Name: ______________________________________ Student #:____________

Date:_______________________________________ T.A. #:_______________

Mathematics 12 Pre-Calculus

LEARNING GUIDE 10/11 TEST – TRIG IDENTITIES

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When using a calculator, you should provide a decimal answer that is correct **to at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

1. Determine the non-permissible values of the following expression in radians: (2 marks)

$$\frac{\tan x}{\sin x}$$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} + n\pi,\ n \in I$

$\sin x \neq 0$

$x \neq n\pi,\ n \in I$

2. Write the expression $\sin 32° \cos 21° - \cos 32° \sin 21°$ as a single trig function. (1 mark)

$= \sin(32° - 21°)$

$= \sin 11°$

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3. Given the identity $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$:
   a) verify the identity for the particular case when x = $\frac{\pi}{3}$. (1 mark)
   b) prove the identity algebraically. (2 marks)

a)

$$\frac{\sin\frac{\pi}{3}}{1-\cos\frac{\pi}{3}} = \frac{1+\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}}$$

$$\frac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = \frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(2\right) = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}}$$

$$\sqrt{3} = \sqrt{3}$$

or $1.73 = 1.73$

b) L.S. $\dfrac{\sin x}{1-\cos x}\dfrac{(1+\cos x)}{(1+\cos x)}$

$$\frac{\sin x(1+\cos x)}{1-\cos^2 x}$$

$$\frac{\cancel{\sin x}(1+\cos x)}{\sin^{\cancel{2}} x}$$

$$\frac{1+\cos x}{\sin x} = R.S.$$

4. Write the expression $\cos^2\dfrac{\pi}{3}$ - $\sin^2\dfrac{\pi}{3}$ in terms of a single trig function. (1 mark)

$$\cos\frac{2\pi}{3}$$

5. Prove the following identities. (2 marks each)

a) $\sin^2 x \cot^2 x = 1 - \sin^2 x$

$\sin^2 x \dfrac{\cos^2 x}{\sin^2 x}$ | $\cos^2 x$

$\cos^2 x$ | $\cos^2 x$

b) $\csc x(1 + \sin x) = 1 + \csc x$

$\dfrac{1}{\sin x}(1+\sin x)$ | $1 + \dfrac{1}{\sin x}$

$\dfrac{1}{\sin x} + 1$

c) $\dfrac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$

$$\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x - 1)} = \frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x = R.S.$$

d) $\dfrac{\sin\theta}{1+\sin\theta} - \dfrac{\sin\theta}{1-\sin\theta} = -2\tan^2\theta$

$$\frac{\sin\theta}{1+\sin\theta}\frac{(1-\sin\theta)}{(1-\sin\theta)} - \frac{\sin\theta}{1-\sin\theta}\frac{(1+\sin\theta)}{1+\sin\theta}$$

$$\frac{\sin\theta - \sin^2\theta - \sin\theta - \sin^2\theta}{1-\sin^2\theta} \rightarrow \frac{-2\sin^2\theta}{\cos^2\theta}$$

$= -2\tan^2\theta = R.S.$

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6. Prove the following identity. (2 marks)

FACTOR

$$\frac{cos^2x - sin^2x}{\sin x + \cos x} = \cos x - \sin x$$

$$\frac{(\cos x - \sin x)\cancel{(\cos x + \sin x)}}{\cancel{\sin x + \cos x}}$$

$$\cos x - \sin x \qquad = R.S.$$

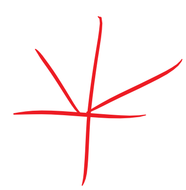
7. Solve the following equation. $0 \leq x < 2\pi$ (2 marks)

$\sin 2x - \cos x = 0$

$$2\sin x \cos x - \cos x = 0$$

$$\cos x\,(2\sin x - 1) = 0$$

$$\cos x = 0 \qquad \sin x = \frac{1}{2}$$



$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

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8. Solve the following equation. Give the general solution expressed in radians. (3 marks)

$$3\cos x + 2 = 5\sec x$$

$3\cos x + 2 = \frac{5}{\cos x}$

$3\cos^2 x + 2\cos x - 5 = 0$

$(3\cos x + 5)(\cos x - 1) = 0$

$\cancel{\cos x = -\frac{5}{3}}$ $\quad \cos x = 1$

$x = n2\pi,\ n \in I$

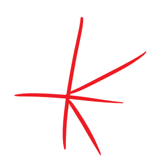
9. Solve the following equation. Give the general solution expressed in degrees. (3 marks)

$$\cos 2x + 1 = \cos x$$

$2\cos^2 x - 1 + 1 = \cos x$

$2\cos^2 x - \cos x = 0$

$\cos x\,(2\cos x - 1) = 0$



$\cos x = 0 \qquad \cos x = \frac{1}{2}$

$x = 90^\circ + n180^\circ,\ n \in I \qquad x = 60^\circ + n360^\circ,\ n \in I$

$x = 300^\circ + n360^\circ,\ n \in I$

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