Math 10F Unit 2 LG 4-5 Multiplying Polynomials & Factoring

Classifying and Simplifying Multiplication of Polynomials

Click here to watch the video on Polynomials or use the QR code.

Term

- A **number or a product of a number** with **one or more variables** which can be **raised to a power**.

Examples of Terms

Term	$5y$	$-2a^3$	$\frac{1}{2}x^2yz^4$	x	10
Coefficients	5	-2	$\frac{1}{2}$	1	10
Variables	y	a	x, y, z	x	No variable

Polynomial

A term or sum of terms, in which all variables have **whole number exponents**, and in which **variables appear only in the numerator**.

Examples and Non-Examples of Polynomials

Polynomial	Non-Polynomial
5	$x^{\frac{1}{2}}$
$\sqrt{2}x$	$2x + \sqrt{y}$
$3a^2 - 2a$	$\frac{1}{2x} + 4$
$\frac{3}{4}y + 3y^2 - 4$	$x^{-3} - 2x$

Classifying Polynomials

Monomial	A polynomial with one term	$3,\ \ 2x^2y,\ \ -3a$
Binomial	A Polynomial with two terms	$x+2,\ 2x^2y+3,\ x^2-y$
Trinomial	A polynomial with three terms	$3x^2+2x-3,\ \sqrt{2}x+y-z$
Polynomial	General term for expressions with more than three terms (can also be used to describe monomial, binomial, and trinomial)	$x^5-2x^4+3x^3-4x^2$

Degree of a Polynomial

The **degree** of a term in a polynomial is the **sum of the exponents** of the variables in the term.

The **degree of a polynomial** is the **term with the highest degree**.

Examples:

In $\mathbf{3x^2+2x-3}$, the term of the highest degree is $3x^2$, so the degree is 2.

In $\mathbf{4x^2y^3 + z^4}$, the term of the highest degree is $4x^2y^3$, so the degree is 5. $(2+3=5)$

In $\mathbf{-2x^2yz^2 + y^4}$, the term of the highest degree is $-2x^2yz^2$, so the degree is 5. $(2+1+2=5)$

Leading Term

The **term** with the **highest** degree.

Consider: $3x^2y^3 - 2xy^2 + 2$

- The **degree** of the terms are: $5, 3$, and 0 respectively.
- This means that the **leading term** is the term with the **highest degree** so: $3x^2y^3$
- It is good routine to always put polynomials in **descending order** with respect to **degree.**

Combining Like Terms

- Like terms are either **constant terms**, or **terms** that contain the **same variables** to the **same power**.

Example:

- $3x^2, 5x^2$ are like terms, they have the same variable and exponent.
- $2xy^2, 3x^2y$ are not like terms because they have different exponents for each variable.

- To **combine like terms, add or subtract** the **coefficients** of the terms.

Example 1: Simplify the expression $4x^2 - 3y - x^2 + 5y$

Solution 1: $4x^2$ and $-x^2$ are like terms, so the coefficients can be added.

$-3y$ and $5y$ are like terms, so the coefficients can be added.

Therefore: $4x^2 - 3y - x^2 + 5 = \mathbf{3x^2 + 2y}$

Evaluating Polynomials

- When a **constant** is **substituted** for a **variable** in a polynomial, the polynomial is evaluated for that constant.

Example:

When $3x^2 - 2$ is evaluated for $x = 4$,

the result is: $3(4)^2 - 2 = 48 - 2 = 46$

When $5x^3 + 6$ is evaluated for $x = 2$,

the result is: $5(2)^3 + 6 = 40 + 6 = 46$

When $4xy^2 - 2x^2y$ is evaluated for $x = 3 \ and \ y = 4$,

the result is: $4(3)(4)^2 - 2(3)^2(4) = 192 - 72 = 120$

Multiplying a Monomial by a Monomial

- To multiply two **monomials** (one term), first multiply the constant factors, and then multiply the variable factors.

Example 2: Multiply

a) $(2x^3)(-3x^4)$
b) $(-3a^2b^3)(-2a^2b^5)$
c) $(-3y^2z^3)(2yz^2)(4y^3z^2)$

Solution 2:

a) $(2)(-3)(x^3)(x^4) = -6x^{3+4} = -6x^7$

b) $(-3)(-2)(a^2)(a^2)(b^3)(b^5) = 6a^{2+2}b^{3+5} = 6a^4b^8$

c) $(-3)(2)(4)(y^2)(y)(y^3)(z^3)(z^2)(z^2) = -24y^{2+1+3}z^{3+2+2} = -24y^6z^7$

Multiplying a Monomial by a Binomial

- When multiplying a **monomial by a binomial**, use the **distributive property.**
- **DISTRIBUTIVE PROPERTY or WATERBOMB** is: $\boldsymbol{a(b + c) = a \cdot b + a \cdot c}$

*******DON'T FORGET YOUR EXPONENT RULES*******

Example 3: Multiply

a) $2(3 + 4)$
b) $2x^2(3x^2 - 4y)$
c) $-3y(x^4 + 2y^3)$

Solution 3:

a) $2 \cdot 3 + 2 \cdot 4 = 6 + 8 = 14$

b) $(2x^2)(3x^2) - (2x^2)(4y) = 6x^4 - 8x^2y$

c) $(-3y)(x^4) + (-3y)(2y^3) = -3x^4y - 6y^4$

Practice Problems

For each polynomial, find the number of terms, the degree, and coefficients of each term.

1. $3x^5$ # OF TERMS : 1 DEGREE : 5 COEFFICIENT : 3	2. $-2y^4$ # OF TERMS : 1 DEGREE : 4 COEFFICIENT : -2	3. $4x^3 - 2x^2$ # OF TERMS : 2 DEGREE : 3 COEFFICIENTS : 4, -2
4. $-3a^3 + 3a - 4^0$ # OF TERMS : 3 DEGREE : 3 COEFFICIENTS : -3, 3	5. $2x^3y^2 - 3x^2$ # OF TERMS : 2 DEGREE : 5 COEFFICIENTS : 2, -3	6. $2^3b^3 - 3^2$ $= 8b^3 - 9$ # OF TERMS : 2 DEGREE : 3 COEFFICIENTS : 8
7. $-x^3y^2z + \sqrt{2}xyz + 4z^3$ # OF TERMS : 3 DEGREE : 6 COEFFICIENTS : $-1, \sqrt{2}, 4$	8. $4x^4y^3z^2p + p$ # OF TERMS : 2 DEGREE : 10 COEFFICIENTS : 4, 1	9. x^3 # OF TERMS : 1 DEGREE : 3 COEFFICIENTS : 1

For each polynomial, simplify, then write the answer in descending order.

10. $3x^2 - 2x + 5x - x^2$

$= 2x^2 + 3x$

11. $\frac{2}{3}x^4 + \frac{4}{3}x^4$

$= \frac{6}{3}x^4$

$= 2x^4$

12. $2.3x^2 + 3 - 4.1x^2 + 3x$

$= -1.8x^2 + 3x + 3$

13. $3y^4 - 2y^2 - y^4 - 2y$

$2y^4 - 2y^2 - 2y$

14. $-2x^3 - 2x^2 - 2x^3 + 2x^2$

$-4x^3$

15. $x^2 - 2x + x^3 + x$

$x^3 + x^2 - x$

16. $2x^2 - \frac{3}{4}x^3 + 6x^2 + \frac{2}{3}x^3$

$-\frac{1}{12}x^3 + 8x^2$

17. $-4y^3 - \frac{1}{2}y^5 + 5y^3 - \frac{1}{3}y^5$

$\frac{-5}{6}y^5 + y^3$

Find the value of the polynomial when $x = -2$

18. $-3x^2 + 2x - 1$

$= -3(-2)^2 + 2(-2) - 1$

$= -12 - 4 - 1$

$= -17$

19. $-3x^2 - 2x + 1$

$-3(-2)^2 - 2(-2) + 1$

$= -12 + 4 + 1$

$= -7$

20. $2x^2 - 3x + 4$

$= 2(-2)^2 - 3(-2) + 4$

$= 8 + 6 + 4$

$= 18$

21. $-2x^2 - 3x - 4$

$= -2(-2)^2 - 3(-2) - 4$

$= -8 + 6 - 4$

$= -6$

Find each product.

22. $3x^3(2x^4)$

$= 6x^7$

23. $-2a^2b^4(4ab^2)$

$= -8a^3b^6$

24. $(3xy)(-4x^2y^2)$

$= -12x^3y^3$

25. $(2ab)(-2ab)(2ab)$

$= -8a^3b^3$

26. $(5x^3)(-2y^3)$

$= -10x^3y^3$

27. $(-4a^4b^3)(2a^3b^2)(3ab)$

$= -24a^8b^6$

28. $(a^2b^4)(a^3b)(-3b^2)$

$= -3a^5b^7$

29. $(-r^4s^2t)(r^3st^2)(-rst)$

$= r^8s^4t^4$

30. $(-3ab^2)(2a^3b)(-a^2b^2)(-2a^3b^2)$

$= -12a^9b^7$

31. $(-5a^3b^3c^2d^3)(-2ab^2cd^2)(-4a^2bc^3d)$

$= -40a^6b^6c^6d^6$

32. $3(x-4)$

$= 3x - 12$

33. $-2x^2(x+3)$

$= -2x^3 - 6x^2$

34. $5x(4x-3)$

$= 20x^2 - 15x$

35. $2x^3(4x^5-3x)$

$= 8x^8 - 6x^4$

36. $2y(3x-5y)$

$= 6xy - 10y^2$

37. $2y^2(5y-x)$

$= 10y^3 - 2xy^2$

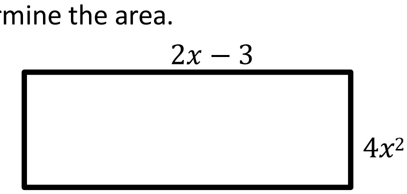
38. $a^2bc(ab^2c^2 - a^2bc^2 - 2a^3b^3c)$

$= a^3b^3c^3 - a^4b^2c^3 - 2a^5b^4c^2$

39. $-abc^2(-a^2bc^3 + ab^2c - a^3c^2)$

$= a^3b^2c^5 - a^2b^3c^3 + a^4bc^4$

40. Determine the area.



$A = L \times W$

$= 4x^2(2x-3)$

$= 8x^3 - 12x^2$

Multiplication of Binomials and Trinomials

Click here to watch the video on Multiplication of Polynomials or use the QR code.

- There are **two methods** that can effectively be used for multiplying polynomials.
- **Method 1: Distributive Method** *(BEST FOR LARGE POLYNOMIALS*
- **Method 2: Horizontal Method (FOIL)** *(BEST FOR TWO BINOMIALS)*

Method 1: Distributive Method

Example 1: Multiply $(2x - 3)(x + 2)$

Solution 1: $(2x - 3)(x + 2) = (2x - 3)(x) + (2x - 3)(2)$

$$= 2x^2 - 3x + 4x - 6$$

$$= \mathbf{2x^2 + x - 6}$$

Example 2: Multiply $(x^2 - 2)(3x^2 - 4x + 3)$

Solution 2: $(x^2 - 2)(3x^2 - 4x + 3) = (x^2 - 2)(3x^2) + (x^2 - 2)(-4x) + (x^2 - 2)(3)$

$$= 3x^4 - 6x^2 - 4x^3 + 8x + 3x^2 - 6$$

$$= \mathbf{3x^4 - 4x^3 - 3x^2 + 8x - 6}$$

Example 3: Multiply $(3x + y)(2x^2 - 5xy + 4y^2)$

Solution 3: $(3x + y)(2x^2 - 5xy + 4y^2) = (3x + y)(2x^2) + (3x + y)(-5xy) + (3x + y)(4y^2)$

$$= 6x^3 + 2x^2y - 15x^2y - 5xy^2 + 12xy^2 + 4y^3$$

$$= \mathbf{6x^3 - 13x^2y + 7xy^2 + 4y^3}$$

Method 2: Horizontal Method (FOIL)

- $FOIL$ is the acronym for: **First, Outside, Inside, Last.**

In $(a + b)(c + d)$:

First refers to $\boldsymbol{a \cdot c}$

Outside refers to $\boldsymbol{a \cdot d}$

Inside refers to $\boldsymbol{b \cdot c}$

Last refers to $\boldsymbol{b \cdot d}$

$$(a + b)(c + d)$$

Example 4: Multiply $(x - 6)(x + 4)$

Solution 4: $(x - 6)(x + 4) = \underbrace{(x)(x)}_{\textit{First}} + \underbrace{(x)(4)}_{\textit{Outside}} + \underbrace{(-3)(x)}_{\textit{Inside}} + \underbrace{(-6)(4)}_{\textit{Last}}$

$= x^2 + 4x - 3x - 24$

$= \boldsymbol{x^2 + x - 24}$

Example 5: Multiply $(2x - 3)(3x + 1)$

Solution 5: $(2x - 3)(3x + 1) = \underbrace{(2x)(3x)}_{\textit{First}} + \underbrace{(2x)(1)}_{\textit{Outside}} + \underbrace{(-3)(3x)}_{\textit{Inside}} + \underbrace{(-3)(1)}_{\textit{Last}}$

$= 6x^2 + 2x - 9x - 3$

$= \boldsymbol{6x^2 - 7x - 3}$

- **FOIL DOES NOT WORK WITH LARGER POLYNOMIALS**
- **IT IS DESIGNED TO USE THE DISTRIBUTIVE METHOD WITH BINOMIALS ONLY**
- **WITH LARGER POLYNOMIALS THE DISTRIBUTIVE METHOD IS THE BEST OPTION**

Verifying your Multiplication

- The process of multiplying Polynomials has multiple steps and can have potential error spots.
- The beauty here is that it is possible to verify your result.
- Simply take any number (0 or 1 is usually a good choice) and substitute it in for your variable.
- The left and right side of the equations should match up.
- If they do not an error has been made

Consider:

$$(2x - 3)(3x + 1) = 6x^2 - 7x - 3$$

$$(2(0) - 3)(3(0) + 1) = 6(0)^2 - 7(0) - 3$$

$$(-3)(1) = -3$$

$$(-3) = (-3)$$

Since both sides of the equation equals -3, the answer is verified for $x = 0$.

Common Errors when Multiplying Polynomials

1. $(x+3)^2 \neq x^2 + 9 \rightarrow \boldsymbol{(x+3)^2} = (x+3)(x+3) = \boldsymbol{x^2 + 6x + 9}$
2. $(x-8)^2 \neq x^2 - 64 \rightarrow \boldsymbol{(x-8)^2} = (x-8)(x-8) = \boldsymbol{x^2 - 16x + 64}$
3. $(x+2)(x-2) \neq x^2 - 4x + 4 \rightarrow \boldsymbol{(x+2)(x-2)} = \boldsymbol{x^2 - 4}$

General Rules

Square of a Binomial

$(a + b)^2 = a^2 + 2ab + b^2$ $\quad$ $(a - b)^2 = a^2 - 2ab + b^2$

Product of Sum and Difference

$$(a - b)(a + b) = a^2 - b^2$$

Practice Problems

Multiply, leave all answers in descending order.

1. $(2x + 1)(3x + 2)$

$= 6x^2 + 4x + 3x + 2$

$= 6x^2 + 7x + 2$

2. $(3y - 4)(2y + 3)$

$= 6y^2 + 9y - 8y - 12$

$= 6y^2 + y - 12$

3. $(3x^2 + 4)(x + 1)$

$= 3x^3 + 3x^2 + 4x + 4$

4. $(4y^2 + 3)(3y - 1)$

$= 12y^3 - 4y^2 + 9y - 3$

5. $(-3y - 4)(2y - 3)$

$= -6y^2 + 9y - 8y + 12$

$= -6y^2 + y + 12$

6. $(5x - y)(5x - y)$

$= 25x^2 - 5xy - 5xy + y^2$

$= 25x^2 - 10xy + y^2$

7. $(2x - 1)(2x^2 + 3x - 1)$

$= 4x^3 + 6x^2 - 2x - 2x^2 - 3x + 1$

$= 4x^3 + 4x^2 - 5x + 1$

8. $(5x - 6)(2x^2 + 7x - 3)$

$= 10x^3 + 35x^2 - 15x - 12x^2 - 42x + 18$

$= 10x^3 + 23x^2 - 57x + 18$

9. $(3y - 2x)(4y^2 - 3xy + x^2)$

$= 12y^3 - 9xy^2 + 3x^2y - 8xy^2 + 6x^2y - 2x^3$

$= 12y^3 - 17xy^2 + 9x^2y - 2x^3$

10. $(x^2 + 2y)(2x^2 + 3xy - y^2)$

$= 2x^4 + 3x^3y - x^2y^2 + 4x^2y + 6xy^2 - 2y^3$

Multiply, leave all answers in descending order.

11. $(a+b)^2$

$= (a+b)(a+b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

12. $(2x+1)^2$

$= (2x+1)(2x+1)$

$= 4x^2 + 2x + 2x + 1$

$= 4x^2 + 4x + 1$

13. $(x-y)^2$

$= (x-y)(x-y)$

$= x^2 - xy - xy + y^2$

$= x^2 - 2xy + y^2$

14. $(3x+2y)^2$

$= (3x+2y)(3x+2y)$

$= 9x^2 + 6xy + 6xy + 4y^2$

$= 9x^2 + 12xy + 4y^2$

15. $(-x-2y)^2$

$(-x-2y)(-x-2y)$

$= x^2 + 2xy + 2xy + 4y^2$

$= x^2 + 4xy + 4y^2$

16. $(-3x^2+2y^2)^2$

$= (-3x^2+2y^2)(-3x^2+2y^2)$

$= 9x^4 - 6x^2y^2 - 6x^2y^2 + 4y^4$

$= 9x^4 - 12x^2y^2 + 4y^4$

17. $(-a^3b + c^2d^2)^2$

$= (-a^3b + c^2d^2)(-a^3b + c^2d^2)$

$= a^6b^2 - a^3bc^2d^2 - a^3bc^2d^2 + c^4d^4$

$= a^6b^2 - 2a^3bc^2d^2 + c^4d^4$

18. $(2a^2b^2 - 4c^3d)^2$

$(2a^2b^2 - 4c^3d)(2a^2b^2 - 4c^3d)$

$= 4a^4b^4 - 8a^2b^2c^3d - 8a^2b^2c^3d + 16c^6d^2$

$= 4a^4b^4 - 16a^2b^2c^3d + 16c^6d^2$

Multiply, leave all answers in descending order.

19. $(a-b)(a+b)$

$= a^2 + ab - ab - b^2$

$= a^2 - b^2$

20. $(2x+1)(2x-1)$

$= 4x^2 - 2x + 2x - 1$

$= 4x^2 - 1$

21. $(x-y)(x+y)$

$= x^2 + xy - xy - y^2$

$= x^2 - y^2$

22. $(3x+2y)(3x-2y)$

$= 9x^2 - 6xy + 6xy - 4y^2$

$= 9x^2 - 4y^2$

23. $(-x-2y)(-x+2y)$

$= x^2 - 2xy + 2xy - 4y^2$

$= x^2 - 4y^2$

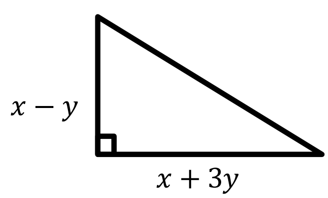
24. $(-x^2-2y^2)(-x^2+2y^2)$

$= x^4 - 2x^2y^2 + 2x^2y^2 - 4y^4$

$= x^4 - 4y^4$

Calculate the area of the following figures.

25. *Right Triangle*

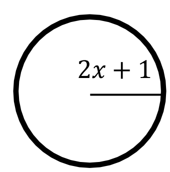


$A = \frac{b \times h}{2}$

$= \frac{(x+3y)(x-y)}{2}$

$= \frac{x^2 - xy + 3xy - 3x^2}{2} = \frac{x^2 + 2xy - 3x^2}{2}$

26. *Circle*



$A = \pi r^2$

$A = \pi (2x+1)(2x+1)$

$= \pi (4x^2 + 2x + 2x + 1)$

$= \pi (4x^2 + 4x + 1)$

Factoring The Greatest Common Factor

Click here to watch the video on Factoring the GCF or use the QR code.

Removing Common Factors

- To **factor** a polynomial is to express it as a **product (things MULTIPLYING together)**
- Removing a **common factor** is the most basic form of factoring.
- If every term has at least one factor that is the same, it is known as the **common factor.**
- Removing the **greatest common factor** is the best approach.

Example 1: Factor

a) $5x + 10$
b) $3x^2 - 6x$
c) $12x^4 - 8x^3 + 4x^2$

Solution 1:

a) $5x + 10 = 5(x) + 5(2) \quad = \quad \mathbf{5(x + 2)}$

b) $3x^2 - 6x = (3x)(x) - 3x(2) \quad = \quad \mathbf{3x(x - 2)}$

c) $12x^4 - 8x^3 + 4x^2 \quad = \quad 4x^2(3x^2) - 4x^2(2x) + 4x^2(1)$

$= \mathbf{4x^2(3x^2 - 2x + 1)}$

- **NOTE:** If an entire term is the greatest common factor, then a "1" must be used to hold its place (as seen in example c)

- **NOTE:** To make sure the solution is correct, you can check your answer by multiplying in

Example 2: Factor $x(x+1) + 3(x+1)$

Solution 2: A common factor does not have to be a monomial, $(x+1)$ is common to both terms and can be taken out.

$$x(x+1) + 3(x+1) = \mathbf{(x + 1)(x + 3)}$$

Practice Problems

Factor out the greatest common factor.

1. $18x^3 - 27x$

$9x(2x^2 - 3)$

2. $24a^3 + 18a$

$= 6a(4a^2 + 3)$

3. $\frac{1}{3}y^2 - \frac{4}{3}y$

$= \frac{1}{3}y(y - 4)$

4. $3x^2 + 3x - 6$

$= 3(x^2 + x - 2)$

5. $8b^2 - 4b + 20$

$4(2b^2 - b + 5)$

6. $5c^5 - 10c^3 + 15c$

$5c(c^4 - 2c^2 + 3)$

7. $12x^3y + 6xy^2$

$6xy(2x^2 + y)$

8. $6x^7 - 9x^6 - 57x^5 + 3x^4$

$3x^4(2x^3 - 3x^2 - 19x + 1)$

Factor

9. $x(2x+1)+3(2x+1)$

$(2x+1)(x+3)$

10. $x(x+1)+(x+1)$

$(x+1)(x+1)$

11. $3y(3y+1)-2(3y+1)$

$(3y+1)(3y-2)$

12. $6y^2(y-3)+5(y-3)$

$(y-3)(6y^2+5)$

13. $a(a+2b)+b(a+2b)$

$(a+2b)(a+b)$

14. $3c(1-2d)-2d(1-2d)$

$(1-2d)(3c-2d)$

Factoring Trinomials

Click here to watch the video on Factoring Trinomials or use the QR code.

Factoring Quadratics (Polynomials of degree 2): $\boldsymbol{x^2 + bx + c}$

Consider this: $(x + a)(x + b) = x^2 + bx + ax + ab$

$$x^2 + (b + a)x + ab$$

- By looking at this we see that:
    - The **first term** is the **product** of x and x
    - The **coefficient** of the **middle term** is the **sum** of a and b
    - The **last term** is the **product** of a and b
- This leads us to the **general rule**:

When factoring $x^2 + bx + c$, look for **two factors of c**, that **multiply** to the **coefficient** of the **last term**, and **add** to the **coefficient** of the middle term.

Example 1: Factor $x^2 + 7x + 12$

Solution 1: What two numbers **add to 7** and **multiply to 12**?

- Integers that multiply to 12: $(1, 12)$ $(2, 6)$ $(3, 4)$ $(-1, -12)$ $(-2, -6)$ $(-3, -4)$
- Only integers $+3$ and $+4$ add to 7
- Therefore $x^2 + 7x + 12 = (x + 3)(x + 4)$
- We can **check our answer** using **FOIL**: $(x + 3)(x + 4)$

$$= x^2 + 3x + 4x + 12$$

$$= \boldsymbol{x^2 + 7x + 12}$$

Example 2: Factor $x^2 + 8 - 6x$

Solution 2: First **arrange** the polynomial in **descending order** of powers.

- $x^2 + 8 - 6x = \boldsymbol{x^2 - 6x + 8}$
- -4 and -2 add to -6 and multiply to $+8$
- Therefore: $x^2 - 6x + 8 = (x - 4)(x - 2)$
- We can **check using FOIL.**

Example 3: Factor $5x^2 + 35x + 60$

Solution 3: Always look for a **common factor** first. The **largest common factor** is $\mathbf{5}$

- Therefore: $5x^2 + 35x + 60 = 5(x^2 + 7x + 12)$
- Now we can factor the Quadratic like we did previously:
- Two numbers that **multiply** to $+12$ and **add** to $+7$
- $+4$ and $+3$ get the job done
- So $5(x^2 + 7x + 12) = 5(x+4)(x+3)$
- **Check** your answer **using FOIL**

Example 4: Factor $-x^2 + 5x + 6$

Solution 4: First factor out -1, so that the **coefficient** of x^2 becomes $+1$.

- So $-x^2 + 5x + 6$ becomes $-(x^2 - 5x - 6)$, now factor $(x^2 - 5x - 6)$
- -6 and 1 **multiply** to -6 and **add** to -5
- Therefore $-x^2 + 5x + 6 = -(x^2 - 5x - 6) \quad = -(x-6)(x+1)$
- **Note the factors are: $(\boldsymbol{x} - \mathbf{6})(\boldsymbol{x} + \mathbf{1})$ and $-\mathbf{1}$**

Example 5: Factor $-3x^4 - 18x^3 - 27x^2$

Solution 5: First look for a common factor. The largest here is $-3x^2$, factor it out

- So $-3x^4 - 18x^3 - 27x^2$ becomes $-3x^2(x^2 + 6x + 9)$, now factor $(x^2 + 6x + 9)$
- $+3$ and $+3$ **multiply** to $+9$ and **add** to $+6$
- Therefore $-3x^4 - 18x^3 - 27x^2 = -3x^2(x^2 + 6x + 9) = -3x^2(x+3)^2$
- **Note the factors are: $(\boldsymbol{x} + \mathbf{3})(\boldsymbol{x} + \mathbf{3})$ and $-\mathbf{3}\boldsymbol{x}^{\mathbf{2}}$**

Difference of Squares

- Whenever we **see subtraction**, **two square terms** only, and **no term with degree one**, we have the possibility of a **difference of squares.**

Example: $x^2 - y^2$ or $4x^2 - 25$

Subtraction

Two perfect squares

Difference of Squares

$$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$$

Example:

$$\boldsymbol{4x^2 - 25 = (2x + 5)(2x - 5)}$$

Check using FOIL.

The middle (degree 1 term), cancels out!

Practice Problems

Give four examples for b so that the following trinomials can be factored.

1. $x^2 + bx + 6$ MUST MULTIPLY TO 6 SO 1 & 6, -1 & -6, 2 & 3, -2 & -3
SO b = 7, -7, 5, -5

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2. $x^2 + bx + 4$ MUST MULTIPLY TO 4 SO 1 & 4, -1 & -4, 2 & 2, -2 & -2
SO b = 5, -5, 4, -4

---

3. $x^2 + bx - 8$ MUST MULTIPLY TO -8 SO -1 & 8, 1 & -8, 2 & -4, -2 & 4
SO b = 7, -7, -2, 2

---

4. $x^2 + bx - 6$ MUST MULTIPLY TO -6 SO -1 & 6, 1 & -6, 3 & -2, -3 & 2
SO b = 5, -5, 1, -1

Give positive and negative examples for c so that the following trinomials can be factored.

5. $x^2 + 6x + c$ ANY 2 #'S THAT ADD TO 6.
4 + 2 & 8 - 2 SO C = 8 AND -16

---

6. $x^2 - 4x + c$ ANY 2 #'S THAT ADD TO -4
-6 + 2 & -3 - 1 SO C = -12 & 3

---

7. $x^2 + 2x + c$ ANY 2 #'S THAT ADD TO 2
1 + 1 & 4 - 2 SO C = 1 & -8

---

8. $x^2 - 5x + c$ ANY 2 #'S THAT ADD TO -5.
-7 + 2 & -3 - 2 SO C = -14 & 6

9. A student factored $x^3 - 5x^2 - 14x$ into $(x-7)(x+2)$. Explain the error that was made.

THERE SHOULD HAVE BEEN A COMMON FACTOR OF X FACTORED OUT. SO $x(x-7)(x+2)$

Factor

10. $a^2 + 9a + 8$

× +
8 9
1, 8

$(a+1)(a+8)$

11. $b^2 + 16b + 15$

× +
15 16
15 1

$(b+15)(b+1)$

12. $c^2 + 10c + 24$

× +
24 10
6, 4

$(c+6)(c+4)$

13. $d^2 + 7d + 10$

× +
10 7
5, 2

$(d+5)(d+2)$

14. $x^2 - 18x + 72$

× +
72 -18
-6, -12

$(x-6)(x-12)$

15. $y^2 - 20y + 91$

× +
91 -20
-7 -13

$(y-7)(y-13)$

16. $z^2 - 13z + 36$

× +
36 -13
-9, -4

$(z-9)(z-4)$

17. $u^2 - 4u + 4$

× +
4 -4
-2, -2

$(u-2)(u-2)$

18. $l^2 + 7l - 30$

× +
-30 7
10, -3

$(l+10)(l-3)$

19. $m^2 + 4m - 12$

× +
-12 4
6, -2

$(m+6)(m-2)$

Factor Completely

20. $3x^2 + 15x + 12$

$3(x^2+5x+4)$

× +
4 5
4, 1

$3(x+4)(x+1)$

21. $4y^2 + 20y + 24$

$4(y^2+5y+6)$

× +
6 5
3, 2

$4(y+3)(y+2)$

22. $-5x^2 + 25x - 20$

$-5(x^2 - 5x + 4)$

× +
4 −5
−4, −1

$-5(x-4)(x-1)$

23. $-2y^2 + 58y - 200$

× +
100 −29
−25, −4

$-2(y^2 - 29y + 100)$

$-2(y-25)(y-4)$

24. $-x^2 - 6x + 27$

$-(x^2 + 6x - 27)$

× +
−27 6
9, −3

$-(x+9)(x-3)$

25. $-x^2 + 7x + 44$

$-(x^2 - 7x - 44)$

× +
−44 −7
−11, 4

$-(x-11)(x+4)$

26. $x^3 + 8x^2 - 20x$

× +
−20 8
10, −2

$x(x^2 + 8x - 20)$

$x(x+10)(x-2)$

27. $-2x^4 - 4x^3 + 30x^2$

× +
−15 2
5, −3

$-2x^2(x^2 + 2x - 15)$

$-2x^2(x+5)(x-3)$

28. $-x^3y - x^2y^2 + 6xy^3$

× +
−6 −1
−3, 2

$-xy(x^2 - xy - 6y^2)$

$-xy(x-3y)(x+2y)$

29. $2x^4 - 16x^3y + 32x^2y^2$

× +
16 −8
−4, −4

$2x^2(x^2 - 8xy + 16y^2)$

$2x^2(x-4y)(x-4y)$

Factor each binomial completely.

30. $x^2 - 1$

$(x+1)(x-1)$

31. $4x^2 - 1$

$(2x+1)(2x-1)$

32. $y^2 - 25$

$(y+5)(y-5)$

33. $25y^2 - 9$

$(5y+3)(5y-3)$

34. $4 - 9z^2$

$(2+3z)\ (2+3z)$

35. $16 - 25y^2$

$(4+5y)(4-5y)$

36. $16x^2 - 9y^2$

$(4x+3y)(4x-3y)$

37. $25x^4 - 81y^6$

$(5x^2+9y^3)(5x^2-9y^3)$

38. $16x^2y^8 - 4$

$4(4x^2y^8 - 1)$

$= 4\,(2xy^4+1)(2xy^4-1)$

39. $20x^2 - 5y^2$

$5(4x^2 - y^2)$

$= 5\,(2x-y)(2x+y)$

40. $(x+1)^2 - y^2$

$(x+1+y)(x+1-y)$

41. $4 - (x+2)^2$

$(2+(x+2))\,(2-(x+2))$

$(x+4)(4-x)$