UNIT 1: RATIOS, UNIT RATES, & PROPORTIONS

- Watch the following screencast video before beginning this section:

https://youtu.be/RDNrC_RDmgo

Ratios andPatterns

You know that when you multiply both numerator and denominator of a fraction or ratio by the same number, the result is an equivalent fraction or ratio.

Example: $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

Try to find a pattern in these values. Can you fill in the denominators?

$\frac{1}{2} = \frac{2}{\ } = \frac{3}{6} = \frac{4}{\ } = \frac{5}{\ } = \frac{6}{\ } = \frac{7}{\ } = \frac{8}{\ }$ Type equation here.

You should have put a 4 in the first box above because $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$

What other patterns do you see in the fractions above?

- The numerators increase by ___ .
- The denominators increase by ____.

You can see that as the numerators increase by 1, the denominators increase by 2.

Let's look at another pattern.

Example: Out of every $5 that Martha earns, she saves $1. If Martha earns $15, how much will she save?

$Earned	$5	$10	$15	$20	$25
$**Saved**	$1				

Martha will save **$3** because $\frac{\$5}{\$1} = \frac{\$15}{\$3}$

What other patterns do you see in the chart above?

- The dollars earned increase by ______.

- The dollars saved increase by ______.

Can you see that as the dollars earned increase by 5, the dollars saved increase by 1?
No matter how much Martha earns, her ratio of *earnings to savings* is 5:1

or $\frac{5}{1}$

Unit Rates

Rates are ratios that compare different units of measurement. For example, *miles per hour* is a rate; *miles* are compared to *hours.* Rates are often expressed with the word *per,* which means "for each."

Example 1:

A car travelled 108 km on 8 litres of gas. How many km per L does the car get?

Step 1:	Step 2:	Step 3:
Write a ratio	Simplify to get a denominator of 1	Write the ratio as a rate
$\frac{108\ km}{8\ L}$	$\frac{108}{8}\ \frac{\div 8}{\div 8} = \frac{13.5\ km}{1\ L}$	The car gets 13.5 km per Litre

Write the following values as rates. The first one is done for you.

1. The River Bridge raises 2 times an hour to let boats pass. — 2 times per hour
2. The average human heart beats 70 times in 1 minute. ______________________
3. The building manager uses 1 gallon of paint to cover 3 walls. ______________________
4. Rowanda paid $13.50 for a ticket to last night's concert. ______________________
5. In 1 day, Mike's Muffler Shop installs 80 car mufflers. ______________________

Find the unit rate.

6. Chana drives 220 miles in 4 hours. How many *miles per hour* does she drive?

7. There are 12 cups in 3 quarts of liquid. How many *cups per quart* are there?

8. Jim drove 80 miles in 2½ hours. How many *miles per hour* did he drive?

9. There are 54 ounces total in 3 boxes of cereal. How many *ounces per box* are there?

Unit Pricing

Another common example of unit rate is **unit pricing.** If you know the cost of several units, you can figure out the cost of one unit. Similarly, if you know the cost of one unit, you can find the cost of several units.

Example 1:

Suppose you buy 3 bottles of juice for $5.55. What is the unit price of the juice.

Step 1:	Step 2:	Step 3:
Write a ratio	Simplify to get a denominator of 1	Write the ratio as a rate
$\frac{\$5.55}{3\ bottles}$	$\frac{5.55}{3}\ \frac{\div 3}{\div 3} = \frac{\$1.85}{1\ bottle}$	The unit price is $1.85 per bottle.

If you know the cost of one item, you can use ratios to find the cost of several.

Example 2: Deanna bought 8 scarves that were on sale for $4.50 each. How much did she spend in all?

$\frac{\$4.50}{1} = \frac{?}{8}$ Since you multiplied the denominator to get 8, multiply the numerator by 8 as well:

$4.50 x 8 = $36.00 for 8 scarves

Find the costs of the products below by using the chart.

SANFORD GROCERIES
Ground sirloin ....$2.99 per pound
Beef tips ....$7.98 for a 2 pound pack
Potatoes ....$2.10 for a 3 pound bag
Onions ....$0.59 per pound
Lettuce ....$0.79 per head
Tomatoes ....$1.19 per pound

1. What would you pay for 4 pounds of ground sirloin?

2. Suppose you bought a 3-pound bag of potatoes. What did you pay per pound?

3. What is the unit price for beef tips at Sanford Groceries?

4. **a.** A customer bought 3 pounds of onions and 1 pound of beef tips. What did he pay?

b. A customer pajd for the purchases in problem 4a with a $10 bill. What change should he get back?

Understanding Proportion

A proportion is made up of two equal ratios. When you work with equivalent fractions, you are working with a proportion.

$\frac{2}{3} = \frac{4}{6}$ is a proportion. The two ratios are equal.

Two equal ratios make a proportion. Two ratios are equal if their **cross products** are equal.

Example 1: Is $\frac{3}{4} = \frac{6}{8}$
Are the cross products equal?

$\frac{3}{4} \times \frac{6}{8}$

3×8 = 24 and 4× 6 =24, so
yes ratios are equal.

Example 2: Is $\frac{1}{3} = \frac{2}{5}$
Are the cross products equal?

$\frac{1}{3} \times \frac{2}{5}$

1×5 = 5 and 3× 2 =26, so
No ratios are not equal.

If one number is missing in a proportion, you can use cross products and equations to find out what it is.

Solving Proportions

Example 1: $\frac{2}{3} = \frac{12}{?}$

Step 1:
Use a variable for the number you don't know

$\frac{2}{3} = \frac{12}{x}$

Step 2:
Cross Multiply

$\frac{2}{3} \times \frac{12}{x}$

Step 3:
Solve for the variable:
$2x = (3)(12)$

$x = \frac{36}{2}$

$\boldsymbol{x = 18}$

Practice:

Use cross products to decide if the following ratios are equal.

1. $\frac{3}{4} \stackrel{?}{=} \frac{6}{8}$
24 =24, yes

2. $\frac{1}{6} \stackrel{?}{=} \frac{12}{2}$
$2 \neq 72$, no

3. $\frac{3}{10} \stackrel{?}{=} \frac{12}{15}$

4. $\frac{1}{3} \stackrel{?}{=} \frac{12}{36}$

5. $\frac{1}{4} \stackrel{?}{=} \frac{5}{8}$

6. $\frac{5}{6} \stackrel{?}{=} \frac{10}{12}$

7. $\frac{6}{7} \stackrel{?}{=} \frac{12}{15}$

8. $\frac{3}{8} \stackrel{?}{=} \frac{6}{16}$

9. $\frac{2}{3} \stackrel{?}{=} \frac{14}{21}$

10. $\frac{5}{8} \stackrel{?}{=} \frac{25}{40}$

11. $\frac{1}{7} \stackrel{?}{=} \frac{14}{21}$

12. $\frac{4}{7} \stackrel{?}{=} \frac{16}{28}$

Use cross products and equations to find the missing number in each proportion.

1. $\frac{1}{5} = \frac{r}{40}$

2. $\frac{7}{8} = \frac{a}{40}$

3. $\frac{4}{q} = \frac{12}{15}$

4. $\frac{x}{3} = \frac{12}{18}$

5. $\frac{w}{6} = \frac{30}{36}$

6. $\frac{5}{9} = \frac{s}{27}$

7. $\frac{d}{8} = \frac{12}{16}$

8. $\frac{2}{9} = \frac{8}{x}$

9. $\frac{2}{5} = \frac{r}{20}$

10. $\frac{1}{8} = \frac{a}{200}$

11. $\frac{6}{q} = \frac{30}{15}$

12. $\frac{x}{7} = \frac{12}{21}$

13. $\frac{w}{10} = \frac{40}{100}$

14. $\frac{2}{9} = \frac{s}{45}$

15. $\frac{d}{6} = \frac{12}{18}$

16. $\frac{7}{8} = \frac{35}{x}$

17. $\frac{r}{5} = \frac{60}{100}$

18. $\frac{3}{7} = \frac{s}{49}$

19. $\frac{s}{5} = \frac{18}{90}$

20. $\frac{9}{10} = \frac{45}{x}$

Making a Table

It can be difficult to decide what to do with all the numbers in a math problem. By making a chart or a table, you can often tell whether a proportion can be used to solve a problem.

Example: Mary Lou can complete 48 circuit boards during an 8-hour shift. How many boards can she complete in 6 hours?

Can a proportion be used to solve this problem? Make a table to find out. Use labels that go with the numbers to set it up.

Circuit Boards	**48**	**n**
Hours	**8**	**6**

If you can make a table that compares two things (circuit board and hours in this case), and you can fill in 3 of the 4 values, you can write a proportion.

$$\frac{48\ boards}{8\ hours} = \frac{n\ boards}{6\ hours}$$

$$48 \times 6 = 8 \times n$$

$$\frac{288}{8} = n$$

$$36 = n$$

Put the correct numbers in the tables below. Then use the tables to help you write a proportion. Solve for the unknown.

1. The Cantin family drove 320 miles in an 8-hour trip. At this rate, how many miles could they drive in 10 hours?

Miles		
Hours		

2. It takes 2.5 gallons of paint to cover 6 walls at Garden Apartments. How many gallons are needed to cover 12 walls?

Walls		
Gallons		

Centimeters		
Inches		

3. There are 2.54 centimeters in 1 inch. How many inches are there in 101.6 centimeters?

4. One yard is equal to 36 inches. How many inches are there in 2 yards?

Yards		
Inches		

Some of the problems below can be solved using a proportion. Some cannot. Make a table for each problem and use it to decide whether you can write a proportion. Then solve each problem.

Problem	Can I use a proportion?	
5. A farmer planted corn in 2.5 of his 18 acres of fields. How many acres are not used for corn?	Yes	No
6. Jeffrey paid $11 for a 4-pound roast. At this same rate, how much would a 3-pound roast cost?	Yes	No
7. Out of every 1,000 newspapers that come off the press, 12 of them are usually defective. How many papers would be defective out of a daily run of 40,000 newspapers?	Yes	No
8. A square has a side that measures 3 ½inches. What is the perimeter (distance around) the square?	Yes	No
9. Ramon bought a sandwich for $4.35 and a soda for $.75. How much did he spend before sales tax?	Yes	No
10. The Aces usual!y are successful in 3 of every 4 foul shots they attempt. How many attempts did they make if they scored on 15 foul shots?	Yes	No

Solving Problems with Proportions

You can use proportions to solve many kinds of problems. First you must know how to write a proportion.

Example : If 5 boxes of matches cost \$1.25, then 3 boxes of matches would cost \$0.75. Write this relationship as a proportion.

$$\frac{5\ boxes}{\$1.25} = \frac{3\ boxes}{\$0.75}$$

Can you see that the first ratio compares boxes to dollars, and that the second ratio uses the same order? This is a correct proportion. What is wrong with the proportion below?

$$\frac{5\ boxes}{\$1.25} = \frac{\$0.75}{3\ boxes}$$

The two ratios do not compare the same unit in their numerators or the same unit in their denominators. This is *not* a true proportion.

When you write a proportion, be sure that the two ratios have corresponding units on top and corresponding units on the bottom.

Writing a Proportion

Example: At Bruce's Market, 2 pounds of greens cost \$1.40. At this rate, how much would 3 pounds cost?

Step 1
Write a ratio with two numbers in the problem.
Include labels

$$\frac{2\ pounds}{\$1.40}$$

Step 2
Write a proportion, using a variable for the number you do not know

$$\frac{2\ pounds}{\$1.40} = \frac{3\ pounds}{\$p}$$

Step 3
Multiply to find the cross products and find the unknown.

$$2 \times p = 1.40 \times 3$$
$$2 \times p = 4.20$$
$$p = 4.20 \div 3$$
$$\boldsymbol{p} = \$\mathbf{2.10}$$

Step 4
Check your proportion by using cross products

$$\frac{2}{1.40} = \frac{3}{\mathbf{2.10}}$$

$$2 \times 2.10 = 1.40 \times 3$$
$$4.20 = 4.20$$

The cost of 3 pounds of greens is \$2.10

Here is another way to set up a proportion for this problem:

$$\frac{\$1.40}{2\ pounds} = \frac{\$p}{3\ pounds}$$

Can you see that it does not matter which number goes on top or bottom in the *first* ratio? What *does* matter is that the second ratio has the *same* units on top and bottom as the first ratio.

Write *two* correct proportions that could be used to solve each problem below. (Be sure there are corresponding units on the top and corresponding units on the bottom) Do not solve.

1. Tom drove 80 miles in 3.5 hours. At this same rate, how far could he drive in 4.5 hours?

2. If 5 yards of silk cost $21.09, how much would 6 yards cost?

3. For every 3 women in the town of Milton, there are 2 men. If there are 2,400 women in all of Milton, how many men are there

4. In 6 square yards of her garden, Mrs. Tanaka can produce about 80 pounds of squash. How many square yards would it take to produce 100 pounds.

Write a proportion and solve for the unknown in each problem below.

5. Ricardo can travel 250 miles on 10 gallons of gas. At this rate, how many gallons does he need to travel 300 miles?

6. Carol picked 8 quarts of strawberries and paid $10.00 for them. How much did she pay per quart?

7. A recipe calls for 1½ cups of sugar to make 24 brownies. How much sugar is needed for 36 brownies?

8. A survey stated that 2 out of 3 residents were against the addition of a fast-food restaurant in their town. If 3,240 people were against the restaurant, how many people were surveyed in all?

9. If 2½ pints of bone meal is enough to fertilize 10 square feet of soil, how many square feet can 10 pints fertilize?

10**. Multiple Solutions** Write two correct proportions for this problem, then solve: At a rate of $15.00 per dozen, how much will 30 roses cost?

- Watch the following screencast video before beginning this section:

https://youtu.be/uvQVJxDECa4

Unit Conversion Proportion Problems

We can use proportions to solve some unit conversion problems. Use a conversion chart to get the ratio. Then set up labels, fill in numbers and solve for the desired unit.

Example: We ran 100 meters. How far is that in feet?

Step one: What is asked? Underline the question. We ran 100 meters. How far is that in feet?

Step two: What information is given? Highlight the information. We ran 100 meters. How far is that in feet?

Step three: Is a ratio given? If so, set up the problem with three lines and an equal sign.

---------- ----- = ------ Look up on a chart the ratio of meters to feet.

Conversion Factors

Length:
1 inch = 25.4 millimeters
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 mile = 1.609 kilometers
1 millimeter = 0.0394 inches
1 centimeter = 0.3937 inches
1 meter = 3.281 feet

1 mile = 1760 yards = 5280 feet

1 yard = 3 feet = 36 inches

1 foot = 12 inches

Use either conversion. The answer will be the same within a few decimal places.

Step four: Fill in the labels. At least one label comes from the question. We ran 100 meters. How far is that in feet?

"How far" says to use "feet" for one label. Use meters for the other label.

$$\frac{meters}{feet}\ \frac{100}{x} = \frac{1}{3.281}$$

OR

$$\frac{meters}{feet}\ \frac{100}{x} = \frac{0.3048}{1}$$

Step five: Fill in numbers. Put the given ratio on one side of the equal sign. Put the other piece of information on the same level as its label. Put an "x" in the empty space. Then solve.

$$\frac{100}{x} = \frac{1}{3.281}$$

(100)(3.281) = (x)(1)

328.1 feet = x

$$\frac{100}{x} = \frac{0.3048}{1}$$

(100)(1) = (x)(0.3048)

100 = 0.3048 x

100 ÷ 0.3048 = x

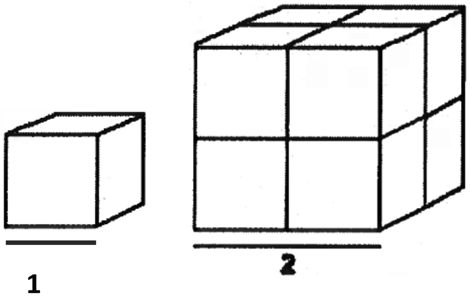
328.1 feet = x

Practice Questions:

Use the conversion chart in your formula sheet.

1. Convert 100cm into metres.
2. Convert 2.3l into ml.
3. Convert 72350mg into kg.
4. Convert 15m into km.
5. Convert 100000m into dm.
6. Convert 220km into Gm.
7. Convert 75cm into Mm.
8. How many inches in 23 cm?
9. How many inches in 0.7 cm?
10. How many centimeters in 5 inches?
11. How many centimeters in 0.45 inches?
12. How many centimeters in 23 inches?
13. How many feet in 43 meters?
14. How many meters in 2.5 feet?
15. How many feet in 0.5 meters?
16. How many miles in 42 km?
17. How many kilometers in 42 miles?
19. How many kilometers in 10 miles?
20. How many yards in 42 feet?
21. How many miles in 300 feet?
22. How many feet in 12 miles?

Area and Volume Scale Factors



Here's a simple box. Beside it is one **twice as big.**

You can tell it's twice as big because we've put a scale beside it.

Actually, to be more precise, the second box is twice as long as the first one. The correct way to say this is that it has been **scaled up** by a **factor of two.**

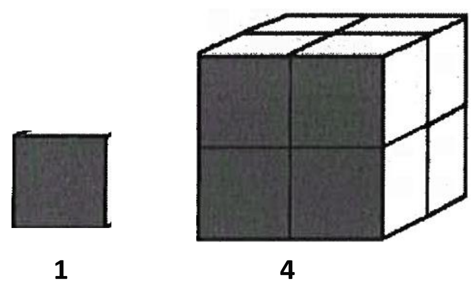
By making the box twice as long, we've also doubled the width and height.

Here's the same diagram, only this time we've shaded one face.

The second box is still twice as long as the first one.

Notice what happens to the area of one face of the box when it's twice as long.

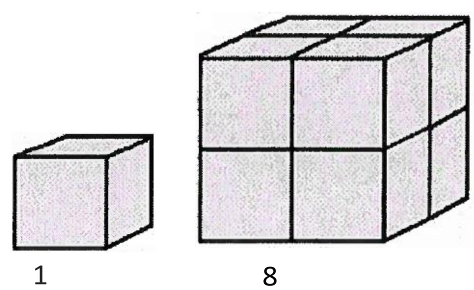
The AREA has been increased by a factor of FOUR.



Here's the same diagram again, only this time we've shaded all the boxes.

The second box is still twice as long as the first one.

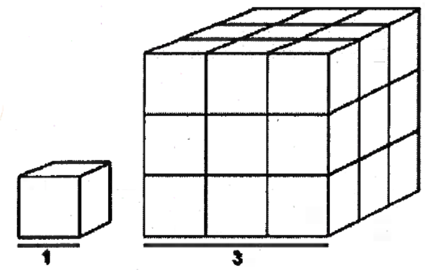
Notice what happens to the volume of the box when it's twice as long



The VOLUME has been increased by a factor of EIGHT.

Increasing the length of an object by a factor of 2 increases the area by a factor of 4 and the volume by a factor of 8.

Let’s start with another set of boxes



Here's a simple box. Beside it is one **three times as big.**
You can tell it's three times as big because we've put a scale beside it.

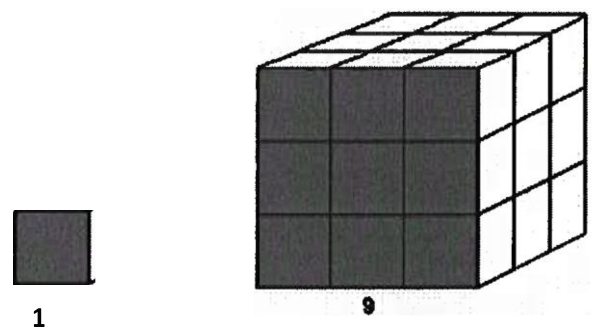
To be more precise, the second box is three times as long as the first one. The correct way to say this is that it has been **scaled** up by a **factor of three.**

By making the box three times as long, we've also tripled the width and height. Here's the same diagram, only this time we've shaded one face.

The second box is still three times as long as the first one.

Notice what happens to the area of one face of the box when it's three times as long.

The AREA has been increased by a factor of NINE.



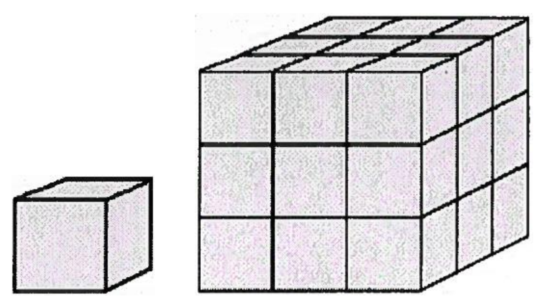
1

Here's the same diagram again, only this time we've shaded all the boxes.

The second box is still three times as long as the first one.

Notice what happens to the volume of the box when it's three times as long.

The VOLUME has been increased by a factor of TWENTY-SEVEN.



1 **27**

Increasing the <u>length of an object by a factor of 3</u> increases the <u>area by a factor of 9</u> and the <u>volume by a factor of 27</u>.

Do you see the pattern? Let's summarize what happened (and add a few more examples) in a table:

SCALE FACTOR	LENGTH INCREASE	AREA INCREASE	VOLUME INCREASE
2	2 TIMES	4 TIMES	8 TIMES
3	3 TIMES	9 TIMES	27 TIMES
4	4 TIMES	16 TIMES	64 TIMES
5	5 TIMES	25 TIMES	125 TIMES
x	x TIMES	x^2 TIMES	x^3 TIMES

Practice Questions

1. A stage director needs a large chess pawn for a scene. The pawn in her chess set is 35 mm tall and she estimates that the height of the enlarged pawn must be 700 mm. How many times greater will the surface area of the larger pawn be?

2. Cylinder A has a radius of 5 mm and a height of30 mm. Cylinder B has a radius of20 mm and a height of 120 mm. These two cylinders are similar.
By what factor is the surface area of cylinder B greater than the surface area of cylinder A?

3. Rectangle A is 6 cm high, 9 cm long, and 15 cm wide. Rectangle B is 14 cm high, 21 cm long, and 35 cm wide.

By what factor is the volume of rectangle B greater than the volume of rectangle A?

4. A cylindrical oil tank is filled with 500 m^3 of oil. A similar oil tank has dimensions that are reduced by a scale factor of $\frac{2}{3}$. What volume of oil will fill the smaller tank?

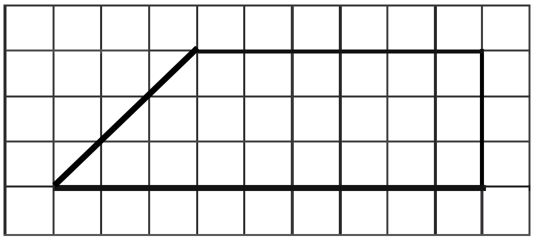
5. A pool in the shape of a rectangular prism is filled with 15 m^3 of water. A similar pool has dimensions that are increased by a scale factor of $\frac{4}{3}$. What volume of water will fill the larger swimming pool?

6. A cylindrical oil tank has a surface area of 1800 m^2. A similar oil tank has dimensions that are reduced by a scale factor of $\frac{2}{3}$. What is the surface area of the smaller tank?

7. A large city map book will be changed so that it can be used as a street guide. To keep the same number of pages, the page dimensions will be halved and the maps will be less detailed. The area of each page in the original book is 3000 cm^2. What is the area of each page in the smaller map book?

8. The sides of a square with an area of 49 cm^2 will be reduced by a scale factor of $\frac{5}{8}$. Determine the area of the reduced square to the nearest square centimetre.

9. Dexter enlarges this figure by a scale factor of 2. Determine the area of the enlarged figure in square units.



10. Sooki enlarges this figure by a scale factor of 2.Determine the area of the enlarged figure, to the nearest square unit.