# Chapter 9 Solving Systems of Linear Equations Algebraically

## 9.1 Solving Systems of Linear Equations by Substitution

**1.** **a)** $x = 13.5$ and $y = 11.5$
**b)** $x = -11$ and $y = -44$
**c)** $x = -1$ and $y = 20$
**d)** $x = 2$ and $y = -3$
**e)** $x = 5$ and $y = -1$
**f)** $x = 2$ and $y = 3$

**2.** Method 1: Isolate the variable $x$ in the equation $2x - y = -5$.

$$2x - y = -5$$
$$2x - y + y = -5 + y$$
$$2x = -5 + y$$
$$\frac{2x}{2} = \frac{-5 + y}{2}$$
$$x = \frac{-5 + y}{2}$$

Substitute for $x$ in the equation $5x + y = -2$. Then, solve for $y$.

$$5x + y = -2$$
$$5\left(\frac{-5 + y}{2}\right) + y = -2$$
$$\frac{-25 + 5y}{2} + y = -2$$
$$2\left(\frac{-25 + 5y}{2} + y\right) = 2(-2)$$
$$-25 + 5y + 2y = -4$$
$$-25 + 7y = -4$$
$$-25 + 25 + 7y = -4 + 25$$
$$7y = 21$$
$$\frac{7y}{7} = \frac{21}{7}$$
$$y = 3$$

Substitute $y = 3$ in the equation $2x - y = -5$ and solve for $x$.

$$2x - y = -5$$
$$2x - 3 = -5$$
$$2x - 3 + 3 = -5 + 3$$
$$2x = -2$$
$$\frac{2x}{2} = \frac{-2}{2}$$
$$x = -1$$

The solution is $x = -1$ and $y = 3$.

Method 2: Isolate the variable $y$ in the equation $5x + y = -2$.

$$5x + y = -2$$
$$5x - 5x + y = -2 - 5x$$
$$y = -2 - 5x$$

Substitute for $y$ in the equation $2x - y = -5$. Then, solve for $x$.

$$2x - y = -5$$
$$2x - (-2 - 5x) = -5$$
$$2x + 2 + 5x = -5$$
$$7x + 2 = -5$$
$$7x + 2 - 2 = -5 - 2$$
$$7x = -7$$
$$\frac{7x}{7} = \frac{-7}{7}$$
$$x = -1$$

Substitute $x = -1$ in the equation $5x + y = -2$ and solve for $y$.

$$5x + y = -2$$
$$5(-1) + y = -2$$
$$-5 + y = -2$$
$$-5 + 5 + y = -2 + 5$$
$$y = 3$$

The solution is $x = -1$ and $y = 3$.
Example: I prefer the second method where I solved for $y$. The coefficient on $y$ was 1, so there were no fractions involved. In the first method, where I solved for $x$, there were fractions. Fractions involve more complicated computations than integers do.

**3.** The point (2, 4) is not the solution to the system $3x - y = 2$ and $x + y = 5$. The point (2, 4) lies on the line of the first equation because substituting 2 for $x$ and 4 for $y$ results in a true statement.

$$3x - y = 2$$
$$3(2) - 4 = 2$$
$$6 - 4 = 2$$
$$2 = 2$$

However, the point (2, 4) does not lie on the line $x + y = 5$ because replacing $x$ with 2 and $y$ with 4 does not result in a true statement.

$$x + y = 5$$
$$2 + 4 \neq 5$$

4. 6000

5. 400 shares of the \$4.50 stock and 880 shares of the \$2.50 stock

6. Let $C$ represent the number of cans. Let $B$ represent the number of bottles.
Write an equation to represent the total number of cans and bottles collected.
$C + B = 900$
Write an equation to represent the total amount of money, in dollars, received.
$0.1C + 0.25B = 145.20$
Solve the system of linear equations by substitution. Solve for $C$ in the first equation.
$$C + B = 900$$
$$C + B - B = 900 - B$$
$$C = 900 - B$$
Substitute for $C$ in the second equation and solve for $B$.
$$0.1C + 0.25B = 145.20$$
$$0.1(900 - B) + 0.25B = 145.20$$
$$90 - 0.1B + 0.25B = 145.20$$
$$90 + 0.15B = 145.20$$
$$90 - 90 + 0.15B = 145.20 - 90$$
$$0.15B = 55.20$$
$$\frac{(0.15B)}{0.15} = \frac{(55.20)}{0.15}$$
$$B = 368$$
Substitute $B = 368$ in the first equation and solve for $C$.
$$C + B = 900$$
$$C + 368 = 900$$
$$C + 368 - 368 = 900 - 368$$
$$C = 532$$
The team brought in 532 cans for recycling.

7. 14

8. 15 new wave songs and 3 hip-hop songs; It is assumed that the station plays the same number of each song in each hour.

9. Jane is 26. Tim is 14.

10. 42

11. music video: \$12.50; CD: \$4.75

12. Team A: 16 wins, 4 losses;
Team B: 4 wins, 16 losses

13. No. The correct system of linear equations is $T + Q + 45 = 125$ and $2T + 0.25Q + 45 = 184$. $T = 68$ and $Q = 12$. The coins in the machine consisted of 12 quarters, 45 \$1 coins, and 68 \$2 coins.

14. Step 2; The correct line is $2x - 15x + 10 = 23$. The solution is $x = -1$ and $y = 5$.

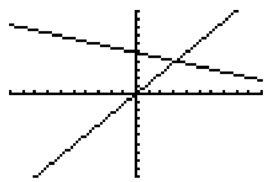
15. Let $x$ represent the distance over which Mandy drove at a speed of 80 km/h.
Let $y$ represent the distance over which Mandy drove at a speed of 100 km/h.
Write an equation to represent the total distance driven.
$x + y = 400$
Write a second equation to express the time of the trip in terms of speed and distance travelled. (Recall that time equals distance divided by speed.)
$\frac{x}{80} + \frac{y}{100} = 4.5$
Isolate the variable $y$ in the equation $x + y = 400$.
$$x + y = 400$$
$$x - x + y = 400 - x$$
$$y = 400 - x$$
Substitute for $y$ in the equation $\frac{x}{80} + \frac{y}{100} = 4.5$.
$$\frac{x}{80} + \frac{y}{100} = 4.5$$
$$\frac{x}{80} + \frac{400 - x}{100} = 4.5$$
Multiply each term of the equation by the lowest common multiple of the denominators to clear out the fractions. The lowest common multiple of 80 and 100 is 400.
$$400\left(\frac{x}{80} + \frac{400 - x}{100}\right) = 400(4.5)$$
$$\frac{400x}{80} + \frac{400(400 - x)}{100} = 1800$$
$$5x + 4(400 - x) = 1800$$
solve for $x$.
$$5x + 4(400 - x) = 1800$$
$$5x + 1600 - 4x = 1800$$
$$x + 1600 = 1800$$
$$x + 1600 - 1600 = 1800 - 1600$$
$$x = 200$$
Substitute $x = 200$ in the equation $x + y = 400$ and solve for $y$.
$$x + y = 400$$
$$200 + y = 400$$
$$200 - 200 + y = 400 - 200$$
$$y = 200$$
Mandy drove a distance of 200 km at a speed of 100 km/h.

**16.** $y = 4$

**17. a)** There is one solution. The slopes of the lines are different, which means that the lines intersect.

**b)** The point of intersection is located in the first quadrant. Rewrite each equation to isolate the variable $y$ and enter the new form of the equations into a graphing calculator.

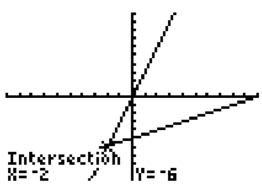
$$\begin{aligned} 5x - 4y &= 0 \\ 5x - 5x - 4y &= 0 - 5x \\ -4y &= -5x \\ -\frac{4y}{4} &= \frac{-5x}{4} \\ y &= \frac{5}{4}x \end{aligned} \qquad \begin{aligned} x + 3y &= 15 \\ x - x + 3y &= 15 - x \\ 3y &= 15 - x \\ \frac{3y}{3} &= \frac{15}{3} - \frac{x}{3} \\ y &= 5 - \frac{x}{3} \end{aligned}$$



The point of intersection is in the first quadrant.

**18.** $x = 14$, $y = 19$, and $z = -8$

**19. a)**




**b)** $x = -2$ and $y = -6$

**c)** Example: The methods are similar in that they produce the same solution. The difference is that the graphing method allows you to see how the two variables relate.

**d)** Example: I prefer the substitution method because there is a coefficient of 1 in one of the equations. This makes it simple to solve for the variable so that it can be substituted into the second equation.

## 9.2 Solving Systems of Linear Equations by Elimination

**1. a)** $x = -1$ and $y = 3$ **b)** $x = 1$ and $y = 1$
**c)** $x = 1$ and $y = 2$ **d)** $x = 8$ and $y = -2$
**e)** $x = -3$ and $y = 12$

**2. a)** $x + 3y = -1$, $2x + 4y = 12$ **b)** $2x + 3y = 1$, $4x - 2y = 10$
**c)** $3x - 2y = 5$, $-5x + 4y = 1$ **d)** $x - 3y = -4$, $4x + 2y = 12$
**e)** $3x + 2y = -9$, $2x + 3y = 9$

**3. a)** $x = 20$ and $y = -7$ **b)** $x = 2$ and $y = -1$
**c)** $x = 11$ and $y = 14$ **d)** $x = 2$ and $y = 2$
**e)** $x = -9$ and $y = 9$

**4. a)** $x = 2$ and $y = 3$

**b)** Multiply each term of the equation $\frac{1}{2}x - \frac{1}{3}y = 1$ by the lowest common multiple of the denominators to clear out the fractions. The lowest common multiple of 2 and 3 is 6.

$$\begin{aligned} 6\left(\frac{1}{2}x - \frac{1}{3}y\right) &= 6(1) \\ \frac{6}{2}x - \frac{6}{3}y &= 6 \\ 3x - 2y &= 6 \end{aligned}$$

Multiply each term of the equation $x + \frac{1}{4}y = 2$ by the lowest common multiple of the denominators to clear out the fractions. The lowest common multiple of 1 and 4 is 4.

$$\begin{aligned} 4\left(x + \frac{1}{4}y\right) &= 4(2) \\ 4x + \frac{4}{4}y &= 8 \\ 4x + y &= 8 \end{aligned}$$

Eliminate variable $y$. The lowest common multiple of 1 and 2 is 2. Multiply the equation $3x - 2y = 6$ by 1 and multiply the equation $4x + y = 8$ by 2.

$$\begin{aligned} 3x - 2y &= 6 \\ 1(3x - 2y) &= 1(6) \\ 3x - 2y &= 6 \end{aligned} \qquad \begin{aligned} 4x + y &= 8 \\ 2(4x + y) &= 2(8) \\ 8x + 2y &= 16 \end{aligned}$$

Add the two equations to eliminate $y$.

$$\begin{array}{r} 3x - 2y = 6 \\ +(8x + 2y = 16) \\ \hline 11x \quad\quad = 22 \end{array}$$

Solve for $x$.

$$\begin{aligned} \frac{11x}{11} &= \frac{22}{11} \\ x &= 2 \end{aligned}$$

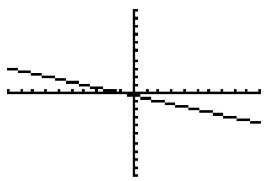
Substitute $x = 2$ into the second equation and solve for $y$.

$$x + \frac{1}{4}y = 2$$
$$2 + \frac{1}{4}y = 2$$
$$2 - 2 + \frac{1}{4}y = 2 - 2$$
$$\frac{1}{4}y = 0$$
$$y = 0$$

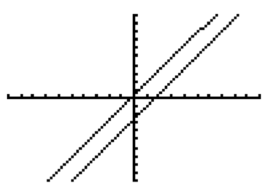
The solution to the system is $x = 2$ and $y = 0$.

**c)** $x = -6$ and $y = 10$

**5. a)** The two equations are the same. Therefore, the solution is an infinite number of points. Another method of finding the solution is to write each equation in "$y =$" form and enter it into a graphing calculator.



**b)** The two equations represent lines with the same slope. The lines are parallel and do not intersect. Therefore, there is no solution. Another method of finding the solution is to write each equation in "$y =$" form and enter it into a graphing calculator.



**6.** 24 \$2 coins and 112 \$1 coins

**7.** The initiation fee is \$45 and the monthly fee is \$25.

**8.** The daily rental charge was \$28 and the charge per kilometre was \$0.25.

**9.** \$300 at 12% and \$360 at 10%

**10.** 175 motorcycles and 325 cars

**11.** adult: \$19; child: \$11

**12.** 0.9 miles

**13.** 2050 vehicles

**14.** Let $C$ represent the number of oranges purchased.
Let $G$ represent the number of granola bars purchased.
Write an equation to represent the total number of snacks purchased.
$C + G = 50$
Express the cost of the snacks in terms of the price of each item. Since oranges cost \$2.40 per dozen, the price of one orange is $\frac{\$2.40}{12} = \$0.20$. Since the price of a 5-bar box of granola is \$3.25, the price of one bar is $\frac{\$3.25}{5} = \$0.65$.
Write an equation to represent the total amount of money that Shanice spent.

$0.2C + 0.65G = 19$

Eliminate the variable $C$. Multiply the second equation by 5 to make the coefficient of $C$ equal to 1.

$$5(0.2C + 0.65G) = 5(19)$$
$$C + 3.25G = 95$$

Subtract this new form of the second equation from the first equation.

$$\begin{array}{r} C + \quad G = \ 50 \\ -(C + 3.25G = \ 95) \\ \hline -2.25G = -45 \end{array}$$

Solve for $G$.

$$-2.25G = -45$$
$$\frac{-2.25G}{-2.25} = \frac{-45}{-2.25}$$
$$G = 20$$

Therefore, Shanice bought 20 granola bars, or 4 boxes of bars.

**15.** Cashews cost \$2.60 per pound. Peanuts cost \$1.50 per pound.

**16. a)** Let $T$ represent the 10s digit. Let $D$ represent the 1s digit.
The original number may be expressed as $10T + D$.
The number with the digits of the first number reversed is represented as $10D + T$.
Write an equation to represent the sum of the digits.
$T + D = 14$

Write an equation to represent the number formed by reversing the digits that is 36 more than the original number.

$10D + T = 36 + 10T + D$

Rewrite the equation to be in the same form as the first equation.

$$10D + T = 36 + 10T + D$$
$$10D + T - 10T - D = 36 + 10T - 10T + D - D$$
$$-9T + 9D = 36$$

Solve the system using elimination. Eliminate the variable $T$.

The lowest common multiple of 1 and 9 is 9. Multiply the first equation by 9 and multiply the second equation by 1.

| | |
|---|---|
| $T + D = 14$ | $-9T + 9D = 36$ |
| $9(T + D) = 9(14)$ | $1(-9T + 9D) = 1(36)$ |
| $9T + 9D = 126$ | $-9T + 9D = 36$ |

Add the two equations.

$$\begin{array}{r} 9T + 9D = 126 \\ +(-9T + 9D = \phantom{1}36) \\ \hline 18D = 162 \end{array}$$

Solve for $D$.

$$18D = 162$$
$$\frac{18D}{18} = \frac{162}{18}$$
$$D = 9$$

Substitute $D = 9$ in the first equation and solve for $T$.

$$T + D = 14$$
$$T + 9 = 14$$
$$T + 9 - 9 = 14 - 9$$
$$T = 5$$

The solution is $T = 5$ and $D = 9$. Therefore, the original number is 59.

**b)** 56

**17.** $m = 7$ and $n = 2$

**18.** Example: **a)** $4x + 2y = 6$ **b)** $2x + y = 5$ **c)** $y = -10$

**19.** $A = 3$

**20. a)** $x = -1$ and $y = 1$ **b)** $x = -1$ and $y = 1$

**c)** Example: I prefer the elimination method because using substitution involved using fractions.

**d)** Example: When using the substitution method, I look for a coefficient of 1 in one of the equations. If there is no coefficient of 1, then I use the elimination method.

## 9.3 Solving Problems Using Systems of Linear Equations

**1. a)** $x = -2$ and $y = -6$ **b)** $x = 0$ and $y = 3$ **c)** $x = 2$ and $y = -2$ **d)** $x = 0$ and $y = -5$ **e)** $x = -20$ and $y = 7$

**2. a)** Solve by elimination.

Eliminate the variable $x$. The lowest common multiple of 2 and 8 is 8. Multiply the first equation by 4 and multiply the second equation by 1.

| | |
|---|---|
| $2x - 5y = -18$ | $8x - 13y = -58$ |
| $4(2x - 5y) = 4(-18)$ | $1(8x - 13y) = 1(-58)$ |
| $8x - 20y = -72$ | $8x - 13y = -58$ |

Subtract the second equation from the new form of the first equation.

$$\begin{array}{r} 8x - 20y = -72 \\ -(8x - 13y = -58) \\ \hline -7y = -14 \end{array}$$

Solve for $y$.

$$-7y = -14$$
$$\frac{-7y}{-7} = \frac{-14}{-7}$$
$$y = 2$$

Substitute $y = 2$ in the first equation and solve for $x$.

$$2x - 5y = -18$$
$$2x - 5(2) = -18$$
$$2x - 10 = -18$$
$$2x - 10 + 10 = -18 + 10$$
$$2x = -8$$
$$\frac{2x}{2} = \frac{-8}{2}$$
$$x = -4$$

The solution is $x = -4$ and $y = 2$.

**b)** $x = \frac{3}{14}$ and $y = \frac{15}{14}$

**c)** $x = 3$ and $y = -3$

**3.** The width is 1080 m and the length is 2120 m.

**4.** He should invest \$6000 at 9% and \$6000 at 11%.

**5.** 15 mph

**6.** \$1.50

7. Let $H$ represent the amount of hay. Let $G$ represent the amount of grain.
Write an equation to represent the daily amounts of hay and grain that are fed to the horse.
$H + G = 20$
The total cost to feed the horse for 60 days is \$702. Therefore, the daily cost is
$\frac{\$702}{60} = \$11.70$.
Write an equation to represent the cost of feeding the horse each day.
$0.08H + 2.10G = 11.70$
Solve by substitution. Solve for $H$ in the first equation.
$H + G = 20$
$H + G - G = 20 - G$
$H = 20 - G$
Substitute $H = 20 - G$ into the second equation and solve for $G$.

$$\begin{aligned} 0.08H + 2.10G &= 11.70 \\ 0.08(20 - G) + 2.10G &= 11.70 \\ 1.6 - 0.08G + 2.10G &= 11.70 \\ 2.02G &= 10.10 \\ \frac{2.02G}{2.02} &= \frac{10.10}{2.02} \\ G &= 5 \end{aligned}$$

Substitute $G = 5$ into the first equation and solve for $H$.

$$\begin{aligned} H + G &= 20 \\ H + 5 &= 20 \\ H + 5 - 5 &= 20 - 5 \\ H &= 15 \end{aligned}$$

The solution is $H = 15$ and $G =$ 5.Therefore, the horse is fed 15 lbs of hay and 5 lbs of grain per day.

8. The team made 32 field goals and 6 three-point shots.

9. 250

10. Tom spent 25 h swimming and 45 h biking.

11. The hourly wage for outdoor work is \$18 and the hourly wage for indoor work is \$15.

12. **a)** $y = 0.15x + 25$, $y = 0.10x + 30$
**b)** 100 km

13. **a)** $2W + T = 26$, $3T = W + 1$
**b)** 11

14. **a)** 458 km **b)** 38.93 L

15. The slower snowmobile rider rides at 40 km/h. The faster snowmobile rider rides at 55 km/h. The assumption is that both riders are riding at a constant rate of speed.

16. 771.4 mL

17. Let $C$ represent the number of correct answers. Let $I$ represent the number of incorrect answers.
Write an equation to represent the total number of answers on the test. The assumption is that all questions were answered.
$C + I = 76$
Write an equation to represent the total score achieved.
$C + (-0.2)I = 58$
Solve for $C$ in the first equation.

$$\begin{aligned} C + I &= 76 \\ C + I - I &= 76 - I \\ C &= 76 - I \end{aligned}$$

Substitute $C = 76 - I$ into the second equation and solve for $I$.

$$\begin{aligned} C + (-0.2)I &= 58 \\ 76 - I + (-0.2)I &= 58 \\ 76 - 1.2I &= 58 \\ 76 - 76 - 1.2I &= 58 - 76 \\ -1.2I &= -18 \\ \frac{-1.2I}{1.2} &= \frac{-18}{1.2} \\ I &= 15 \end{aligned}$$

Substitute $I = 15$ into the first equation and solve for $C$.

$$\begin{aligned} C + I &= 76 \\ C + 15 &= 76 \\ C + 15 - 15 &= 76 - 15 \\ C &= 61 \end{aligned}$$

Therefore, the student answered 61 questions correctly.

18. 62.5 square units

19. **a)–c)** Answers may vary.