Math 10 Foundations Unit 1 LG 1-2 Answers

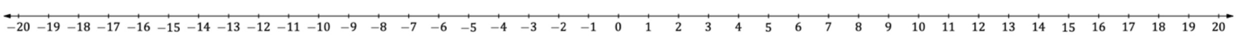
Welcome to the start of Math 10 Foundations! The first part of the unit will review adding/subtracting/multiplying and dividing integers and fractions. There are lots of practice questions here. You should do enough practice so you can do them in your sleep!

Adding and Subtracting Integers.

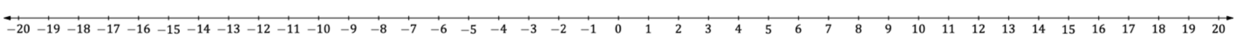
Click here to watch the video on Adding Integers or use the QR code.

Put a dot for your starting value, then move to the:
- **RIGHT if *adding* positivity (+)**
- **LEFT if *adding* negativity (-)**

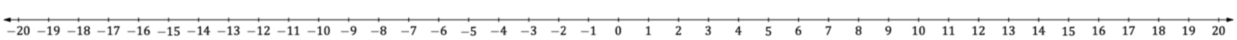
Example 1. (+4) + (+7) =



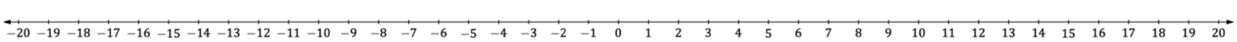
Example 2. (-8) + (+5) =



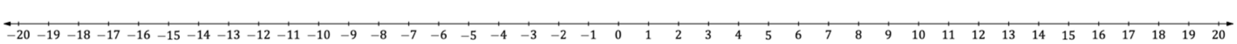
Example 3. (+4) + (-7) =



Example 4. (-10) + (-5) =



Example 5. (+20) + (-20) =



Answer the questions by using a number line or by doing the calculations in in your head.

Put a dot for your starting value, then move to the:

- **RIGHT if adding positivity (+)**
- **LEFT if adding negativity (-)**

$(-8) + (-6) =$ -14	$(+7) + (+5) =$ 12
$(+6) + (+9) =$ 15	$(-9) + (-9) =$ -18
$(+6) + (+8) =$ 14	$(-5) + (-3) =$ -8
$(-6) + (-6) =$ -12	$(-1) + (-4) =$ -5
$(-7) + (-7) =$ -14	$(-3) + (-1) =$ -4
$(+9) + (-6) =$ 3	$(-9) + (+2) =$ -7
$(-9) + (-7) =$ -16	$(-4) + (+5) =$ 1
$(-8) + (+9) =$ 1	$(+5) + (-7) =$ -2
$(+8) + (+7) =$ 15	$(-8) + (+2) =$ -6
$(+9) + (-1) =$ 8	$(-2) + (-2) =$ -4
$(+6) + (-7) =$ -1	$(-2) + (-8) =$ -10
$(-1) + (-5) =$ -6	$(+3) + (-8) =$ -5
$(+8) + (+5) =$ 13	

Modelling: Subtracting negative & positive integers using a number line.

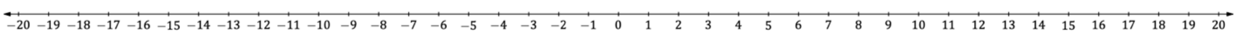
We all like to feel pretty good. Imagine that someone comes along and takes away your positivity – that would make you more negative right? It is the same with numbers. If you are taking away positivity (+ number) then it becomes more negative. Same if you take away someone's negativity (- number) then it will be more positive!

[Click here to watch the video on Subtracting Integers or use the QR code.]

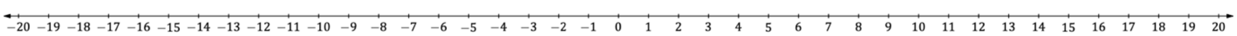
Put a dot for your starting value, then move to the:

- **LEFT if *subtracting* positivity (+)**
- **RIGHT if *subtracting* negativity (-)** **- (-) subtracting a (-) means we are adding**

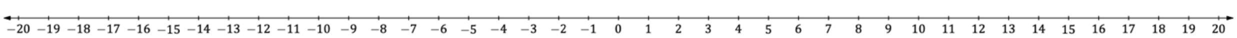
Example 1. (+4) - (+7) =



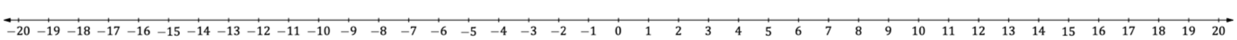
Example 2. (-8) - (+5) =



Example 3. (+4) - (-7) =



Example 4. (-10) - (-5) =



Example 5. (-10) - (-10) =

Answer the questions by using a number line or by doing the calculations in in your head.

Put a dot for your starting value, then move to the:

- **<u>LEFT if *subtracting* positivity (+)</u>**
- **<u>RIGHT if *subtracting* negativity (-)   - (-)   subtracting a (-) means we are adding</u>**

$(+7)-(+3)=$ 4	$(+4)-(-7)=$ 11
$(-9)-(-1)=$ -8	$(+5)-(-7)=$ 12
$(+7)-(-9)=$ 16	$(+9)-(+7)=$ 2
$(-8)-(-7)=$ -1	$(+9)-(+4)=$ 5
$(+7)-(+2)=$ 5	$(-5)-(+3)=$ -8
$(-4)-(-8)=$ 4	$(-6)-(+3)=$ -9
$(+1)-(-7)=$ 8	$(-2)-(+4)=$ -6
$(-6)-(+8)=$ -14	$(+4)-(+9)=$ -5
$(-6)-(+2)=$ -8	$(+3)-(-7)=$ 10
$(-8)-(+1)=$ -9	$(-4)-(-5)=$ 1
$(+6)-(-4)=$ 10	$(-7)-(-7)=$ 0
$(-9)-(+5)=$ -14	$(+7)-(+4)=$ 3
$(-7)-(-6)=$ -1	

$(+6) + (+6) =$ 12

$(+8) - (-5) =$ 13

$(-5) - (-9) =$ 4

$(-9) + (-8) =$ -17

$(+4) - (-9) =$ 13

$(-5) + (+7) =$ 2

$(-7) + (-6) =$ -13

$(+6) - (-7) =$ 13

$(-6) - (-1) =$ -5

$(+4) - (-7) =$ 11

$(-8) - (-2) =$ -6

$(-4) - (-6) =$ 2

$(-3) + (+9) =$ 6

$(-6) + (+5) =$ -1

$(-7) - (+4) =$ -11

$(-4) + (+8) =$ 4

$(+5) + (+8) =$ 13

$(-7) - (+7) =$ -14

$(+8) - (+3) =$ 5

$(+8) - (-7) =$ 15

$(+6) - (-2) =$ 8

$(-3) - (-1) =$ -2

$(-5) - (+6) =$ -11

$(-2) - (-9) =$ 7

$(+4) + (+4) =$ 8

Multiplying & Dividing Integers

$+ \times + = +$
$- \times - = +$
$+ \div + = +$
$- \div - = +$

Multiplication & Division

Same signs: Positive answer	Different signs: Negative answer
(-6) x (-5) = (+30)	(-12) ÷ (+3) = (-4)

$+ \times - = -$
$- \times + = -$
$+ \div - = -$
$- \div + = -$

[Click here to watch the video on Multiplying and Dividing Integers or use the QR code.]

Using a multiplication table, let's try these together:

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

$8 \times (-12) =$

$7 \div (-7) =$

$44 \div 4 =$

$-6 \times 6 =$

$27 \div (-9) =$

$-4 \times (-7) =$

$24 \div 8 =$

$-9 \times 2 =$

$6 \times 9 =$

$-3 \times (-5) =$

Circle all equations that will have a **positive** answer:

$64 \div 8 =$	$-11 \div (-1) =$	$-2 \times (-8) =$	$-80 \div (-10) =$
$-120 \div 10 =$	$-36 \div 4 =$	$-6 \div 1 =$	$-42 \div (-6) =$
$-8 \times (-9) =$	$7 \times (-9) =$	$120 \div (-12) =$	$-6 \times (-11) =$

Circle all equations that will have a **negative** answer:

$-9 \times 10 =$	$-54 \div (-6) =$	$10 \div 1 =$	$-56 \div 7 =$
$9 \times 8 =$	$5 \times 3 =$	$20 \div 5 =$	$-10 \div (-10) =$
$11 \times (-11) =$	$2 \div (-1) =$	$-24 \div 3 =$	$-9 \times 3 =$

Let's practice all the multiplication & division rules together now.

$12 \times (-11) = -132$

$-9 \times (-12) = 108$

$-10 \times 12 = -120$

$-11 \times 8 = -88$

$-10 \times (-10) = 100$

$108 \div (-9) = -972$

$121 \div 11 = 11$

$-132 \div 12 = -11$

$10 \times 9 = 90$

$80 \div 10 = 8$

$10 \times 11 = 110$

$8 \times (-9) = -72$

$90 \div 10 = 9$

$11 \times 10 = 110$

$-96 \div 12 = -8$

$-120 \div (-10) = 12$

$-80 \div (-8) = 10$

$1 \times (-8) = -8$

$48 \div (-12) = -4$

$11 \times (-7) = -77$

$84 \div (-7) = -12$

$45 \div (-5) = -9$

$-7 \div (-7) = 1$

$-21 \div (-3) = 7$

$-25 \div (-5) = 5$

$15 \div (-3) = -5$

$-6 \times (-5) = 30$

$-72 \div (-12) = 6$

$24 \div (-6) = -4$

$-18 \div 9 = -2$

$-81 \div (-9) = 9$

$12 \times (-4) = -48$

$18 \div (-2) = -9$

$-4 \times (-11) = 44$

$-3 \times (-11) = 33$

$-10 \times (-3) = 30$

$3 \div (-1) = -3$

$-2 \times (-3) = 6$

$6 \times (-7) = -42$

$2 \times (-12) = -24$

$99 \div (-9) = -11$

$12 \times 12 = 144$

$-5 \times 2 = -10$

$-5 \times (-7) = 35$

$-77 \div (-11) = 7$

$-14 \div (-2) = 7$

$9 \times (-8) = -72$

$-5 \times (-10) = 50$

$99 \div (-11) = -9$

$-3 \div (-3) = 1$

Adding and subtracting fractions.

Click here to watch the video on Adding and Subtracting Fractions or use the QR code.

When adding and subtracting fractions, the denominator NEEDS to be the SAME! Use the butterfly method to solve.

Butterfly method:

1. **Multiply denominators to find common denominator:**

$$\frac{2}{6} + \frac{1}{2} = \frac{\square}{12} + \frac{\square}{12}$$

6 x 2 = 12

2. **Then multiply the across.**

2 x 2 = 4 1 x 6 = 6

$$\frac{2}{6} + \frac{1}{2} = \frac{4}{12} + \frac{6}{12}$$

*always multiply the numerator in the 1st fraction 1st

3. **Now add the fractions.**

$$\frac{4}{12} + \frac{6}{12} = \frac{10}{12}$$

4. **Now reduce.**

$$\frac{10}{12} = \frac{5}{6}$$

You can use the same method for subtraction too!

1) $\frac{1^{\times 5}}{4_{\times 5}} + \frac{9^{\times 2}}{10_{\times 2}} = \frac{5}{20} + \frac{18}{20} = \frac{23}{20}$

2) $\frac{1^{\times 2}}{2_{\times 2}} + \frac{2}{4} = \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$

3) $\frac{4^{\times 4}}{5_{\times 4}} + \frac{1^{\times 5}}{4_{\times 5}} = \frac{16}{20} + \frac{5}{20} = \frac{21}{20}$

4) $\frac{1^{\times 3}}{2_{\times 3}} + \frac{2^{\times 2}}{3_{\times 2}} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

5) $\frac{1^{\times 5}}{2_{\times 5}} + \frac{3^{\times 2}}{5_{\times 2}} = \frac{5}{10} + \frac{6}{10} = \frac{11}{10}$

6) $\frac{8}{10} - \frac{1^{\times 5}}{2_{\times 5}} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$

7) $\frac{3^{\times 5}}{4_{\times 5}} - \frac{5^{\times 2}}{10_{\times 2}} = \frac{15}{20} - \frac{10}{20} = \frac{5}{20} = \frac{1}{4}$

8) $\frac{3}{4} - \frac{1^{\times 2}}{2_{\times 2}} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

9) $\frac{3}{10} - \frac{1^{\times 2}}{5_{\times 2}} = \frac{3}{10} - \frac{2}{10} = \frac{1}{10}$

10) $\frac{3^{\times 2}}{5_{\times 2}} - \frac{6}{10} = \frac{6}{10} - \frac{6}{10} = 0$

Adding and subtracting mixed numbers.

Step 1. Change to improper.
Step 2. Use butterfly method to solve.

[Click here to watch the video on Adding and Subtracting Mixed Numbers or use the QR code.]

Example 1. $1\frac{2}{7} + 4\frac{4}{5} =$

Example 2. $4\frac{2}{3} - 2\frac{5}{6} =$

1. $\frac{15^{\times 6}}{7_{\times 6}} - \frac{2^{\times 7}}{6_{\times 7}} =$ $\frac{90}{42} - \frac{14}{42} = \frac{76}{42} = \frac{38}{21}$

2. $\frac{9^{\times 9}}{5_{\times 9}} - \frac{5^{\times 5}}{9_{\times 5}} =$ $\frac{81}{45} - \frac{25}{45} = \frac{56}{45}$

3. $\frac{11^{\times 10}}{9_{\times 10}} - \frac{2^{\times 9}}{10_{\times 4}} =$ $\frac{110}{90} - \frac{18}{90} = \frac{92}{90} = \frac{46}{45}$

4. $\frac{13^{\times 9}}{7_{\times 9}} - \frac{1^{\times 7}}{9_{\times 7}} =$ $\frac{117}{63} - \frac{7}{63} = \frac{110}{63}$

5. $\frac{33^{\times 4}}{9_{\times 4}} - \frac{7^{\times 9}}{4_{\times 9}} =$ $\frac{132}{36} - \frac{63}{36} = \frac{69}{36} = \frac{23}{12}$

6. $\frac{10^{\times 11}}{4_{\times 11}} + \frac{10^{\times 4}}{11_{\times 4}} =$ $\frac{110}{44} + \frac{40}{44} = \frac{150}{44} = \frac{75}{22}$

7. $\frac{10^{\times 7}}{3_{\times 7}} + \frac{26^{\times 3}}{7_{\times 3}} =$ $\frac{70}{21} + \frac{78}{21} = \frac{148}{21}$

8. $\frac{7^{\times 9}}{8_{\times 9}} + \frac{31^{\times 8}}{9_{\times 8}} =$ $\frac{63}{72} + \frac{248}{72} = \frac{311}{72}$

9. $\frac{14^{\times 17}}{9_{\times 17}} + \frac{57^{\times 9}}{17_{\times 9}} =$ $\frac{238}{153} + \frac{513}{153} = \frac{751}{153}$

10. $\frac{15^{\times 17}}{6_{\times 17}} + \frac{22^{\times 6}}{17_{\times 6}} =$ $\frac{255}{102} + \frac{132}{102} = \frac{387}{102} = \frac{129}{34}$

Multiplying Fractions

Simple: multiply the top (numerator), then multiply the bottom (denominator)!

Click here to watch the video on Multiplying Fractions or use the QR code.

1. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

2. $\frac{1}{3} \times \frac{1}{7} = \frac{1}{21}$

3. $\frac{6}{7} \times \frac{5}{7} = \frac{30}{49}$

4. $\frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$

5. $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

6. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

7. $\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

8. $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

9. $\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$

10. $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

11. $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

12. $\frac{1}{8} \times \frac{3}{4} = \frac{3}{32}$

13. $\frac{2}{5} \times \frac{4}{9} = \frac{8}{45}$

14. $\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$

15. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

16. $\frac{6}{7} \times \frac{6}{7} = \frac{36}{49}$

17. $\frac{1}{7} \times \frac{3}{4} = \frac{3}{28}$

18. $\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$

19. $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

20. $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$

Dividing Fractions

Keep *1st fraction* **Flip** *multiplication to division* **Flip** *2nd fraction*

[Click here to watch the video on Dividing Fractions or use the QR code.]

1. $\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$

Inversion Result

2. $\frac{1}{8} \div \frac{2}{3} = \frac{1}{8} \times \frac{3}{2} = \frac{3}{16}$

3. $\frac{1}{5} \div \frac{4}{9} = \frac{1}{5} \times \frac{9}{4} = \frac{9}{20}$

4. $\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \times \frac{3}{2} = \frac{9}{16}$

5. $\frac{3}{7} \div \frac{2}{3} = \frac{3}{7} \times \frac{3}{2} = \frac{9}{14}$

6. $\frac{3}{4} \div \frac{8}{9} = \frac{3}{4} \times \frac{9}{8} = \frac{27}{32}$

7. $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$

8. $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

9. $\frac{1}{3} \div \frac{5}{7} = \frac{1}{3} \times \frac{7}{5} = \frac{7}{15}$

10. $\frac{2}{9} \div \frac{1}{2} = \frac{2}{9} \times \frac{2}{1} = \frac{4}{9}$

11. $\frac{5}{6} \div \frac{6}{7} = \frac{5}{6} \times \frac{7}{6} = \frac{35}{36}$

12. $\frac{1}{4} \div \frac{4}{5} = \frac{1}{4} \times \frac{5}{4} = \frac{5}{16}$

13. $\frac{1}{8} \div \frac{2}{7} = \frac{1}{8} \times \frac{7}{2} = \frac{7}{16}$

14. $\frac{1}{2} \div \frac{5}{7} = \frac{1}{2} \times \frac{7}{5} = \frac{7}{10}$

15. $\frac{1}{7} \div \frac{1}{5} = \frac{1}{7} \times \frac{5}{1} = \frac{5}{7}$

16. $\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$

17. $\frac{1}{4} \div \frac{2}{7} = \frac{1}{4} \times \frac{7}{2} = \frac{7}{8}$

18. $\frac{1}{5} \div \frac{2}{3} = \frac{1}{5} \times \frac{3}{2} = \frac{3}{10}$

19. $\frac{6}{7} \div \frac{7}{8} = \frac{6}{7} \times \frac{8}{7} = \frac{48}{49}$

20. $\frac{1}{5} \div \frac{1}{4} = \frac{1}{5} \times \frac{4}{1} = \frac{4}{5}$

Operation with fractions word problems:

1. **Fruit Salad Mix**
   Sarah used $\frac{3}{8}$ of a cup of strawberries and $\frac{5}{8}$ of a cup of blueberries to make a fruit salad.
   - How many cups of fruit did she use in total?

     $\frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1$ SHE USED 1 CUP.

   - If she eats $\frac{1}{4}$ of the salad, how much is left?

     $1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}$ $\frac{3}{4}$ IS LEFT.

2. **Pizza Party**
   At a party, $\frac{2}{3}$ of a pizza was eaten by the kids, and $\frac{1}{5}$ was eaten by the adults.
   - How much of the pizza was eaten altogether?

     $\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$ $\frac{13}{15}$ WAS EATEN.

   - How much of the pizza remains?

     $1 - \frac{13}{15} = \frac{15}{15} - \frac{13}{15} = \frac{2}{15}$ $\frac{2}{15}$ IS LEFT.

3. **Milk Recipe**
   A recipe calls for $\frac{2}{5}$ of a liter of milk. If Maria wants to make 3 batches of the recipe, how many liters of milk will she need?

   $3 \times \frac{2}{5} = \frac{3}{1} \times \frac{2}{5} = \frac{6}{5}$ SHE WILL NEED $\frac{6}{5}$ OR $1\frac{1}{5}$ L OF MILK.

4. **Garden Plot**

A rectangular garden is $\frac{3}{4}$ meters wide and $\frac{2}{3}$ meters long.

- What is the area of the garden?

$$A = L \times W$$

$$A = \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

THE AREA IS $\frac{1}{2}$ m^2.

5. **Sharing Cookies**

A baker has $\frac{5}{6}$ of a batch of cookies left. He wants to divide them equally among 3 friends.

- How much of the batch does each friend get?

$$\frac{5}{6} \div \frac{3}{1} = \frac{5}{6} \times \frac{1}{3} = \frac{5}{18}$$

EACH PERSON GETS $\frac{5}{18}$ OF THE BATCH.

6. **Juice Bottles**

A jug contains $\frac{7}{8}$ of a liter of juice. If each bottle holds $\frac{1}{4}$ of a liter, how many bottles can be filled?

$$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \times \frac{4}{1} = \frac{28}{8} = \frac{7}{2}$$

$\frac{7}{2}$ OR 3.5 BOTTLES CAN BE FILLED.

Number Systems Refresher

Natural Numbers: $\{1, 2, 3 \ldots\}$

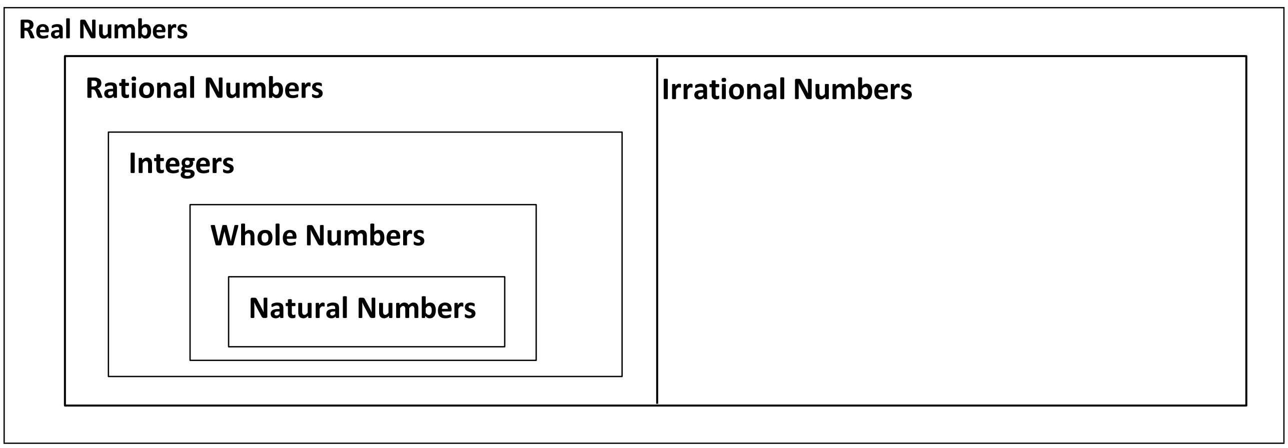
Whole Numbers: $\{0, 1, 2, 3 \ldots\}$

Integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$

Rational Numbers: All numbers that **can be written as a fraction** with the denominator not equal to zero.

Irrational Numbers: All the numbers that **cannot be written as a fraction, a terminating decimal, or a repeating decimal.**

Real Numbers: All the rational and irrational numbers combined.



Prime Numbers and Prime Factors

- **Consider two types of whole numbers called prime numbers and composite numbers.**
- **A prime number is a whole number that has exactly 2 factors: 1 and itself.**
- **A composite number is a whole number greater than 1 that has a divisor other than itself, or in other words, is not prime. Every composite number is made up of a product of purely prime factors.**

Here is a list of the prime numbers less than 100.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

ZERO AND ONE!!

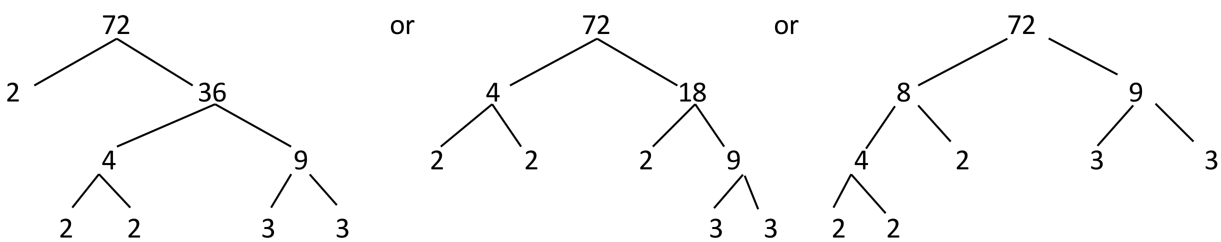
The whole numbers 0 and 1 are neither prime nor composite, why?

- Zero is not a prime factor because it has an infinite number of divisors.
- One is not a prime number because it does not have two different positive whole number divisors

Finding Prime Factors of Composite Numbers

- **There are multiple methods in breaking down a number into its factors.**
- **My preferred method is a prime factor tree.**
- **Break the number down so that every branch has a prime factor on it.**

Factor tree – It doesn't matter how you start; the result should always be the same.



Therefore, the Prime Factors of 72 are: $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

- **Finding the Prime Factor or just factor breakdown of numbers can be beneficial.**
- **We can use it to:**
    - **Simplify Fractions**
    - **Remove Common Factors**
    - **Find Lowest Common Multiples**
    - **Find Greatest Common Factors**
    - **Simplify Radicals**

Click here to watch the video on GCF and LCM or use the QR code.

Example 1: Find the Greatest Common Factor (GCF) and Lowest Common Multiple (LCM) of: 4 and 6

Solution 1:

$4 = 2 \cdot 2$
$6 = 2 \cdot 3$

What Factors are Common to all?	**The Lowest Common Multiple is the smallest number that 4 and 6 can divide into. We do the 4 and 6 times tables until we find a common number.**
Just a 2 The GCF is: 2	4: 4 , 8, **12**, 16, 24, ... 6: 6, **12**, 18, 24..... 12 is the first number that is common to both so the LCM is 12.

Example 2: Find the Greatest Common Factor and Lowest Common Multiple of: 12 and 18

Solution 2:

$12 = 2 \cdot 2 \cdot 3$
$18 = 2 \cdot 3 \cdot 3$

What Factors are Common to all?	**To find the Lowest Common Multiple we will need the 12 and 18 times tables.**
2 and 3 The GCF is: $2 \times 3 = 6$	12: 12, 24, **36**, 48, 60, 72, ... 18: 18, **36**, 54, ... The LCM is: 36

Example 3: Find the Greatest Common Factor and Lowest Common Multiple of: $21, 28, 42$

Solution 3:

$21 = 3 \cdot 7$
$28 = 2 \cdot 2 \cdot 7$
$42 = 2 \cdot 3 \cdot 7$

What Factors are Common to all?	To find the Lowest Common Multiple we will need the 21, 28 and 42 times tables.
7 The GCF is: 7	21: 21, 42, 63, **84**, 105, ... 28: 28, 56, **84**, 112, ... 42: 42, **84**, 126, ... The LCM is: 84

Practice Questions

1. Consider the list of numbers; -12, -2.7, 0, $\frac{2}{3}$, π, 4.21, 50

List All:

a) Natural Numbers 50	b) Whole Numbers 0, 50
c) Integers -12, 0, 50	d) Rational Numbers -12, -2.7, 0, $\frac{2}{3}$, 4.21, 50
e) Irrational Numbers π	f) Real Numbers -12, -2.7, 0, $\frac{2}{3}$, π, 4.21, 50

2. Consider the list of numbers: $-4, -0.3, 0, 0.121121112 \ldots, 2.3535 \ldots, 12, \sqrt{10}$.

List all:

a) Natural Numbers 12	b) Whole Numbers 0, 12
c) Integers $-4, 0, 12$	d) Rational Numbers $-4, -0.3, 0, 2.3535\ldots, 12$
e) Irrational Numbers $0.121121112\ldots, \sqrt{10}$	f) Real Numbers $-4, -0.3, 0, 0.121121112\ldots, 2.3535\ldots,$ $12, \sqrt{10}$

3. List every number system that the following number belongs to. (Natural, Whole, etc.)

a) $\sqrt{16}$ $= 4$ NATURAL WHOLE INTEGER RATIONAL REAL	b) π IRRATIONAL REAL
c) 0 WHOLE INTEGER RATIONAL REAL	d) 2.34 RATIONAL REAL
e) $4.010010001 \ldots.$ IRRATIONAL REAL	f) $\sqrt{0.0004}$ $= 0.02$ RATIONAL REAL
g) $\sqrt{\frac{27}{12}}$ $= \sqrt{\frac{9}{4}}$ $= \frac{3}{2}$ RATIONAL REAL	h) $-3.181818 \ldots$ RATIONAL REAL

4. Decide whether the number is prime or composite. If it's composite, factor it into primes.

a) 19 PRIME

b) 51 PRIME

c) 87 PRIME

d) 101 PRIME

e) 117

COMPOSITE

13 9

3 3

$117 = 3 \times 3 \times 13$

f) 199

PRIME

g) 611

COMPOSITE

13 47

13×47

h) 997

PRIME

i) 629

COMPOSITE

17 37

$629 = 17 \times 37$

j) 551

PRIME

5. Completely factor each number into a product of primes.

a) 36

3 12

3 4

$36 = 3 \times 3 \times 4$

b) 78

2 39

3 13

$78 = 2 \times 3 \times 13$

c) 84

2 42

2 21

3 7

$84 = 2 \times 2 \times 3 \times 7$

d) 169

13 13

$169 = 13 \times 13$

e) 178

2 89

$178 = 2 \times 89$

f) 425

5 85

5 17

$425 = 5 \times 5 \times 17$

6. Find the Greatest Common Factor of the following numbers.

a) 12, 28

12: 2 × 2 × 3
28: 2 × 2 × 7

GCF = 2 × 2
= 4

b) 54, 66

54: 2 × 3 × 3 × 3
66: 2 × 3 × 11

GCF = 2 × 3
= 6

c) 48, 136

48 = 2 × 2 × 2 × 2 × 3
136 = 2 × 2 × 2 × 17

GCF = 2 × 2 × 2
= 8

d) 65, 169

65 = 5 × 13
169 = 13 × 13

GCF = 13

e) 81, 108

81: 3 × 3 × 3 × 3
108: 2 × 2 × 3 × 3 × 3

GCF = 3 × 3 × 3 = 27

f) 30, 45, 60

30: 2 × 3 × 5
45: 3 × 3 × 5
60: 2 × 2 × 3 × 5

GCF = 3 × 5 = 15

7. Find the Lowest Common Multiple of the following numbers.

a) 3 , 4

3: 3 6 9 12 15 18

4: 4 8 12 16

LCM = 12

b) 6, 8

6: 6 12 18 24 30

8: 8 16 24

LCM = 24

c) 22, 33, 66

22: 22 44 66 88

33: 33 66 99

66: 66

LCM = 66

d) 15, 20, 30

15: 15 30 45 60

20: 20 40 60

30: 30 60

LCM = 60

e) 8, 12

8: 8 16 24 32 40

12: 12 24

LCM = 24

f) 6, 9, 12

6: 6 12 18 24 30 36

9: 9 18 27 36

12: 12 24 36

LCM = 36

Exponent Laws and Operations

Remember How Brackets and Negatives Change the Question

Where it gets tricky is with negative bases, and how the brackets, if any, are used.

Here we go…

$(-2)^2$ this means that everything inside the brackets is multiplied repeatedly.

$$(-2) \cdot (-2)$$

<u>This has a profound effect on the final result</u>

<u>A negative number multiplied an even number of times will always finish POSITIVE.</u>

So...

$$(-2)^4 = (-2)(-2)(-2)(-2)$$

$$= 4 \cdot 4$$

$$= 16$$

So, when we have an **EVEN POWER,** we can **REWRITE** the statement without the brackets as a **POSTIVE statement**.

Watch this:

$$(-2)^4 = 2^4$$

This is a big deal.

<u>A negative number multiplied an odd number of times will always finish NEGATIVE.</u>

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2)$$

$$= 4 \cdot 4 \cdot (-2)$$

$$= 16 \cdot (-2)$$

$$-32$$

So, when we have an **ODD POWER,** we can **REWRITE** the statement without the brackets as a **NEGATIVE statement**.

Watch this:

$$(-2)^5 = -2^5$$

This is a big deal.

Now we have covered when there are brackets.
But what about when there are no brackets?

So far, we know this…

$$(-a)^{Even} = a^{same\ power}$$

$$(-a)^{Odd} = -a^{same\ power}$$

But what does $-a$ mean? Let's look at it with a number.

Example:

$$-2 = (-1)2$$

So that means that…

$-2^3 = (-1)2^3$ also $-2^4 = (-1)2^4$

$= (-1) \cdot 2 \cdot 2 \cdot 2$ $= (-1) \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= -8$ $= -16$

Regardless of the power, even or odd, if there are **no brackets** to begin with the answer is **ALWAYS NEGATIVE**

Summary

If the negative is in brackets, then the result depends on the exponents being odd or even.

$(-2)^4 = 2^4$ Even exponent, the answer is always POSITIVE.

$(-2)^5 = -2^5$ Odd exponent, the answer is always NEGATIVE.

If there are NO BRACKETS, the answer is ALWAYS NEGATIVE

$$-2^5 = (-1)2^5$$

$$-2^4 = (-1)2^4$$

Summary of the Exponents Laws from Grade 9

For any Integers m and n:		
Exponent of 1	$a^1 = a$	$3^1 = 3$
Exponent of 0	$a^0 = 1, \qquad a \neq 0$	$(-5)^0 = 1$
Product Rule	$a^m \cdot a^n = a^{m+n}, \qquad a \neq 0$	$2^3 \cdot 2^4 = 2^{3+4} = 2^7$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}, \qquad a \neq 0$	$\frac{3^5}{3^2} = 3^{5-2} = 3^3$
Power to a Power Rule	$(a^m)^n = a^{m \cdot n}$ $(ab)^n = a^n \cdot b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$ $(2x)^3 = 2^3 \cdot x^3 = 8x^3$ $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$
Negative Exponents	$a^{-2} = \frac{1}{a^2}$	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

[Click here to watch the video on Exponents or use the QR code.]

Example 1: Simplify

$$(4ab^{-1})^3(2a^2b^3)^2$$

Solution 1:

i) **Waterbomb in the outer exponent – Power to a Power – Multiply the Exponents**

$$4^3a^3b^{-3} \cdot 2^2a^4b^6$$

ii) **Multiply the common bases – Add the Exponents**

iii)Compute the numerical situations (rearrange the factors)

$$64 \cdot 4a^7b^3 \qquad \rightarrow \qquad \mathbf{256a^7b^3}$$

Example 2: Simplify

$$\frac{(x^2y^2)^{-2}(x^2y^2)^3}{x^{-1}y^{-2}}$$

Solution 2:

Power to a Power Waterbomb (**Multiply) Exponents**

$$\frac{(x^2y^2)^{-2}(x^2y^2)^3}{x^{-1}y^{-2}} \rightarrow \frac{x^{-4}y^{-4}\cdot x^6y^6}{x^{-1}y^{-2}}$$

Multiply Common Base **Add Exponents**

Divide Common Base **Subtract Exponents**

$$\frac{x^2y^2}{x^{-1}y^{-2}} \rightarrow \boldsymbol{x^3y^4}$$

Here are 3 New Concepts

Changing from Negative to Positive Exponents

Consider the expression: $\frac{1}{2^{-3}}$ By the negative exponent rule, this is the same as saying:

$$2^0 \div 2^{-3}$$

Which is equal to:

$$2^{0-(-3)} = 2^3$$

Changing from Negative to Positive Exponents For any non-zero numbers a and b, with exponents m and n: $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$ and $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Example 3:

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{3^3}{2^3} = \frac{27}{8}$$

Example 4:

$$\frac{3^{-2}}{4^{-3}} = \frac{4^3}{3^2} = \frac{64}{9}$$

Practice Questions

1. Multiply. **Leave answers in exponential form, positive exponents only.**

a) $2^3 \cdot 2^4$ $2^{3+4} = 2^7$	b) $3^5 \cdot 3^7$ $3^{5+7} = 3^{12}$
c) $4^{-3} \cdot 4^2$ $4^{-3+2} = 4^{-1} = \frac{1}{4}$	d) $5^0 \cdot 5^3$ $5^{0+3} = 5^3$

e) $a^2 \cdot a^3 \cdot a^{-5}$ $a^{2+3-5} = a^0 = 1$	f) $y^{-3} \cdot y^2 \cdot y$ $y^{-3+2+1} = y^0 = 1$
g) $8^0 \cdot 8^1 \cdot 8^2$ $8^{0+1+2} = 8^3$	h) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4$ $\left(\frac{2}{3}\right)^{3+4} = \left(\frac{2}{3}\right)^7$
i) $(-3)^4 \cdot (-3)^3 \cdot -3^2$ $(-3)^{4+3+2} = (-3)^9$	j) $\left(-\frac{1}{2}\right)^5 \times \left(-\frac{1}{2}\right)^{-3} \times \left(-\frac{1}{2}\right)^6$ $\left(\frac{-1}{2}\right)^{5-3+6} = 5^8$

2. Divide. **Leave answers in exponential form, positive exponents only.**

a) $\frac{5^6}{5^3}$ $= 5^{6-3} = 5^3$	b) $\frac{4^8}{4^4}$ $4^{8-4} = 4^4$
c) $\frac{2^8}{2^2}$ $2^{8-2} = 2^6$	d) $\frac{3^9}{3^3}$ $3^{9-3} = 3^6$

e) $\frac{t^6}{t^2}$ $t^{6-2} = t^4$	f) $\frac{x^6}{x^6}$ $x^{6-6} = x^0 = 1$
g) $\frac{(-6)^4}{(-6)^{-2}}$ $(-6)^{4-(-2)} = (-6)^6$	h) $\frac{(-9)^{-3}}{(-9)^{-6}}$ $(-9)^{-3-(-6)} = (-9)^3$
i) $\frac{(-2x)^3}{(-2x)^{-4}}$ $= (-2x)^{3-(-4)} = (-2x)^7$	j) $\frac{(z)^{-2}}{(z)^{-6}}$ $(z)^{-2-(-6)} = (z)^4$

3. Simplify. **Express without brackets or negative exponents.**

a) $(2^4)^2$ $2^{4\times 2} = 2^8$	b) $(5^3)^{-2}$ $5^{3\times -2} = 5^{-6} = \frac{1}{5^6}$
c) $(3^{-4})^{-2}$ $3^{-4\times -2} = 3^8$	d) $(-3x^{-2})^0$ $= 1$

e) $(2x)^3$

$= 8x^3$

f) $(3x^{-4})^2$

$9x^{-8} = \frac{9}{x^8}$

g) $(2a^{-4})^3$

$8a^{-12} = \frac{8}{a^{12}}$

h) $(3x^4y^{-2})^4$

$81x^{16}y^{-8} = \frac{81x^{16}}{y^8}$

i) $(-4a^{-3}b^{-2})^2$

$16a^{-6}b^{-4}$

$= \frac{16}{a^6b^4}$

j) $(-2^{-3}x^{-2}y)^3$

$-2^{-9}x^{-6}y^3 = \frac{y^3}{-512x^6}$

4. Simplify. **Express without brackets or negative exponents.**

a) $\frac{3^4 \cdot 3^7}{3^5}$

$= \frac{3^{11}}{3^5} = 3^6$

b) $\frac{2^5}{2^4 \cdot 2^3}$ $= \frac{2^5}{2^7} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

c) $\dfrac{4^{-3}\cdot 4^{1}}{4^{-1}}$ $\dfrac{4^{-2}}{4^{-1}} = 4^{-2-(-1)} = 4^{-1} = \dfrac{1}{4}$	d) $\dfrac{5^{4}\cdot 5^{-2}}{5^{3}\cdot 5^{-1}} = \dfrac{5^{2}}{5^{2}} = 5^{0} = 1$
e) $\dfrac{7^{0}\cdot 7^{-3}}{7^{2}\cdot 7^{-2}}$ $\dfrac{7^{-3}}{7^{0}} = \dfrac{7^{-3}}{1} = \dfrac{1}{7^{3}}$	f) $\dfrac{11^{2}\cdot 11^{3}}{11^{-1}} = \dfrac{11^{5}}{11^{-1}} = 11^{6}$
g) $\dfrac{3(x^{3})^{2}}{x^{-2}}$ $\dfrac{3x^{6}}{x^{-2}} = 3x^{6-(-2)} = 3x^{8}$	h) $\dfrac{(3x^{2})^{-3}}{x^{3}} = \dfrac{3^{-3}x^{-6}}{x^{3}}$ $= \dfrac{x^{-9}}{3^{3}} = \dfrac{1}{27x^{9}}$
i) $\dfrac{(2a^{2}b^{-4}c^{-5})^{3}}{2^{2}}$ $\dfrac{8a^{6}b^{-12}c^{-15}}{4}$ $= \dfrac{2a^{6}}{b^{12}c^{15}}$	j) $\left(\dfrac{2a^{2}}{3b^{4}}\right)^{-3}$ $= \dfrac{2^{-3}a^{-6}}{3^{-3}b^{-12}} = \dfrac{3^{3}b^{12}}{2^{3}a^{6}}$ $= \dfrac{27b^{12}}{8a^{6}}$

5. **Solve.**

a) 3^2 $= 9$	b) 3^{-2} $= \frac{1}{9}$
c) $\left(\frac{1}{3}\right)^2$ $= \frac{1}{9}$	d) $\left(\frac{1}{3}\right)^{-2}$ $= 9$
e) -3^2 $= -9$	f) $(-3)^2$ $= 9$
g) $-\left(-\frac{1}{3}\right)^2$ $= -\frac{1}{9}$	h) $\left(-\frac{1}{3}\right)^2$ $= \frac{1}{9}$
i) $\left(-\frac{1}{3}\right)^{-2}$ $= 9$	j) $-\left(-\frac{1}{3}\right)^{-2}$ $= -\left(\frac{-3}{1}\right)^2 = -9$
k) -2^3 $= -2 \times 2 \times 2 = -8$	l) $-(-2)^3$ $= -(-8) = 8$

6. Simplify. **Express without brackets or negative exponents.**

a) $\frac{(2a^2)^2(2a)^2}{(b^3)^{-4}}$

$= \frac{(4a^4)(4a^2)}{b^{-12}}$

$= 16a^6b^{12}$

b) $\frac{(m^2n^{-1})^{-2}}{(m^{-3}n)^3}$

$= \frac{m^{-4}n^2}{m^{-9}n^3}$

$= m^5n^{-1} = \frac{m^5}{n}$

c) $\frac{(3^2n^3)^3}{(3n^2)^3}$

$= \frac{3^6n^9}{3^3n^6}$

$= 3^3n^3$ or $27n^3$

d) $\frac{(x^2y^2)^3}{(5xy^{-2})^{-2}}$

$= \frac{x^6y^6}{5^{-2}x^{-2}y^4}$

$= 5^2x^8y^2$ or $25x^8y^2$

e) $\left(\frac{5x^2}{4x^3}\right)^{-1}$

$= \frac{4x^3}{5x^2}$

$= \frac{4x}{5}$

f) $\frac{(3a)^2(3ab^2)^2}{(3b^4)^3}$

$= \frac{(3^2a^2)(3^2a^2b^4)}{3^3b^{12}}$

$= \frac{3^4a^4b^4}{3^3b^{12}}$

$= 3a^4b^{-8} = \frac{3a^4}{b^8}$