

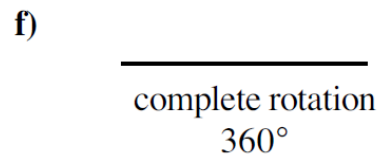
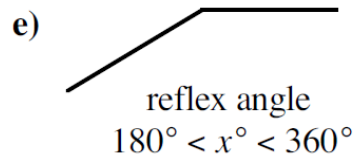
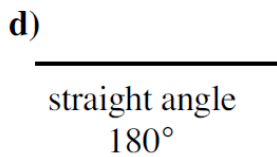
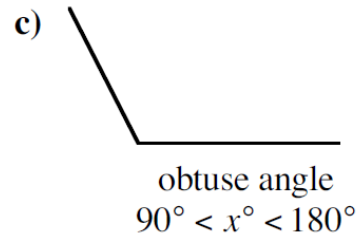
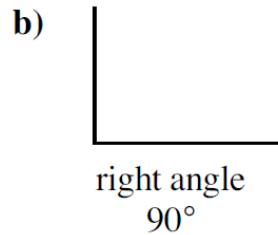
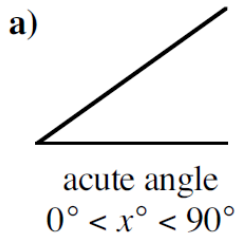
UNIT 2: REASONING WITH ANGLES AND TRIANGLES

- Watch the following screencast video before beginning this section:

<https://youtu.be/Nm6BwUS2UX4>

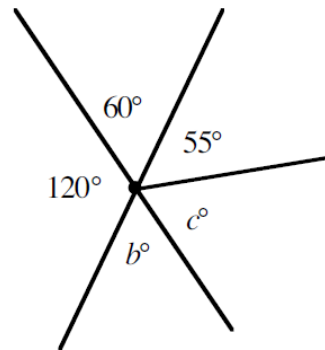
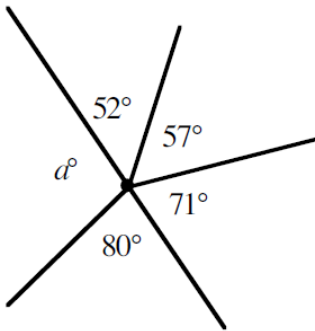
Types of Angle

Draw an appropriate arc to represent the angle x° in each case.
For the figure which represents a right angle, use the symbol \square .



Example 1:

Calculate the angles marked with letters.

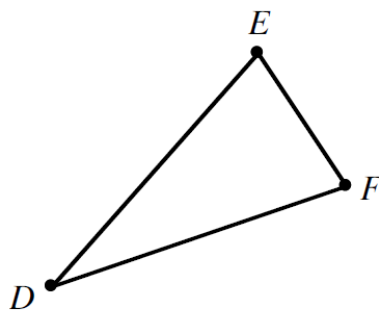
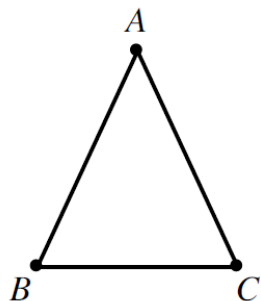


Angles in an Isosceles Triangle

Recall the following triangle property:

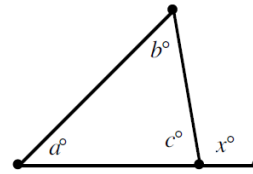
In an isosceles triangle, there are two equal sides and two equal angles.

Verify this property by measuring the sides and angles of the triangles shown.



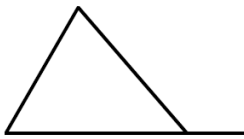
Exterior Angles of a Triangle

In the diagram shown at the right, angle x is an exterior angle. Angles a and b are interior opposite angles.



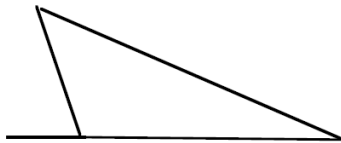
a) In the following examples measure the exterior angle and the interior opposite angles.

i)



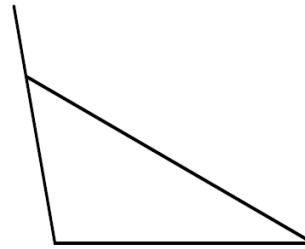
exterior angle is ____°
interior opposite angles
are ____° and ____°

ii)



exterior angle is ____°
interior opposite angles
are ____° and ____°

iii)



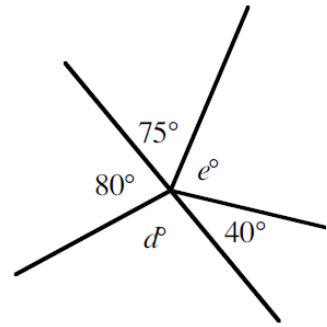
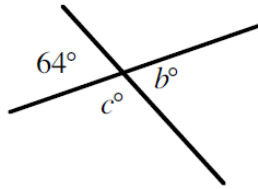
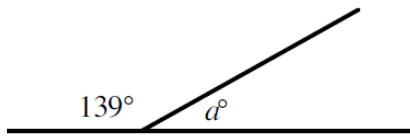
exterior angle is ____°
interior opposite angles
are ____° and ____°

b) Use inductive reasoning to describe any relationship you can see.

Practice:

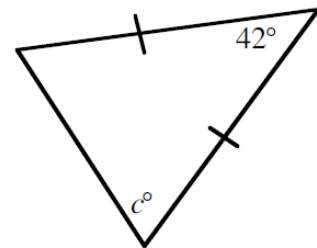
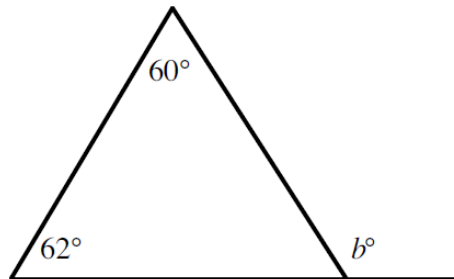
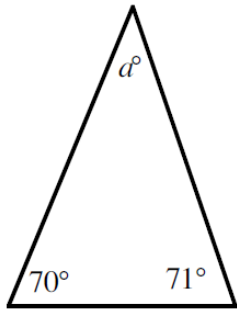
1.

Calculate the angles marked by letters.



2.

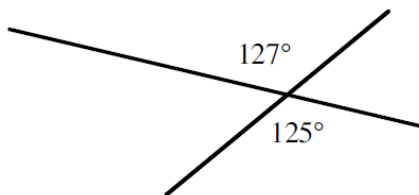
Calculate the angles marked by letters.



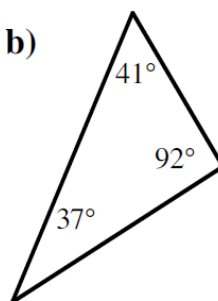
3.

Students have been asked to measure angles. The answers have been recorded on the diagrams. In each case, explain why there must have been a measurement error.

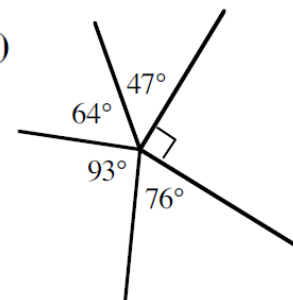
a)



b)



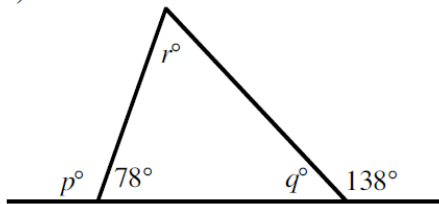
c)



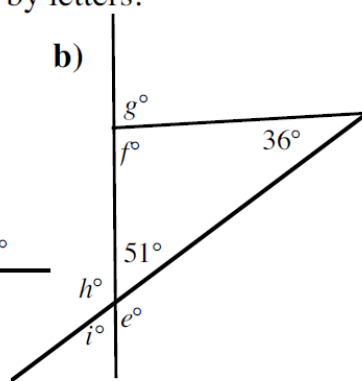
4.

Calculate the angles marked by letters.

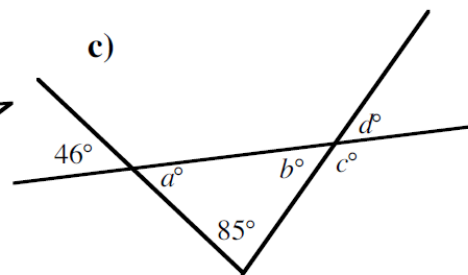
a)



b)

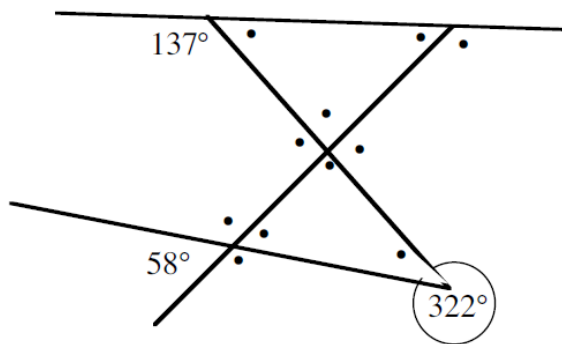


c)



5.

Mark on the diagram the measures of the eleven angles marked with dots.



- Watch the following screencast videos before beginning this next section:

<https://youtu.be/UQ3dnssMlyQ>

<https://youtu.be/atRi9tygdqE>

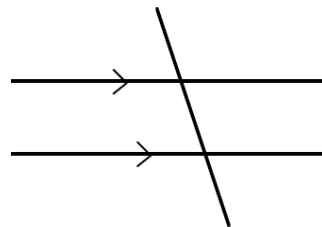
Angles Associated with Parallel Lines

Parallel Lines and a Transversal

On a diagram, parallel lines are often marked by arrows.

A line that crosses parallel lines is called a **transversal**.

When a transversal intersects parallel lines there are several relationships between the angles formed.

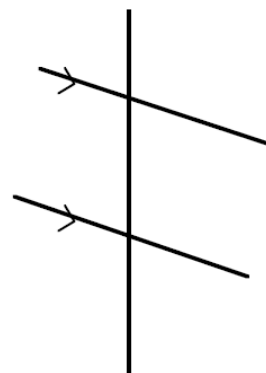
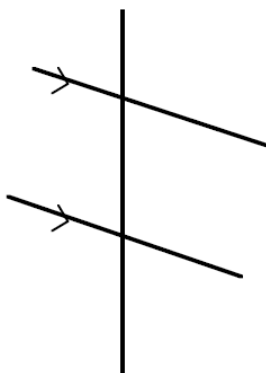
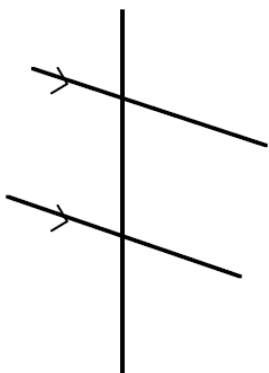
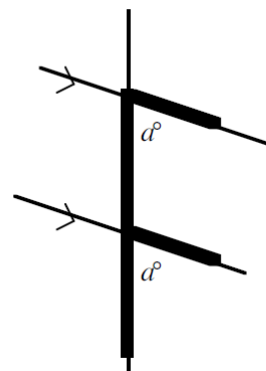


Corresponding Angles - “F angles”

Any pair of parallel lines makes an **F**-shape with a transversal that crosses them.

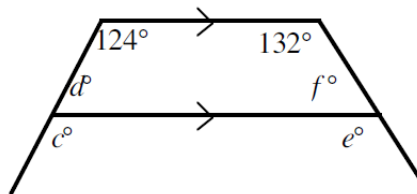
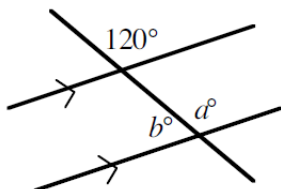
The angles marked a° are equal and are called **corresponding angles**.

- On the diagrams below mark three other pairs of **F**-shaped corresponding angles and label them b° , c° , and d° . (Note that the **F**-shape can be backwards or upside down).



Example 1:

Use corresponding angles and straight angles to calculate the angles marked by letters.

**Alternate Interior Angles - “Z angles”**

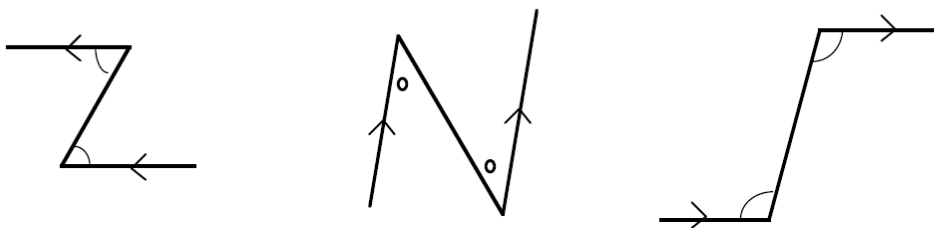
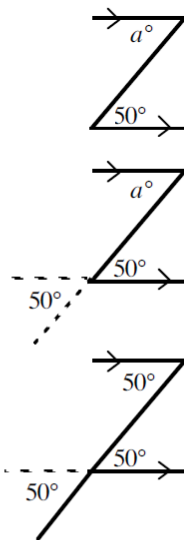
The Z in the diagram is made by two parallel lines and a transversal.

If we extend two of the lines we make an X and have two equal angles that are opposite at a vertex.

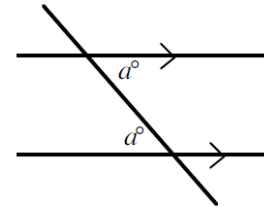
Now we have an F- shape as well, so the top angle a° must be 50° as well.

So both of the angles of the Z are equal.

- Measure the angles in each of the following Z diagrams. Are they equal?



Any pair of parallel lines makes a Z-shape with a transversal that crosses them.
(Note that the Z-shape can be backwards or upside down.)

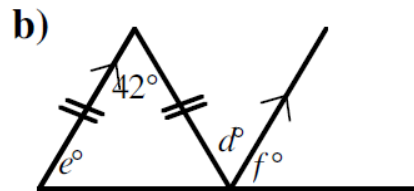
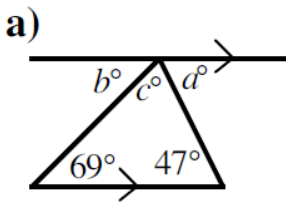


The angles marked a° are equal and are called **alternate interior angles** or sometimes just **alternate angles**.

- On the diagram, mark another pair of Z-shaped alternate angles and label them b° .

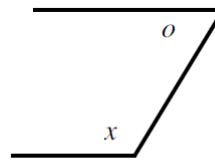
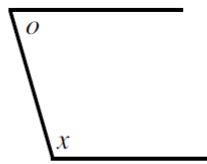
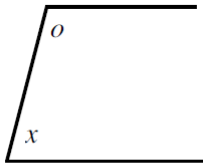
Example 2

Use alternate angles and straight angles to calculate the angles marked by letters.



. Co-interior Angles - “C angles”

The C in each diagram is made by two parallel lines and a transversal.
The angles marked o and x are called **co-interior angles**.

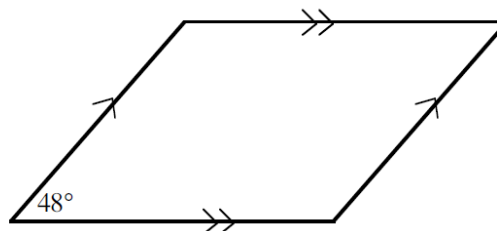


- Measure the angles o and x in each of the diagrams. Are they equal?
- Can you discover a relationship between the values of o and x ?

We can make a conjecture that co-interior angles are supplementary i.e. they add up to 180° .

Example 3

Calculate the measures of the three remaining angles in the parallelogram.



Summary

1. When parallel lines are intersected by a transversal,
 - corresponding angles are equal,
 - alternate angles are equal,
 - co-interior angles are supplementary.
 2. If a transversal intersects two lines such that corresponding angles are equal, alternate angles are equal, and co-interior angles are supplementary, then the two lines are parallel.
-

Practice:

1.

Use the given diagram to answer the following questions.

a) State the number of the angle corresponding to

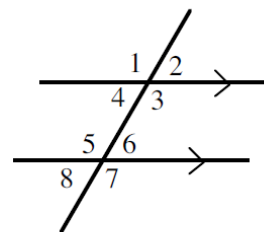
- i) 1 ii) 2 iii) 3 iv) 4

b) State the number of the angle alternate to

- i) 3 ii) 4

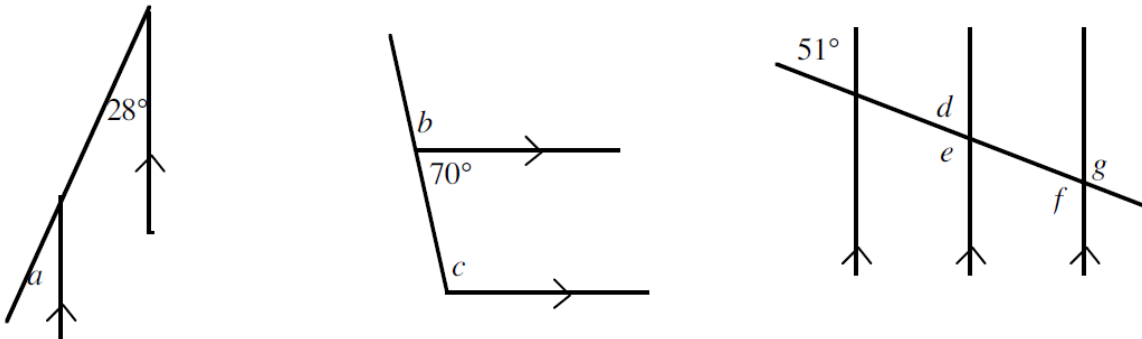
c) State the number of the angle co-interior to

- i) 3 ii) 4



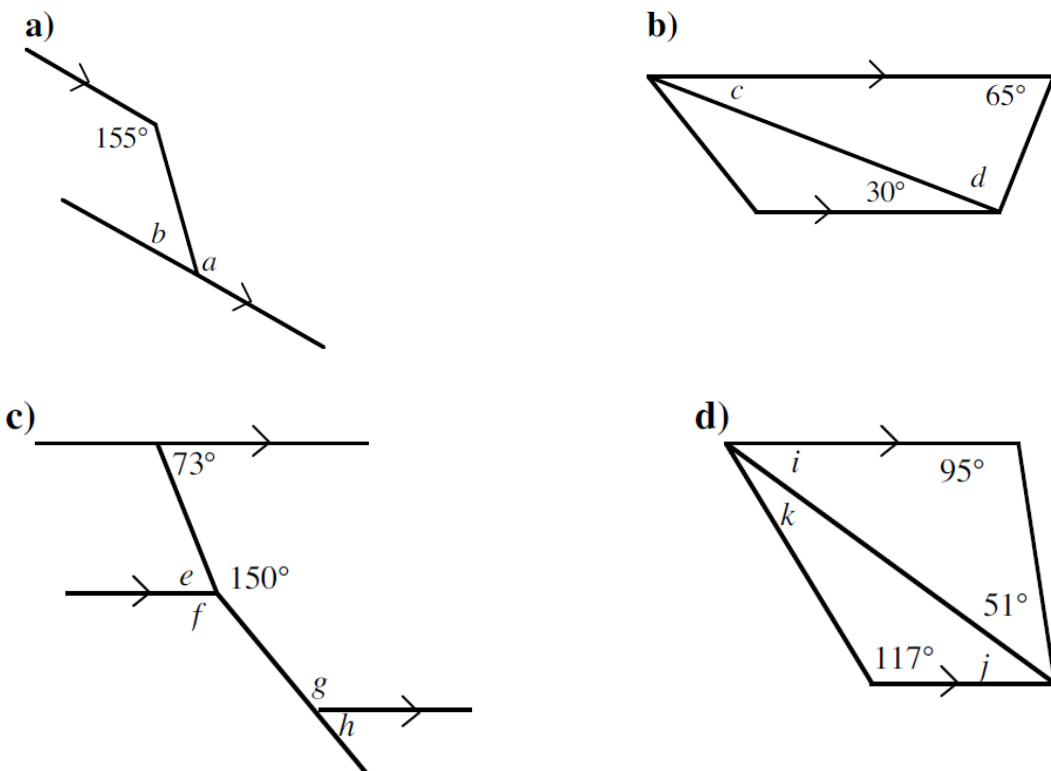
2.

Calculate the degree measure of the angles marked by letters.



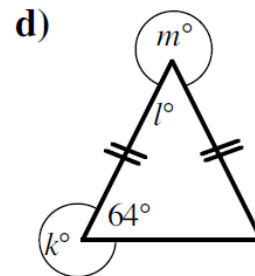
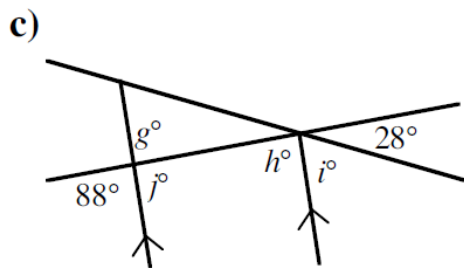
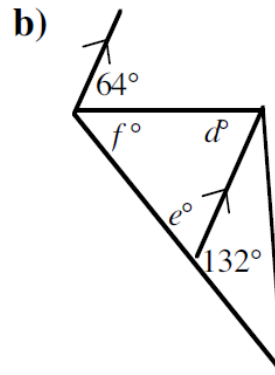
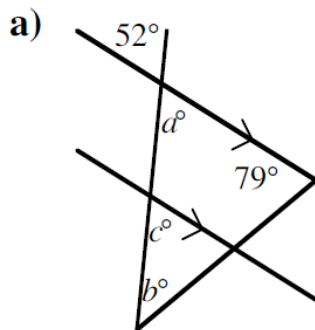
3.

Calculate the degree measure of the angles marked by letters.



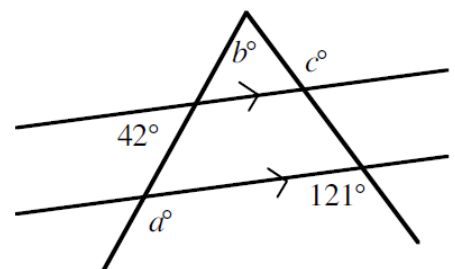
4.

Calculate the angles marked by letters.



5.

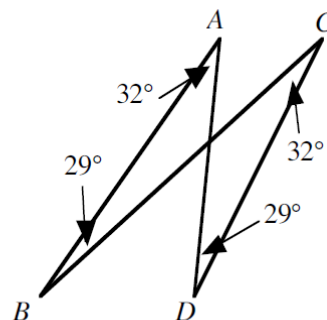
In the following example, you will need to work out other angles before you can determine the value of the marked angles. Calculate the angles marked with letters.



6.

Consider the diagram shown.

Explain why CD is not parallel to AB .



- Watch the following screencast video before beginning this next section:

https://youtu.be/3A0KYhOZm_o

Proving Angle Relationships

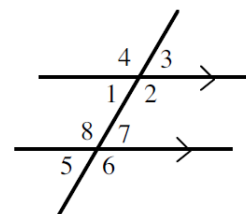
Proving Alternate Angles are Equal

Consider the diagram shown.

Angle 1 and angle 7 are alternate angles formed by the Z-shape.

We need to prove that angle 1 = angle 7.

Complete the following two column proof.



Statement

Reason

angle 1 = angle 3

angle 3 = angle 7

so angle 1 = angle 7

both = angle 3 (transitive property)

Alternate angles are equal.

Example 1

Complete the following two column proof to prove that co-interior angles are supplementary.

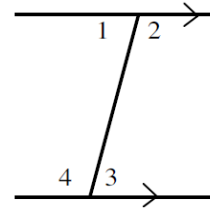
We need to prove that $\angle 1 + \angle 4 = 180^\circ$

Statement

Reason

$$\angle 1 + \angle 2 = 180^\circ$$

$$\angle 2 = \angle 4$$



Assignment

1.

. Consider the following diagram.

a) Name two pairs of alternate angles in the diagram.

b) Complete the following two column proof to prove that the angle sum of a triangle is 180° .

We need to prove that $\angle 2 + \angle 4 + \angle 5 = 180^\circ$

Statement

Reason

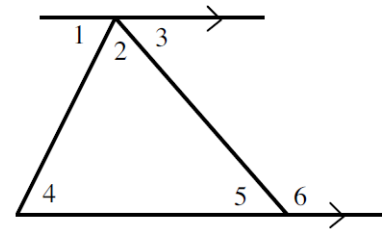
$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle 1 = \angle 4$$

$$\angle 3 = \angle 5$$

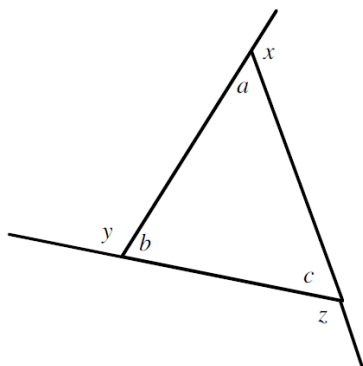
$$\text{so } \angle 2 + \angle 4 + \angle 5 = \angle 2 + \angle 1 + \angle 3$$

so



2.

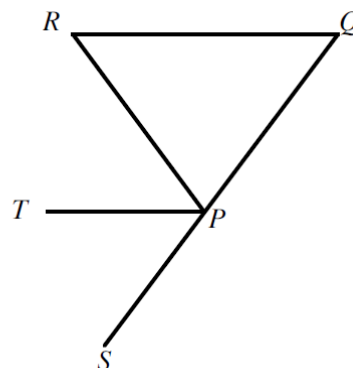
The diagram shows a triangle with three exterior angles. Use deductive reasoning to prove that the sum of the exterior angles is 360° .



3.

In the diagram, PQR is an isosceles triangle with $PQ = PR$. The lines PT and QR are parallel.

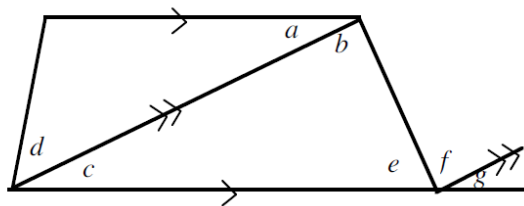
- Name a pair of corresponding angles.
- Name a pair of alternate angles.
- Why does PT bisect angle SPR ?



4.

Consider the diagram shown.

Use a two column proof to show that $a = g$.



- Watch the following screencast video before beginning this next section:

<https://youtu.be/Xr2bKG0oVRE>

Reasoning with Polygons

- Complete the table below to determine the measure of the interior angle of each polygon and the sum of the measures of the interior angles of each polygon.

Name of Regular Polygon	Number of Sides	Measure of the Central Angle	Measure of the Base Angle of the Isosceles Triangle	Measure of the Interior Angle of the Polygon	Sum of the Measures of the Interior Angles in Degrees
Triangle	3	$360 \div 3 = 120^\circ$	$(180 - 120) \div 2 = 30^\circ$	$2 \times 30^\circ = 60^\circ$	$3 \times 60^\circ = 180^\circ$
Square	4	$360 \div 4 = 90^\circ$	$(180 - 90) \div 2 = 45^\circ$	$2 \times 45^\circ = 90^\circ$	$4 \times 90^\circ = 360^\circ$
Pentagon					
Nonagon	9				
Dodecagon	12				
n-sided polygon					

The sum of the interior angles of an n -sided polygon is $180(n - 2)^\circ$.

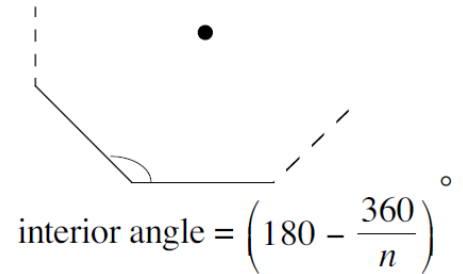
Extension to the above property

- Since there are n equal interior angles in a regular polygon, the measure of each angle is

$$\frac{180(n-2)^\circ}{n} = \left(180 - \frac{360}{n}\right)^\circ$$

The interior angle of an n -sided regular polygon

$$\text{measures } \left(180 - \frac{360}{n}\right)^\circ$$



Extension to the above property

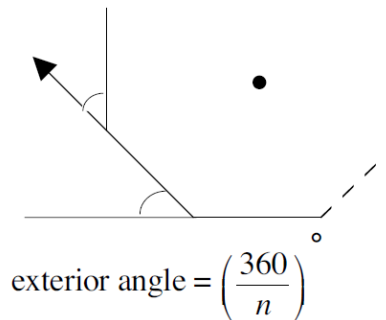
- Since the exterior angle is supplementary to the interior angle, the measure

of the exterior angle = $(180 - \text{interior angle})^\circ$,

$$\text{i.e. exterior angle} = 180^\circ - \left(180 - \frac{360}{n}\right)^\circ = \left(\frac{360}{n}\right)^\circ$$

The exterior angle of an n -sided regular polygon

$$\text{measures } \left(\frac{360}{n}\right)^\circ$$



It follows from the above property that:

The sum of the measures of the exterior angles of an n -sided regular polygon is 360°

Example 1

Calculate, in degrees, the sum of the measures of the interior angles of a polygon with 9 sides.

Example 2

Determine the number of sides of a polygon whose interior angle sum equals 4140° .

Example 3

Determine the number of sides of a regular polygon whose interior angle measures 170° .

Example 4

Determine the number of sides of a regular polygon whose exterior angle measures 60° .

Assignment

1.

Calculate the sum of the measures of the interior angles of a polygon with the given number of sides.

a) 12

b) 15

2.

Determine the number of sides of a polygon whose interior angle sum equals:

a) 720°

b) 1260°

c) 2880°

3.

Determine the number of sides of a regular polygon whose interior angle measures;

- a) 156° b) 175° c) 171°

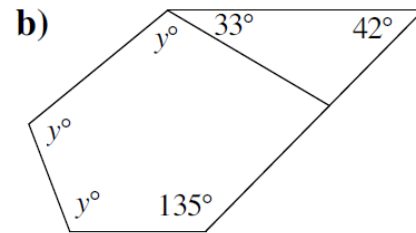
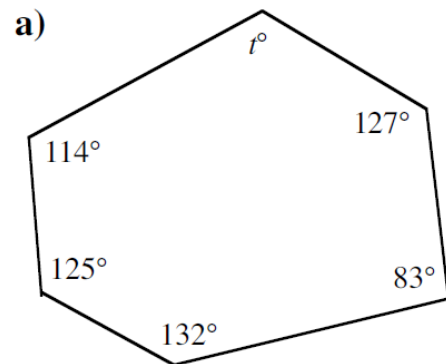
4.

Determine the number of sides of a regular polygon whose exterior angle measures

- a) 15° b) 40° c) 1°

5.

Determine the values of t and y in the diagrams below.



6.

The interior angle, x° of a regular polygon is 5 times as large as the exterior angle, y° .

The number of sides in the regular polygon is _____ .

