

Math 10F Unit 2 LG 4-5 Multiplying Polynomials & Factoring

Classifying and Simplifying Multiplication of Polynomials

[Click here to watch the video on Polynomials or use the QR code.](#)



Term

- A number or a product of a number with **one or more variables** which can be **raised to a power**.

Examples of Terms

Term	$5y$	$-2a^3$	$\frac{1}{2}x^2yz^4$	x	10
Coefficients	5	-2	$\frac{1}{2}$	1	10
Variables	y	a	x, y, z	x	No variable

Polynomial

A term or sum of terms, in which all variables have **whole number exponents**, and in which **variables appear only in the numerator**.

Examples and Non-Examples of Polynomials

Polynomial	Non-Polynomial
5	$x^{\frac{1}{2}}$
$\sqrt{2}x$	$2x + \sqrt{y}$
$3a^2 - 2a$	$\frac{1}{2x} + 4$
$\frac{3}{4}y + 3y^2 - 4$	$x^{-3} - 2x$

Classifying Polynomials

Monomial	A polynomial with one term	$3, 2x^2y, -3a$
Binomial	A Polynomial with two terms	$x + 2, 2x^2y + 3, x^2 - y$
Trinomial	A polynomial with three terms	$3x^2 + 2x - 3, \sqrt{2}x + y - z$
Polynomial	General term for expressions with more than three terms (can also be used to describe monomial, binomial, and trinomial)	$x^5 - 2x^4 + 3x^3 - 4x^2$

Degree of a Polynomial

The **degree** of a term in a polynomial is the **sum of the exponents** of the variables in the term.

The **degree of a polynomial** is the **term with the highest degree**.

Examples:

In $3x^2 + 2x - 3$, the term of the highest degree is $3x^2$, so the degree is 2.

In $4x^2y^3 + z^4$, the term of the highest degree is $4x^2y^3$, so the degree is 5. ($2 + 3 = 5$)

In $-2x^2yz^2 + y^4$, the term of the highest degree is $-2x^2yz^2$, so the degree is 5. ($2 + 1 + 2 = 5$)

Leading Term

The **term** with the **highest** degree.

Consider: $3x^2y^3 - 2xy^2 + 2$

- The **degree** of the terms are: 5, 3, and 0 respectively.
- This means that the **leading term** is the term with the **highest degree** so: $3x^2y^3$
- It is good routine to always put polynomials in **descending order** with respect to **degree**.

Combining Like Terms

- Like terms are either **constant terms**, or **terms** that contain the **same variables** to the **same power**.

Example:

- $3x^2, 5x^2$ are like terms, they have the same variable and exponent.
- $2xy^2, 3x^2y$ are not like terms because they have different exponents for each variable.
- To **combine like terms**, **add or subtract** the **coefficients** of the terms.

Example 1: Simplify the expression $4x^2 - 3y - x^2 + 5y$

Solution 1: $4x^2$ and $-x^2$ are like terms, so the coefficients can be added.
 $-3y$ and $5y$ are like terms, so the coefficients can be added.

Therefore: $4x^2 - 3y - x^2 + 5y = 3x^2 + 2y$

Evaluating Polynomials

- When a **constant** is **substituted** for a **variable** in a polynomial, the polynomial is evaluated for that constant.

Example:

When $3x^2 - 2$ is evaluated for $x = 4$,
the result is: $3(4)^2 - 2 = 48 - 2 = 46$

When $5x^3 + 6$ is evaluated for $x = 2$,
the result is: $5(2)^3 + 6 = 40 + 6 = 46$

When $4xy^2 - 2x^2y$ is evaluated for $x = 3$ and $y = 4$,
the result is: $4(3)(4)^2 - 2(3)^2(4) = 192 - 72 = 120$

Multiplying a Monomial by a Monomial

- To multiply two **monomials** (one term), first multiply the constant factors, and then multiply the variable factors.

Example 2: Multiply

- $(2x^3)(-3x^4)$
- $(-3a^2b^3)(-2a^2b^5)$
- $(-3y^2z^3)(2yz^2)(4y^3z^2)$

Solution 2:

- $(2)(-3)(x^3)(x^4) = -6x^{3+4} = -6x^7$
- $(-3)(-2)(a^2)(a^2)(b^3)(b^5) = 6a^{2+2}b^{3+5} = 6a^4b^8$
- $(-3)(2)(4)(y^2)(y)(y^3)(z^3)(z^2)(z^2) = -24y^{2+1+3}z^{3+2+2} = -24y^6z^7$

Multiplying a Monomial by a Binomial

- When multiplying a **monomial by a binomial**, use the **distributive property**.
- DISTRIBUTIVE PROPERTY** or **WATERBOMB** is: $a(b + c) = a \cdot b + a \cdot c$

*****DON'T FORGET YOUR EXPONENT RULES*****

Example 3: Multiply

- $2(3 + 4)$
- $2x^2(3x^2 - 4y)$
- $-3y(x^4 + 2y^3)$

Solution 3:

- $2 \cdot 3 + 2 \cdot 4 = 6 + 8 = 14$
- $(2x^2)(3x^2) - (2x^2)(4y) = 6x^4 - 8x^2y$
- $(-3y)(x^4) + (-3y)(2y^3) = -3x^4y - 6y^4$

Practice Problems

For each polynomial, find the number of terms, the degree, and coefficients of each term.

1. $3x^5$	2. $-2y^4$	3. $4x^3 - 2x^2$
4. $-3a^3 + 3a - 4^0$	5. $2x^3y^2 - 3x^2$	6. $2^3b^3 - 3^2$
7. $-x^3y^2z + \sqrt{2}xyz + 4z^3$	8. $4x^4y^3z^2p + p$	9. x^3

For each polynomial, simplify, then write the answer in descending order.

10. $3x^2 - 2x + 5x - x^2$

11. $\frac{2}{3}x^4 + \frac{4}{3}x^4$

12. $2.3x^2 + 3 - 4.1x^2 + 3x$

13. $3y^4 - 2y^2 - y^4 - 2y$

14. $-2x^3 - 2x^2 - 2x^3 + 2x^2$

15. $x^2 - 2x + x^3 + x$

16. $2x^2 - \frac{3}{4}x^3 + 6x^2 + \frac{2}{3}x^3$

17. $-4y^3 - \frac{1}{2}y^5 + 5y^3 - \frac{1}{3}y^5$

Find the value of the polynomial when $x = -2$

18. $-3x^2 + 2x - 1$

19. $-3x^2 - 2x + 1$

20. $2x^2 - 3x + 4$

21. $-2x^2 - 3x - 4$

Find each product.

22. $3x^3(2x^4)$

23. $-2a^2b^4(4ab^2)$

24. $(3xy)(-4x^2y^2)$

25. $(2ab)(-2ab)(2ab)$

26. $(5x^3)(-2y^3)$

27. $(-4a^4b^3)(2a^3b^2)(3ab)$

28. $(a^2b^4)(a^3b)(-3b^2)$

29. $(-r^4s^2t)(r^3st^2)(-rst)$

30. $(-3ab^2)(2a^3b)(-a^2b^2)(-2a^3b^2)$

31. $(-5a^3b^3c^2d^3)(-2ab^2cd^2)(-4a^2bc^3d)$

32. $3(x - 4)$

33. $-2x^2(x + 3)$

$$34. 5x(4x - 3)$$

$$35. 2x^3(4x^5 - 3x)$$

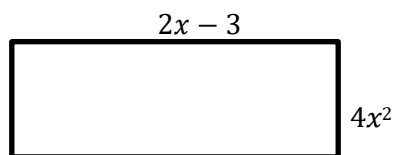
$$36. 2y(3x - 5y)$$

$$37. 2y^2(5y - x)$$

$$38. a^2bc(ab^2c^2 - a^2bc^2 - 2a^3b^3c)$$

$$39. -abc^2(-a^2bc^3 + ab^2c - a^3c^2)$$

40. Determine the area.



Multiplication of Binomials and Trinomials

[Click here to watch the video on Multiplication of Polynomials](#) or use the QR code.

- There are **two methods** that can effectively be used for multiplying polynomials.
- **Method 1: Distributive Method** *(BEST FOR LARGE POLYNOMIALS)*
- **Method 2: Horizontal Method (FOIL)** *(BEST FOR TWO BINOMIALS)*



Method 1: Distributive Method

Example 1: Multiply $(2x - 3)(x + 2)$

Solution 1:

$$\begin{aligned}(2x - 3)(x + 2) &= (2x - 3)(x) + (2x - 3)(2) \\ &= 2x^2 - 3x + 4x - 6 \\ &= 2x^2 + x - 6\end{aligned}$$

Example 2: Multiply $(x^2 - 2)(3x^2 - 4x + 3)$

Solution 2:

$$\begin{aligned}(x^2 - 2)(3x^2 - 4x + 3) &= (x^2 - 2)(3x^2) + (x^2 - 2)(-4x) + (x^2 - 2)(3) \\ &= 3x^4 - 6x^2 - 4x^3 + 8x + 3x^2 - 6 \\ &= 3x^4 - 4x^3 - 3x^2 + 8x - 6\end{aligned}$$

Example 3: Multiply $(3x + y)(2x^2 - 5xy + 4y^2)$

Solution 3:

$$\begin{aligned}(3x + y)(2x^2 - 5xy + 4y^2) &= (3x + y)(2x^2) + (3x + y)(-5xy) + (3x + y)(4y^2) \\ &= 6x^3 + 2x^2y - 15x^2y - 5xy^2 + 12xy^2 + 4y^3 \\ &= 6x^3 - 13x^2y + 7xy^2 + 4y^3\end{aligned}$$

Method 2: Horizontal Method (FOIL)

- *FOIL* is the acronym for: **F**irst, **O**utside, **I**nside, **L**ast.

In $(a + b)(c + d)$:

First refers to $a \cdot c$

Outside refers to $a \cdot d$

Inside refers to $b \cdot c$

Last refers to $b \cdot d$



Example 4: Multiply $(x - 6)(x + 4)$

Solution 4:

$$\begin{aligned}(x - 6)(x + 4) &= \underset{\text{First}}{(x)(x)} + \underset{\text{Outside}}{(x)(4)} + \underset{\text{Inside}}{(-6)(x)} + \underset{\text{Last}}{(-6)(4)} \\&= x^2 + 4x - 6x - 24 \\&= x^2 - 2x - 24\end{aligned}$$

Example 5: Multiply $(2x - 3)(3x + 1)$

Solution 5:

$$\begin{aligned}(2x - 3)(3x + 1) &= \underset{\text{First}}{(2x)(3x)} + \underset{\text{Outside}}{(2x)(1)} + \underset{\text{Inside}}{(-3)(3x)} + \underset{\text{Last}}{(-3)(1)} \\&= 6x^2 + 2x - 9x - 3 \\&= 6x^2 - 7x - 3\end{aligned}$$

- **FOIL DOES NOT WORK WITH LARGER POLYNOMIALS**
- **IT IS DESIGNED TO USE THE DISTRIBUTIVE METHOD WITH BINOMIALS ONLY**
- **WITH LARGER POLYNOMIALS THE DISTRIBUTIVE METHOD IS THE BEST OPTION**

Verifying your Multiplication

- The process of multiplying Polynomials has multiple steps and can have potential error spots.
- The beauty here is that it is possible to verify your result.
- Simply take any number (0 or 1 is usually a good choice) and substitute it in for your variable.
- The left and right side of the equations should match up.
- If they do not an error has been made

Consider:

$$\begin{aligned}(2x - 3)(3x + 1) &= 6x^2 - 7x - 3 \\ (2(0) - 3)(3(0) + 1) &= 6(0)^2 - 7(0) - 3 \\ (-3)(1) &= -3 \\ (-3) &= (-3)\end{aligned}$$

Since both sides of the equation equals -3 , the answer is verified for $x = 0$.

Common Errors when Multiplying Polynomials

1. $(x + 3)^2 \neq x^2 + 9 \rightarrow (x + 3)^2 = (x + 3)(x + 3) = x^2 + 6x + 9$
2. $(x - 8)^2 \neq x^2 - 64 \rightarrow (x - 8)^2 = (x - 8)(x - 8) = x^2 - 16x + 64$
3. $(x + 2)(x - 2) \neq x^2 - 4x + 4 \rightarrow (x + 2)(x - 2) = x^2 - 4$

General Rules

Square of a Binomial

$$(a + b)^2 = a^2 + 2ab + b^2 \qquad (a - b)^2 = a^2 - 2ab + b^2$$

Product of Sum and Difference

$$(a - b)(a + b) = a^2 - b^2$$

Practice Problems

Multiply, leave all answers in descending order.

1. $(2x + 1)(3x + 2)$

2. $(3y - 4)(2y + 3)$

3. $(3x^2 + 4)(x + 1)$

4. $(4y^2 + 3)(3y - 1)$

5. $(-3y - 4)(2y - 3)$

6. $(5x - y)(5x - y)$

7. $(2x - 1)(2x^2 + 3x - 1)$

8. $(5x - 6)(2x^2 + 7x - 3)$

9. $(3y - 2x)(4y^2 - 3xy + x^2)$

10. $(x^2 + 2y)(2x^2 + 3xy - y^2)$

Multiply, leave all answers in descending order.

11. $(a + b)^2$

12. $(2x + 1)^2$

13. $(x - y)^2$

14. $(3x + 2y)^2$

15. $(-x - 2y)^2$

16. $(-3x^2 + 2y^2)^2$

17. $(-a^3b + c^2d^2)^2$

18. $(2a^2b^2 - 4c^3d)^2$

Multiply, leave all answers in descending order.

19. $(a - b)(a + b)$

20. $(2x + 1)(2x - 1)$

21. $(x - y)(x + y)$

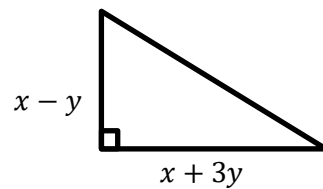
22. $(3x + 2y)(3x - 2y)$

23. $(-x - 2y)(-x + 2y)$

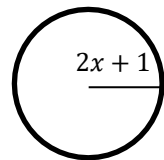
24. $(-x^2 - 2y^2)(-x^2 + 2y^2)$

Calculate the area of the following figures.

25. *Right Triangle*



26. *Circle*



Factoring The Greatest Common Factor

[Click here to watch the video on Factoring the GCF or use the QR code.](#)



Removing Common Factors

- To **factor** a polynomial is to express it as a **product (things MULTIPLYING together)**
- Removing a **common factor** is the most basic form of factoring.
- If every term has at least one factor that is the same, it is known as the **common factor**.
- Removing the **greatest common factor** is the best approach.

Example 1: Factor

- a) $5x + 10$
- b) $3x^2 - 6x$
- c) $12x^4 - 8x^3 + 4x^2$

Solution 1:

$$\text{a) } 5x + 10 = 5(x) + 5(2) = 5(x + 2)$$

$$\text{b) } 3x^2 - 6x = (3x)(x) - 3x(2) = 3x(x - 2)$$

$$\begin{aligned} \text{c) } 12x^4 - 8x^3 + 4x^2 &= 4x^2(3x^2) - 4x^2(2x) + 4x^2(1) \\ &= 4x^2(3x^2 - 2x + 1) \end{aligned}$$

- **NOTE:** If an entire term is the greatest common factor, then a "1" must be used to hold its place (as seen in example c)
- **NOTE:** To make sure the solution is correct, you can check your answer by multiplying in

Example 2: Factor $x(x + 1) + 3(x + 1)$

Solution 2: A common factor does not have to be a monomial, $(x + 1)$ is common to both terms and can be taken out.

$$x(x + 1) + 3(x + 1) = (x + 1)(x + 3)$$

Practice Problems

Factor out the greatest common factor.

1. $18x^3 - 27x$

2. $24a^3 + 18a$

3. $\frac{1}{3}y^2 - \frac{4}{3}y$

4. $3x^2 + 3x - 6$

5. $8b^2 - 4b + 20$

6. $5c^5 - 10c^3 + 15c$

7. $12x^3y + 6xy^2$

8. $6x^7 - 9x^6 - 57x^5 + 3x^4$

Factor

9. $x(2x + 1) + 3(2x + 1)$

10. $x(x + 1) + (x + 1)$

11. $3y(3y + 1) - 2(3y + 1)$

12. $6y^2(y - 3) + 5(y - 3)$

13. $a(a + 2b) + b(a + 2b)$

14. $3c(1 - 2d) - 2d(1 - 2d)$

Factoring Trinomials

[Click here to watch the video on Factoring Trinomials](#) or use the QR code.



Factoring Quadratics (Polynomials of degree 2): $x^2 + bx + c$

Consider this: $(x + a)(x + b) = x^2 + bx + ax + ab$

$$x^2 + (b + a)x + ab$$

- By looking at this we see that:
 - The **first term** is the **product** of x and x
 - The **coefficient** of the **middle term** is the **sum** of a and b
 - The **last term** is the **product** of a and b
- This leads us to the **general rule**:

When factoring $x^2 + bx + c$, look for **two factors of c** , that **multiply** to the **coefficient** of the **last term**, and **add** to the **coefficient** of the middle term.

Example 1: Factor $x^2 + 7x + 12$

Solution 1: What two numbers **add to 7** and **multiply to 12**?

- Integers that multiply to 12: (1, 12) (2, 6) (3, 4) (-1, -12) (-2, -6) (-3, -4)
- Only integers +3 and +4 add to 7
- Therefore $x^2 + 7x + 12 = (x + 3)(x + 4)$
- We can **check our answer** using **FOIL**: $(x + 3)(x + 4)$

$$= x^2 + 3x + 4x + 12$$

$$= x^2 + 7x + 12$$

Example 2: Factor $x^2 + 8 - 6x$

Solution 2: First **arrange** the polynomial in **descending order** of powers.

- $x^2 + 8 - 6x = x^2 - 6x + 8$
- -4 and -2 add to -6 and multiply to +8
- Therefore: $x^2 - 6x + 8 = (x - 4)(x - 2)$
- We can **check using FOIL**.

Example 3: Factor $5x^2 + 35x + 60$

Solution 3: Always look for a **common factor** first. The **largest common factor** is 5

- Therefore: $5x^2 + 35x + 60 = 5(x^2 + 7x + 12)$
- Now we can factor the Quadratic like we did previously:
- Two numbers that **multiply** to +12 and **add** to +7
- +4 and +3 get the job done
- So $5(x^2 + 7x + 12) = 5(x + 4)(x + 3)$
- **Check** your answer **using FOIL**

Example 4: Factor $-x^2 + 5x + 6$

Solution 4: First factor out -1 , so that the **coefficient** of x^2 becomes +1.

- So $-x^2 + 5x + 6$ becomes $-(x^2 - 5x - 6)$, now factor $(x^2 - 5x - 6)$
- -6 and 1 **multiply** to -6 and **add** to -5
- Therefore $-x^2 + 5x + 6 = -(x^2 - 5x - 6) = -(x - 6)(x + 1)$
- **Note the factors are: $(x - 6)(x + 1)$ and -1**

Example 5: Factor $-3x^4 - 18x^3 - 27x^2$

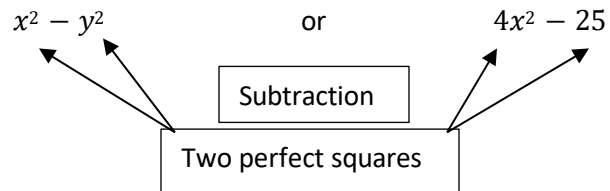
Solution 5: First look for a common factor. The largest here is $-3x^2$, factor it out

- So $-3x^4 - 18x^3 - 27x^2$ becomes $-3x^2(x^2 + 6x + 9)$, now factor $(x^2 + 6x + 9)$
- +3 and +3 **multiply** to +9 and **add** to +6
- Therefore $-3x^4 - 18x^3 - 27x^2 = -3x^2(x^2 + 6x + 9) = -3x^2(x + 3)^2$
- **Note the factors are: $(x + 3)(x + 3)$ and $-3x^2$**

Difference of Squares

- Whenever we **see subtraction, two square terms only**, and **no term with degree one**, we have the possibility of a **difference of squares**.

Example:



Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example:

$$4x^2 - 25 = (2x + 5)(2x - 5)$$

Check using FOIL.

The middle (degree 1 term), cancels out!

Practice Problems

Give four examples for b so that the following trinomials can be factored.

1. $x^2 + bx + 6$

2. $x^2 + bx + 4$

3. $x^2 + bx - 8$

4. $x^2 + bx - 6$

Give positive and negative examples for c so that the following trinomials can be factored.

5. $x^2 + 6x + c$

6. $x^2 - 4x + c$

7. $x^2 + 2x + c$

8. $x^2 - 5x + c$

9. A student factored $x^3 - 5x^2 - 14x$ into $(x - 7)(x + 2)$. Explain the error that was made.

Factor

10. $a^2 + 9a + 8$

11. $b^2 + 16b + 15$

12. $c^2 + 10c + 24$

13. $d^2 + 7d + 10$

14. $x^2 - 18x + 72$

15. $y^2 - 20y + 91$

16. $z^2 - 13z + 36$

17. $u^2 - 4u + 4$

18. $l^2 + 7l - 30$

19. $m^2 + 4m - 12$

Factor Completely

20. $3x^2 + 15x + 12$

21. $4y^2 + 20y + 24$

22. $-5x^2 + 25x - 20$

23. $-2y^2 + 58y - 200$

24. $-x^2 - 6x + 27$

25. $-x^2 + 7x + 44$

26. $x^3 + 8x^2 - 20x$

27. $-2x^4 - 4x^3 + 30x^2$

28. $-x^3y - x^2y^2 + 6xy^3$

29. $2x^4 - 16x^3y + 32x^2y^2$

Factor each binomial completely.

30. $x^2 - 1$

31. $4x^2 - 1$

32. $y^2 - 25$

33. $25y^2 - 9$

34. $4 - 9z^2$

35. $16 - 25y^2$

36. $16x^2 - 9y^2$

37. $25x^4 - 81y^6$

38. $16x^2y^8 - 4$

39. $20x^2 - 5y^2$

40. $(x + 1)^2 - y^2$

41. $4 - (x + 2)^2$