Name: ____ Student #:_____________

Date:____ T.A. #:________________

Mathematics 12 Pre-Calculus
LEARNING GUIDE 10/11 TEST – TRIG IDENTITIES

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When using a calculator, you should provide a decimal answer that is correct **to at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

1. Determine the non-permissible values of the following expression in radians: (2 marks)

$$\frac{\sec x}{\sin x}$$

$\cos x \neq 0$ $\sin x \neq 0$

$x \neq \frac{\pi}{2} + n\pi, n \in I$ $x \neq n\pi, n \in I$

2. Write the expression $\sin 32° \cos 21° + \cos 32° \sin 21°$ as a single trig function. (1 mark)

$\sin(32° + 21°)$

$= \sin(53°)$

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3. Given the identity $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$

a) verify the identity for the particular case when x = $\frac{\pi}{3}$. (1 mark)
b) prove the identity algebraically. (2 marks)

a) $\dfrac{\overset{1}{\cos\frac{\pi}{3}}}{1-\sin\frac{\pi}{3}} = \dfrac{1+\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}$

$\dfrac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}} = \dfrac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\dfrac{\frac{1}{2}}{\frac{2-\sqrt{3}}{2}} = 2+\sqrt{3}$

$\dfrac{1}{2-\sqrt{3}}\dfrac{(2+\sqrt{3})}{(2+\sqrt{3})} = 2+\sqrt{3}$

$2+\sqrt{3} = 2+\sqrt{3}$

or $3.73 = 3.73$

b) $\dfrac{\cos x}{1-\sin x}\dfrac{(1+\sin x)}{(1+\sin x)}$

$\dfrac{\cos x(1+\sin x)}{1-\sin^2 x}$

$\dfrac{\cos x(1+\sin x)}{\cos^2 x}$

$\dfrac{1+\sin x}{\cos x} = R.S.$

4. Write the expression $2sin^2x - 1$ in terms of a single trig function. (1 mark)

$-\cos 2x$

5. Prove the following identities. (2 marks each)

a) $\sec x \cot x \, sin^2 x = \sin x$

$\left(\frac{1}{\cancel{\cos x}}\right)\left(\frac{\cancel{\cos x}}{\cancel{\sin x}}\right)(\sin^2 x) = \sin x$

$\sin x = \sin x$

b) $\csc x(1+\sin x) = 1+\csc x$

$\frac{1}{\sin x}(1+\sin x) = 1+\frac{1}{\sin x}$

$\frac{1}{\sin x}+1 = 1+\frac{1}{\sin x}$

c) $\dfrac{1+\cos 2x}{\sin 2x} = \cot x$

$\frac{1+2\cos^2 x-1}{2\sin x\cos x} = \frac{2\cos^2 x}{2\sin x\cos x} = \frac{\cos x}{\sin x} = \cot x = R.S.$

d) $\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x} = \dfrac{1}{\sin x \cos x}$

$\frac{\sin x}{\cos x}\frac{(\sin x)}{(\sin x)} + \frac{\cos x}{\sin x}\frac{(\cos x)}{(\cos x)}$

$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = R.S.$

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6. Prove the following identity. (2 marks)

$$\frac{cos^2 x - \cos x - 2}{4\cos x - 8} = \frac{\cos x + 1}{4}$$

$$\frac{(\cos x \cancel{-2})(\cos x + 1)}{4(\cos x \cancel{-2})}$$

$$\frac{\cos x + 1}{4}$$

$$\frac{\cos x - 1}{4}$$

7. Solve the following equation. $0 \leq x < 2\pi$ (2 marks)

$\sin x + 1 = 2\csc x$

$$\sin x + 1 = \frac{2}{\sin x}$$

$$\sin^2 x + \sin x - 2 = 0$$

$$(\sin x + 2)(\sin x - 1) = 0$$

$$\cancel{\sin x = -2} \qquad \sin x = 1$$

$$\boxed{x = \frac{\pi}{2}}$$

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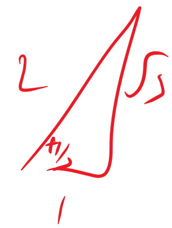
8. Solve the following equation. Give the general solution expressed in radians. (3 marks)

$$tan^2 x + \sqrt{3}\tan x = 0$$

$\tan x(\tan x + \sqrt{3}) = 0$



$\tan x = 0 \qquad \tan x = -\sqrt{3}$

$x = n\pi, n \in I$

$x = \frac{2\pi}{3} + n2\pi, n \in I$

$x = \frac{5\pi}{3} + n2\pi, n \in I$

or $x = \frac{2\pi}{3} + n\pi, n \in I$

9. Solve the following equation. Give the general solution expressed in degrees. (3 marks)

$$\cos 2x - 2 = 3\sin x$$

$1 - 2\sin^2 x - 3\sin x - 2 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = -1$

$x = 270^\circ + n360^\circ, n \in I$

$x = 210^\circ + n360^\circ, n \in I$

$x = 330^\circ + n360^\circ, n \in I$

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