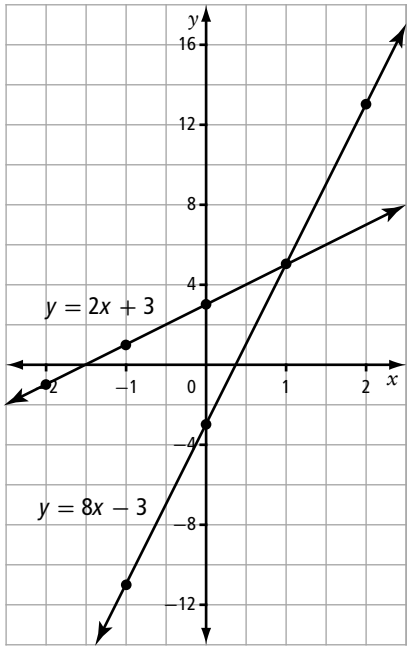
The point of intersection is (5, 17.5).

4. a) table of values for $y = 8x - 3$

x	−1	0	1	2
y	−11	−3	5	13

table of values for $y = 2x + 3$

x	−2	−1	0	1
y	−1	1	3	5



solution is (1, 5)

The solution (1, 5) can be verified by substitution.

$y = 8x - 3$

Left Side	Right Side
y	$8x - 3$
$= 5$	$= 8(1) - 3$
	$= 5$

Left Side = Right Side

$y = 2x + 3$

Left Side	Right Side
y	$2x + 3$
$= 5$	$= 2(1) + 3$
	$= 5$

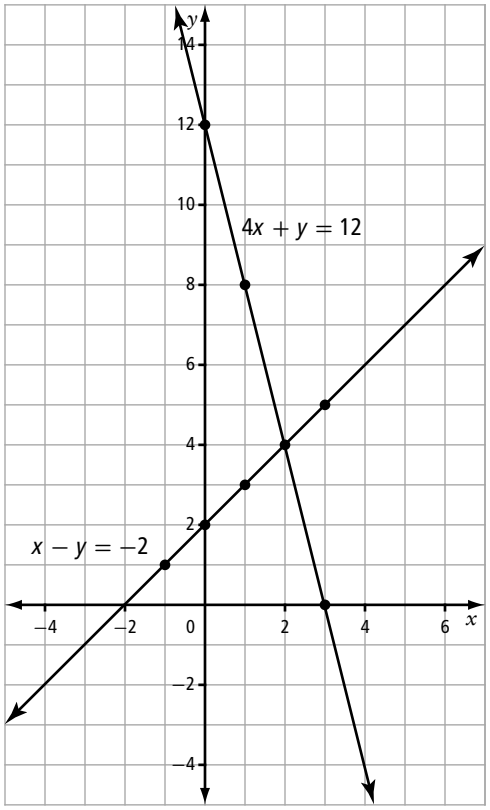
Left Side = Right Side

b) table of values for $x - y = -2$

x	−1	0	1	3
y	1	2	3	5

table of values for $4x + y = 12$

x	0	1	2	3
y	12	8	4	0



solution is (2, 4)

The solution (2, 4) can be verified by substitution.

$x - y = -2$

Left Side	Right Side
$x - y$	-2
$= 2 - 4$	$= -2$
$= -2$	

Left Side = Right Side

$4x + y = 12$

Left Side	Right Side
$4x + y$	12
$= 4(2) + 4$	$= 12$
$= 8 + 4$	
$= 12$	

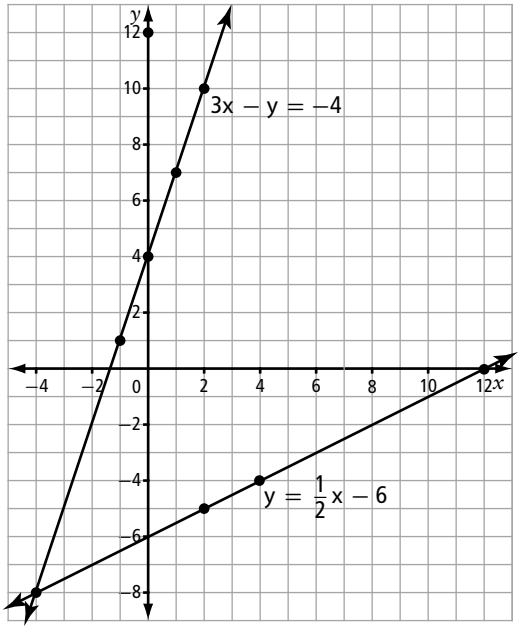
Left Side = Right Side

c) table of values for $y = \frac{1}{2}x - 6$

x	−4	2	4	12
y	−8	−5	−4	0

table of values for $3x - y = -4$

x	−2	−1	0	2
y	−2	1	4	10



solution is (−4, −8)

The solution (−4, −8) can be verified by substitution.

$y = \frac{1}{2}x - 6$

Left Side	Right Side
y	$\frac{1}{2}x - 6$
$= -8$	$= \frac{1}{2}(-4) - 6$
	$= -2 - 6$
	$= -8$

Left Side = Right Side

$3x - y = -4$

Left Side	Right Side
$3x - y$	-4
$= 3(-4) - (-8)$	$= -4$
$= -12 + 8$	
$= -4$	

Left Side = Right Side

5. a) $M = 10 + 0.5d$
$M = 5 + d$
b) Amounts of money are equal on day 10, when both have \$15.

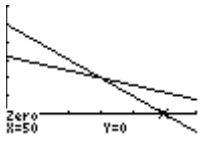
6. a) The initial volume of oil in the first tank is 125 m^3. If the tank is being drained at the rate of 2.5 m^3 per minute, the amount of oil remaining at any time is the initial amount less 2.5 times the number of minutes for which the tank has been emptied. Therefore, the amount of oil remaining in the first tank may be modelled by the equation $A = 125 - 2.5t$. Applying the same analysis in the case of the second tank yields the equation $A = 80 - t$ to model the amount of oil remaining in that tank. Thus, the system of linear equations to model this situation is $A = 125 - 2.5t$ and $A = 80 - t$.

b)

The point of intersection is (30, 50).

c) The point of intersection represents the time when the amounts of oil in the two tanks are equal and describes that amount. At the point (30, 50), both tanks have 50 m^3 of oil in them. This occurs after 30 min of draining.

d) The tank containing 125 m^3 of oil will drain first. From the graph you can see that the line of its equation intersects the x-axis first. At this point, $A = 0$, which means that the tank is empty.



7. a) $a + c = 69$
$15a + 10c = 900$
b) $c = -a + 69$
$c = -1.5a + 90$
c) The point of intersection is (42, 27).
d) The point of intersection describes the numbers of both types of tickets sold: 42 adults' tickets and 27 children's tickets.

8. a) $V = 200 - 8t$
$V = 0 + 8t$
b) The point of intersection is (12.5, 100).
c) The volumes in both tanks are equal at 100 L in each.
d) 25 min
e) no

9. a) x-intercept = 8; x-intercept = -1
b) y-intercept = 4; y-intercept = 1
c) slope is 1; slope is $-\frac{1}{2}$
d) (2, 3)
e) $y = x + 1$; $y = -\frac{1}{2}x + 4$

10. a) $x + y = 62$
$\frac{1}{2}x = 1 + y$
b) Mr. Darwal is 42 years old. His daughter is 20 years old.

11. a) $d = 210t$
$d = 150t$
b) (0, 0)
c) 240 km farther

12. Since two points are given for each line, you can use the slope formula to calculate the slopes of the lines. Slope of the first line passing through points (1, 1) and (4, 7):

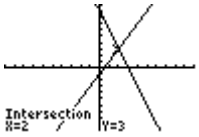
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{7 - 1}{4 - 1}$$
$$m = \frac{6}{3}$$
$$m = 2$$

Slope of the second line passing through the points (1, 6) and (3, 0):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{0 - 6}{3 - 1}$$
$$m = -\frac{6}{2}$$
$$m = -3$$

Using slope-point form, write the equation of each line.
First line: Use the point (1, 1) and slope 2.
$y - y_1 = m(x - x_1)$
$y - 1 = 2(x - 1)$
$y - 1 = 2x - 2$
Isolate y to express the equation in slope-intercept form.

$y - 1 = 2x - 2$
$y - 1 + 1 = 2x - 2 + 1$
$y = 2x - 1$
The equation of the first line is $y = 2x - 1$.
Second line: Use the point (3, 0) and slope -3.
$y - y_1 = m(x - x_1)$
$y - 0 = -3(x - 3)$
$y = -3x + 9$
The equation of the second line is $y = -3x + 9$.
Graphing these lines produces a point of intersection at (2, 3).



13. $-40°C = -40°F$

14. a) $d = 10t$
$d = 40(1 - t)$
or
$d = 40t$
$d = 10(1 - t)$

b) The x-coordinate represents the time required to travel between Ferdinand's home and the school in one of the directions. The y-coordinate represents distance travelled.

c) 0.2 h or 12 min; 0.8 h or 48 min

d) 8 km

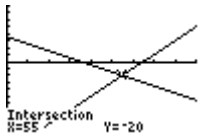
15. Yes, if lines are parallel they will have no point of intersection.

16. 8

17. In a parallelogram, opposite angles are equal and alternate interior angles are equal. Therefore, from the diagram you can see that $x + y = 35$ and $2x - y = 130$.
Express the equations in slope-intercept form:

$x + y = 35$	$2x - y = 130$
$x + y - x = 35 - x$	$2x - y - 2x = 130 - 2x$
$y = 35 - x$	$-y = 130 - 2x$
$y = -x + 35$	$y = 2x - 130$

Graphing these produces two lines that intersect at (55, -20).



Therefore, the value of x is 55 and the value of y is -20.

8.2 Modelling and Solving Linear Systems

1. a) $s = 3c$ and $s + 4 = 2(c + 4)$

b) $V = 5 + 0.9t$ and $V = 3 + 1.2t$

2. a)

x	y	$x - y$
30	138	-108
50	118	-68
70	98	-28
80	88	-8
90	78	12
100	68	32

It can be inferred that $90 < x < 100$ and $68 < y < 78$.

b) $x + y = 168$
$x - y = 18$

c)

$x + y = 168$	$x - y = 18$
$x + y - x = 168 - x$	$x - y - x = 18 - x$
$y = -x + 168$	$-y = 18 - x$
	$y = x - 18$

d)

The point of intersection is (93, 75).

e) Yes, this confirms the inferences drawn from the values in the table in part a) because 93 is between 90 and 100, and 75 is between 68 and 78.

3. a) $x + y = 15\,000$
$0.065x + 0.05y = 885$

b) $y = -x + 15\,000$
$y = -1.3x + 17\,700$

c) The point of intersection is (9000, 6000). Josee invested $9000 at 6.5% interest per year and invested $6000 at 5% interest per year.

4. **a)** $x + y = 5$
$0.4x + 0.25y = 0.32(5)$
b) $0.4x + 0.25y = 0.32(5)$
c) $y = -x + 5$ and $y = -1.6x + 6.4$
d) The point of intersection is $\left(\frac{7}{3}, \frac{8}{3}\right)$ or approximately (2.3, 2.7). The chemist needs about 2.3 L of 40% bromine solution and about 2.7 L of 25% bromine solution.

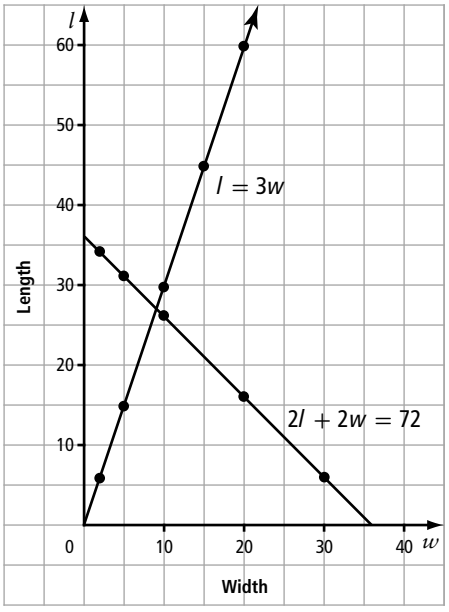
5. **a)** Answers may vary. Example:
$l = 3w$

w	2	5	10	15	20
l	6	15	30	45	60

$2l + 2w = 72$

w	2	5	10	20	30
l	34	31	26	16	6

b)



From reading the graph, the point of intersection can be estimated to be about (8, 27).
c) The actual point of intersection is (9, 27).
d) The width of the rectangle is 9 m and its length is 27 m.

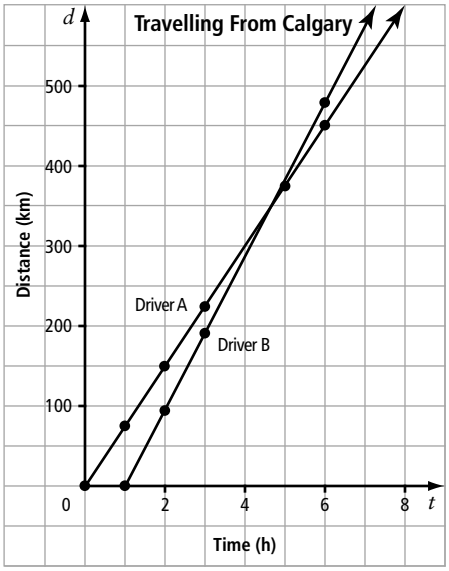
6. **a)** $c + p = 0.5$
$12c + 3p = 0.5(5)$
b) $p = -c + 0.5$
$p = -4c + 0.8\overline{3}$
c) The point of intersection is approximately (0.11, 0.39). The company should use 0.11 kg of cashews and 0.39 kg of peanuts per bag.

7. **a)** $A = 1200 - 20s$
$A = 200 + 30s$
b) 20 s after the time interval begins; 800 m

8. **a)** The first equation means that the difference in elevation between Mount Columbia and Cypress Hills is 2279 m. The second equation means that the sum of the elevations is 5215 m.
b) $s = a - 2279$ and $s = -a + 5215$
c) The point of intersection is (3747, 1468). Mount Columbia has an elevation of 3747 m and Cypress Hills has an elevation of 1468 m.

9. **a)** The first restaurant charges \$175 for room rental and \$20 per person. The second restaurant charges \$100 for room rental and \$22.50 per person.
b) The point of intersection is (30, 775).
c) The first restaurant is cheaper if there are more than 30 guests to a maximum of 100 guests: $30 < n \leq 100$. The second restaurant is cheaper if there are fewer than 30 guests: $0 < n < 30$.

10. **a)** Driver B left one hour later than Driver A.
b) Driver B
c) Driver B has caught up to and passed Driver A.
d)



From the graph, you can estimate that it takes approximately 5 h for the drivers to travel the same distance.

11. a) $2.5(s + c) = 50$; $4(s - c) = 50$

b) $s = -c + 20$; $s = c + 12.5$; the point of intersection is (3.75, 16.25).

c) The boat's speed is 16.25 km/h and the current's speed is 3.75 km/h.

12. a) One tank has 120 L. The other tank has 10 L.

b) The 120-L tank is draining and the 10-L tank is filling.

c) At 10 min, both tanks contain 90 L of water.

d) The slopes of the lines represent the rates at which the tanks are draining or filling. Use two points on each line and the slope formula to calculate slope.

First line: Use points (0, 120) and (10, 90).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{90 - 120}{10 - 0}$

$m = \frac{30}{10}$

$m = -3$

The first tank is draining at a rate of 3 L/min.

Second line: Use points (0, 10) and (10, 90).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{90 - 10}{10 - 0}$

$m = \frac{80}{10}$

$m = 8$

The second tank is filling at a rate of 8 L/min.

e) $V = 120 - 3t$ and $V = 10 + 8t$

13. a) The point of intersection is (1, 70).

b) Company A will charge less.

c) A: $C = 45 + 25t$

B: $C = 30 + 40t$

14. Answers may vary. Example:

a) Charges for Company A are \$100 for a sign-up fee and \$18.50 per month. Charges for Company B are \$75 to sign up, plus \$20 per month. Which company should you choose?

b) A \$900 deposit is put into two different investments. One part earns interest at a rate of 5% per year; the other earns interest at 4.5% per year. If total interest earned in one year is \$74, how much money was invested at each rate?

c) One person is 8 years older than another. Five times the first person's age added to 9 times the other person's age totals 100 years. Find the age of each person.

15. Answers may vary. Examples are: $x + y = 27$ and $x - y = -21$ or $y - 6x = 6$ and $3x + y = 33$

8.3 Number of Solutions for Systems of Linear Equations

1. a) no solution; lines have same slope and different y-intercepts

b) one solution; lines have different slopes

c) infinite number of solutions; lines are multiples of each other (same slope and same y-intercept)

d) no solution; lines have same slope and different y-intercepts

e) one solution; lines have different slopes

2. a) $2x - 3y = C$, $C \neq 8$

b) $x - 3y = 8$ or any other change to the coefficient of x or y

c) $4x - 6y = 16$ or any other multiple of the first equation

3. a) no solution

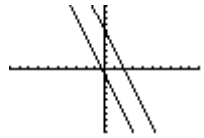
b) no solution

c) one solution

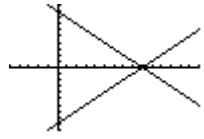
d) one solution

e) one solution

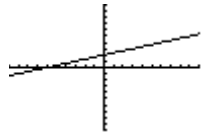
4. a) no solution



b) one solution



c) an infinite number of solutions



5. For a system of linear equations in the form $Ax + By = C$ and $Dx + Ey = F$:

a) If the coefficients D, E, and F are the same multiple of coefficients A, B, and C, respectively, then there will be an infinite number of solutions.

b) If coefficients D and E are the same multiple of coefficients A and B, respectively, but F is not this same multiple of C, then there will be no solution.

c) In all other cases, there will be one solution.

6. a) $7x - 3y = C$, $C \neq 12$

b) $14x - 6y = 24$ or any other multiple of the given equation

c) Example: $7x - 3y = 12$. Only coefficient A in the equation of the second line has to be different as that is the number that dictates the slope.

7. Jocelyn: $E = 1200 + 0.03s$;
Mario: $E = 1000 + 0.045s$;
Kendra: $E = 2000 + 0.03s$;
Pavel: $E = 2000 + 0.03s$

a) Jocelyn and Kendra or Jocelyn and Pavel

b) Kendra and Pavel

c) Jocelyn and Mario, or Mario and Kendra, or Mario and Pavel

8. a) $C = 20 + 0.35t$
$C = 15 + 0.4t$

b) $C = 20 + 0.35t$
$C = 15 + 0.35t$

c) $C = 20 + 0.35t$
$C = 20 + 0.35t$

9. a) Ling is correct. The slopes are similar, but the lines are not parallel and will intersect at (37.5, 25). Since the slopes of the lines are different, the system must have one solution.

b) The system can be solved by comparing coefficients. Because the left sides of the equations are identical, any value of y that is substituted will result in the same value on the left side of each equation. However, only one value of x (37.5) will yield equivalent values on the right side of the equations. Therefore, the system has only one solution.

10. a) Gold Coast: $E = 1.25k - 40$; The Salmon House: $E = 1.00k - 25$

b) one solution

c) The point of intersection is (60, 35). A fisher should bring a catch to The Salmon House when $k < 60$ kg because of the lower processing fee and to Gold Coast Fishery when $k > 60$ kg because of the higher rate of pay per kilogram.

d) There would be no point of intersection and The Salmon House would be the better choice regardless of the size of the catch.

11. a) $C \neq 60$

b) $C = 60$

12. a) $A = 1$

b) $A \neq 1$

13. a) $h = 5w$ and $h = 6w - 24$

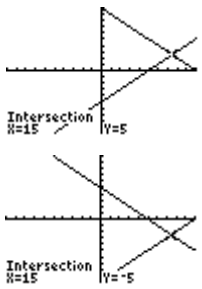
b) one solution

c) (24, 120); The solution represents the time when both towers are the same height. Each tower will be 120 ft in height 24 weeks after construction starts on the first tower.

14. a) The equations would be $x + y = 20$ and $x - y = 10$. This would lead to one solution since natural numbers that satisfy both equations can be found easily.

b) The equations would be $x + y = 10$ and $x - y = 20$. This would lead to no solution since a negative number is needed to satisfy this system of linear equations, but a negative number is not a natural number.

c)



d) The difference is due to the domain being N, which does not include negative numbers.

15. Let d represent distance, in metres Let t represent time, in seconds.

a) $d = 220 + 3.1t$ and $d = 198 + 3.6t$; There is one solution. Boat C catches up to Boat A in 44 s at 356.4 m into the race.

b) $d = 220 + 3.1t$ and $d = 230 + 3.2t$; There is one solution; however, at the point of intersection the value of t is less than zero, which is outside the range for time ($t \geq 0$). Boat D has already overtaken Boat A, which will not catch up.

c) $d = 206 + 3.4t$ and $d = 230 + 3.2t$; There is one solution; however, at the point of intersection the value of d is greater than 500, which is outside the domain for distance ($d \leq 500$). Boat B is gaining on Boat D, but will not catch up before Boat D crosses the finish line.

16. Eva is correct. Vince is correct in saying that the lines will intersect, but the intersection is at a negative time and a negative volume, which is not allowed in the context of the problem. In this problem, the domain of time is $t \geq 0$ and the range of volume is $V \geq 150$.

17. a) $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$ **b)** $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$