Learning Guide 1: Ratios, Rates and Proportional Reasoning.

Writing Ratios

A ratio compares 2 items with the same unit.
The order of the words in a sentence shows the order of the numbers in the ratio.
You can write ratios 3 different ways.

● ● ●   ○ ○ ○ ○ ○ ○

The ratio of the black marbles to the *total number* of marbles is:
1. In **words**: three to nine, or 3 to 9
2. In **ratio notation**: $3:9$
3. As a **fraction**: $\frac{3}{9}$ which reduces to $\frac{1}{3}$

1. Use the diagram above. Write each ratio 3 different ways.
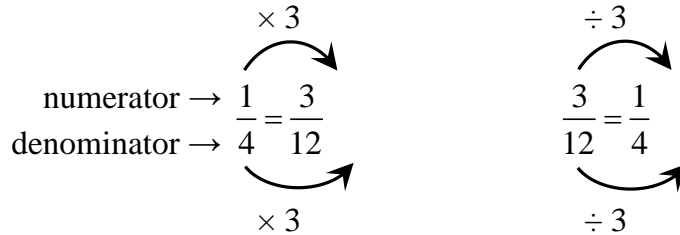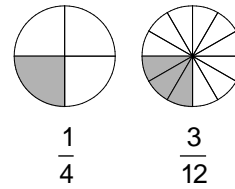
a) black marbles to white marbles

b) white marbles to total number of marbles

_____   ← Words →   _____

_____ : _____   ← Ratio Notation →   _____

← Fraction →

Equivalent Fractions

$\dfrac{1}{4}$ and $\dfrac{3}{12}$ are **equivalent fractions**.

To make equivalent fractions, multiply or divide both the numerator and the denominator by the same number.

$\dfrac{1}{4}$ $\qquad$ $\dfrac{3}{12}$

$$\times 3 \qquad\qquad \div 3$$

numerator $\rightarrow$ $\dfrac{1}{4} = \dfrac{3}{12}$ $\qquad\qquad$ $\dfrac{3}{12} = \dfrac{1}{4}$ $\leftarrow$ denominator

$$\times 3 \qquad\qquad \div 3$$

2. Fill in the blanks and the box to make an equivalent fraction.

a)
$\times \underline{\qquad}$

$\dfrac{5}{8} = \dfrac{\boxed{}}{24}$

$\times 3$

b)
$\times \underline{\qquad}$

$\dfrac{1}{3} = \dfrac{5}{\boxed{}}$

$\times \underline{\qquad}$

Comparing Quantities

a)

5 g $\qquad$ 50 g
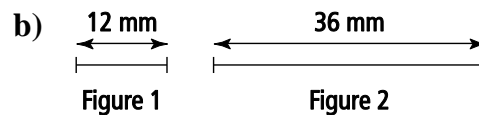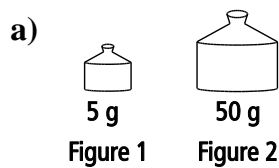
Figure 1 $\quad$ Figure 2

The ratio of Figure 2 to Figure 1 is

$10 : \underline{\qquad\qquad}$ or $\dfrac{\boxed{}}{\boxed{}}$.

b)

12 mm $\qquad\qquad$ 36 mm

Figure 1 $\qquad\qquad$ Figure 2

The ratio of Figure 2 to Figure 1 is

$\underline{\qquad} : \underline{\qquad}$ or $\dfrac{\boxed{}}{\boxed{}}$.

Warm Up
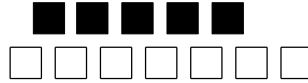
1. Write each ratio 3 different ways.

   ■ ■ ■ ■ ■
   □ □ □ □ □ □ □

   a) black tiles to white tiles   **b)** white tiles to all tiles

   _____   ← Words →   _____

   _____   ← Ratio Notation →   _____

   $\boxed{}$
   ────
   $\boxed{}$
   ← Fraction →
   $\boxed{}$
   ────
   $\boxed{}$

2. Write each fraction in lowest terms.

   ÷ 2

   a) $\dfrac{10}{12} = \dfrac{\boxed{}}{\boxed{}}$

   (cloud: Divide by common factors until the only factor you can divide by is 1.)

   ÷ 2

   ÷ _____

   b) $\dfrac{9}{27} = \dfrac{\boxed{}}{\boxed{}}$

   ÷ _____

3. Write each fraction as a decimal. **Example:** $\dfrac{1}{2} = 1 \div 2 = 0.5$

   a) $\dfrac{4}{5} = 4 \div 5 = 0.$ _____

   b) $\dfrac{3}{4} =$ _____

   c) $\dfrac{10}{16} =$ _____

4. Write each decimal as a percent. **Example:** $0.66 = 0.66 \times 100 = 66\%$
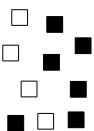
   a) $0.89 =$ _____%   b) $0.9 =$ _____%

Two-Term and Three-Term Ratios

Example 1: Represent Ratios

2-term ratio
- compares 2 quantities with the same units
- write as $a:b$ or a to b

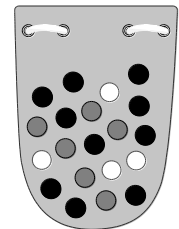- example: □ ■ black : white is $6:4$

A bag holds 20 marbles.

a) What is the **2-term ratio** of black marbles to white marbles?

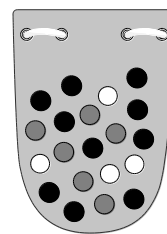*Solution*

Method 1: Represent a Ratio Using Symbols or Words

The ratio of black marbles to white marbles is $10:4$ or 10 _____ 4.
                                                        (symbols)   *(words)*

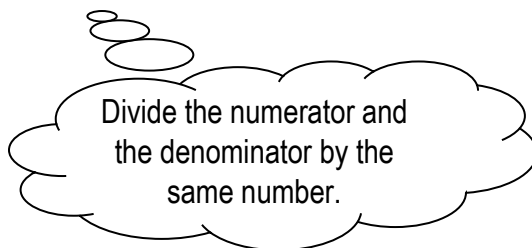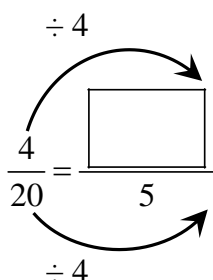**b)** Compare the number of white marbles to the *total* number of marbles. Write the fraction in lowest terms.

Solution

There are ——————— white marbles out of a total of ——————— marbles.

Write as a fraction: $\dfrac{\text{white}}{\text{total}}$ is $\dfrac{4}{20}$

Write the fraction in lowest terms.

$\div 4$

$\dfrac{4}{20} = \dfrac{\boxed{}}{5}$

$\div 4$

Divide the numerator and the denominator by the same number.

c) What marbles show the ratio $6:10$?

Solution
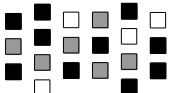
There are ——————— grey marbles and ——————— black marbles.

grey : black is ——————— : ———————
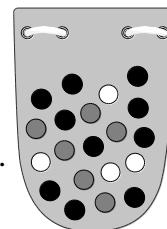
3-term ratio
- compares 3 quantities with the same units

- write as $a:b:c$ or a to b to c

- example: black : white : grey is $10:4:6$

d) Write the **3-term ratio** comparing the white, grey, and black marbles.

Solution

white : grey : black = ————— : ————— : —————

To write this ratio in lowest terms, divide each number by 2.

$= 2 : \underline{} : 5$

1. Write each ratio in ratio notation. Then, write the ratio in lowest terms.

   a) $3 compared to $9

   _____ : _____

   = _____ : _____   (÷ 3)

2. Write each ratio in fraction form.

   a) There are 12 red beads and 3 blue beads in a bag.
   Compare red beads to *total* beads.

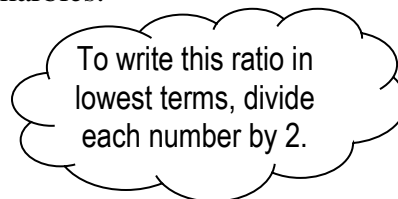 $$\frac{\text{red beads}}{\text{total beads}} = \boxed{}$$

   b) A team won 3 games and lost 6 games. What is the ratio of games won to *total* games played?

   $$\frac{\text{games won}}{\text{total games played}} = \boxed{}$$ (Total = 3 + 6)

3. Find the missing number to make an equivalent fraction.

   a)
   × 4

   $$\frac{1}{2} = \frac{\boxed{}}{8}$$

   × 4

   b)
   ÷ 3

   $$\frac{\boxed{}}{4} = \frac{3}{12}$$

   ÷ 3

4. A class of 32 students has 24 girls.

   a) How many boys are in the class? _____

   b) What is the ratio of boys to total students? Write the ratio as a fraction and a percent.

   c) What is the ratio of girls to boys? Write the ratio in ratio notation.

5. The ratio of the width to the length of the Canadian flag is $1:2$.

a) The flag on the cover of an atlas is 12 cm wide.
How long is the flag?

$$\frac{\text{width}}{\text{length}} \longrightarrow \frac{1}{2} = \frac{12}{\boxed{}}$$

Sentence: _____

b) A Canadian flag outside a school is 4 m long. How wide is the flag?

$$\frac{\text{width}}{\text{length}} \longrightarrow$$

Sentence: _____

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Rates

Rate
- compares 2 quantities measured in different units
- example: $1.69 per 100 g *or* $1.69/100 g of food

Example 1: Determine Unit Rates

Unit rate
- a rate in which the second term is 1
- example: 30 km/h means 30 km travelled in 1 h

Jesse and Brent send text messages to each other.
Jesse can text 187 words in 5 min.
Brent can text 444 words in 10 min.
Calculate Jesse's and Brent's texting speeds.

Solution

$$\text{Speed of texting} = \frac{\text{number of words}}{\text{time}}$$

	Jesse	Brent
Calculate Speed	$\dfrac{187 \text{ words}}{5 \text{ min}}$ **187** ÷ **5** = _____ = 37.4 words/min	$\dfrac{444 \text{ words}}{10 \text{ min}}$ **444** ÷ **10** = _____ = _____ words/min

Jesse can text approximately_____ words/min and Brent can text approximately

_____ words/min.

Find the unit rates.

a) Brandon runs 150 m in 25 s.

$$\frac{150 \text{ m}}{25 \text{ s}} = \underline{\hspace{2cm}} \text{ m/s}$$

150 ÷ 25

= _____

Brandon runs _____ m/s.

b) Kira earns $88 for working 8 h.

$$\frac{\$ \boxed{}}{\boxed{} \text{ h}} = \$ \underline{\hspace{2cm}} / \text{h}$$

Example: Compare Prices Using Unit Rates

Which container of orange juice is the best buy?

$1.25 $2.30 $4.50
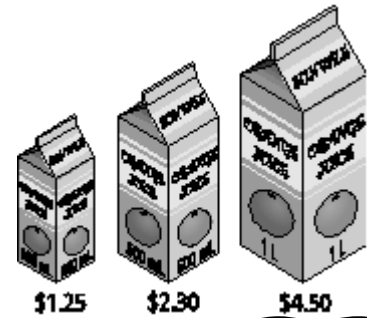
Solution

Calculate the **unit price** of each container of orange juice.

$1 = 100¢
To change dollars to
cents, multiply by 100.

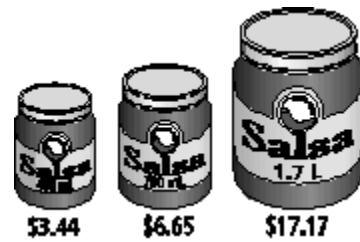| Container Size	Calculations for Unit Price Unit Price = $\dfrac{\text{cost}}{\text{volume}}$	Change to Cents (¢)	Unit Price (¢)
250 mL	$\dfrac{\$1.25}{250 \text{ mL}} = \$0.005/\text{mL}$ C 1.25 ÷ 250 = 0.005	$0.005 × 100 = 0.5	0.5¢/mL
500 mL	$\dfrac{\$2.30}{500 \text{ mL}} = \$\underline{\hspace{1.5cm}}/\text{mL}$ C 2.30 ÷ 500 = 0.0046	$0.0046 × 100 = _____	0.46¢/mL
1 L = 1000 mL	$\dfrac{\$4.50}{1000 \text{ mL}} = \underline{\hspace{1.5cm}}/\text{mL}$		

Compare the prices.
The smallest unit price is the best buy.
So, the best buy is the _____ container with a unit price of $_____.

Which container of salsa is the best buy?

$3.44 $6.65 $17.17

Complete the table. Show your work.

Container Size	Calculations for Unit Price Unit Price = $\dfrac{\text{cost}}{\text{volume}}$	Change to Cents (¢)	Unit Price (¢)
400 mL	$\dfrac{\$3.44}{400 \text{ mL}} = \$$ _____ /mL [C] **3.44** [÷] **400** [=] _____	$_____ × 100 = _____	_____ ¢/mL
700 mL	$\dfrac{\boxed{}}{700 \text{ mL}} = \$$ _____ /mL		
1.7 L = 1.7 × 1000 = _____ mL			

The best buy is _____.

1. Find the unit rate.

a) An orca swims 110 km in 2 h.
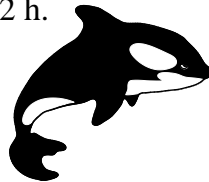
Speed = $\dfrac{\text{distance}}{\text{time}}$

= $\dfrac{\boxed{} \text{ km}}{\boxed{} \text{ h}}$

= _____ km/h

b) A Canada goose flies 800 km in 12.5 h.

10

c) Cathy plants 60 daffodils in 30 min.

d) A blue whale eats 8 tonnes of food in 2 days.

2. a) Gina earns $78.00 for working 6 h. Find her hourly rate of pay.

b) Asad makes $192.50 for working 14 h. Find his hourly rate of pay.

c) Who has a greater hourly rate of pay?

_____

3. The table shows the prices of different-sized packages of nuts.

Nut Package	Mass	Price
1	300 g	$2.19
2	500 g	$3.09
3	700 g	$4.83
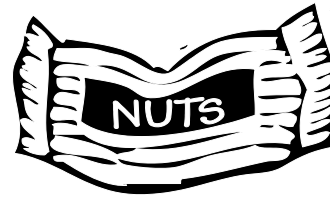
a) What is the unit price per 100 g for each package?

Package 1:

$$\text{Unit price} = \frac{\text{price}}{\text{mass}}$$

$\div 3$

$$\frac{\$2.19}{300 \text{ g}} = \frac{\boxed{}}{100 \text{ g}}$$

$\div 3$

The unit price is

$ _____ /100 g.

Package 2:

Package 3:

_____

_____

b) Which package is the best buy? _____

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4. Trevor rode his bike 84 km in 3 h.
Jillian rode 56 km in 2 h.
Who is the faster cyclist?

Who rode the fastest
in 1 h?

Trevor: $\dfrac{\text{distance}}{\text{time}}$

Jillian: $\dfrac{\text{distance}}{\text{time}}$

Sentence: _____.

5. The table shows the fuel consumption of 2 cars.

Owner	Distance (km)	Fuel Used (L)
Joe	400	28
Sarah	840	60

Fuel consumption measures how many litres of gas a car uses to travel 1 km. It is a unit rate.

Find the fuel consumption in litres per kilometre (L/km) for each car.
Round your answer to 4 decimal places.

$$\text{Fuel consumption} = \dfrac{\text{fuel used (L)}}{\text{distance (km)}}$$

Joe's car:

Sarah's car:

Sentence: The fuel consumption for

Joe's car is _____ L/km.

Sentence:

_____

_____

b) Whose car has the lower fuel consumption?

_____

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Warm Up

1. Round to the nearest cent (¢).

> **Examples:** 34.7¢ = 35¢
> 125.3¢ = 125¢
>
> 34.7 means 34 and 7 tenths cents.
> Round up 5 tenths or more to the next cent.
> Don't round if it is less than 5 tenths.

   a) 84.3¢ = —————¢

   b) 39.9¢ = —————

   c) 173.1¢ = —————¢

   d) 75.5¢ = —————

2. Change cents (¢) to dollars ($).

> **Examples:** 35¢ = 35 ÷ 100
> = $0.35
>
> 158¢ = 158¢ ÷ 100
> = $1.58
>
> 100¢ = $1.00

   a) 58¢ = —————

   b) 185¢ = —————

3. Find the missing number to make an equivalent fraction.

a) $\frac{1}{4} = \frac{\boxed{}}{8}$

$\times \underline{\hspace{1.5em}}$ (top) $\times \underline{\hspace{1.5em}}$ (bottom)

b) $\frac{3}{5} = \frac{9}{\boxed{}}$

$\times \underline{\hspace{1.5em}}$ (top) $\times \underline{\hspace{1.5em}}$ (bottom)

c) $\frac{9}{\boxed{}} = \frac{3}{4}$

$\times \underline{\hspace{1.5em}}$ (top) $\times \underline{\hspace{1.5em}}$ (bottom)

d) $\frac{\boxed{}}{100} = \frac{3}{10}$

$\times \underline{\hspace{1.5em}}$ (top) $\times \underline{\hspace{1.5em}}$ (bottom)

Proportional Reasoning

proportion
- an equation that shows 2 ratios or 2 rates are equal

- examples: $\dfrac{2}{3} = \dfrac{6}{9}$ *or* $\dfrac{2 \text{ km}}{3 \text{ h}} = \dfrac{6 \text{ km}}{9 \text{ h}}$

$\times 3$ (top) $\times 3$ (bottom)

Example 1: Solve a Rate Problem Using Proportional Reasoning

Electricity costs 11.58¢ per 2 kWh.
How much does 30 kWh cost? Round your answer to the nearest cent.
Solution
Use a Proportion

$\times 15$

$\dfrac{11.58¢}{2 \text{ kWh}} = \dfrac{\boxed{}}{30 \text{ kWh}}$

$\times 15$

11.58¢ is about 12¢. **M⁹E**
$12 \times 15 = (10 \times 15) + (2 \times 15)$
$\qquad\quad = 150 + 30$
$\qquad\quad = 180$
The answer will be about 180¢ or $1.80.

So, 30 kWh costs about 174¢ or $_____$.

Practice

1 a) There are 72 players on 8 baseball teams.
Find the number of players on 2 teams.

$$\div \underline{}$$

$$\frac{72 \text{ players}}{8 \text{ teams}} = \frac{\boxed{} \text{ players}}{2 \text{ teams}}$$

$$\div \underline{}$$

Sentence: _____

Example 2: Solve a Ratio Problem Using Proportional Reasoning

A biologist catches and tags 24 trout, and then releases them back into the pond.
Two weeks later, he catches 30 trout and finds that 5 of them are tagged.
Estimate how many trout are in the pond.

Solution

Using proportional reasoning

$$\frac{5}{30} = \frac{24}{t}$$

$$\times 4.8$$

$$\frac{5}{30} = \frac{24}{t}$$

Divide to find what number
multiplied by 5 gives 24.

$$\frac{24}{5} = 4.8$$

$$\times 4.8$$

$t = 30 \times 4.8$

$t =$ _____

I estimate there are _____ trout in the pond.

15

Practice

1. Find the missing value.

a)
$$\times \underline{\qquad}$$
$$\frac{2}{3} = \frac{\boxed{}}{15}$$
$$\times \underline{\qquad}$$

b)
$$\div \underline{\qquad}$$
$$\frac{\boxed{}}{5} = \frac{14}{35}$$
$$\div \underline{\qquad}$$

2. Find the unit rate.

a) Three dinner rolls cost 99¢.

$$\frac{99¢}{3 \text{ rolls}} = \frac{\boxed{}}{1 \text{ roll}}$$

$$\div 3$$
$$\frac{99}{3} = \frac{\boxed{}}{1}$$
$$\div 3$$

The unit rate is _____ ¢/roll.

b) Seven boxes have a mass of 14 kg.

The unit rate is _____.

3. Find the missing value to make each rate equivalent.
Include the units.

a)

$$\times \underline{\quad}$$

$$\frac{60 \text{ km}}{3 \text{ h}} = \frac{\boxed{}}{6 \text{ h}}$$

$$\times \underline{\quad}$$

b) $\dfrac{\$3}{4 \text{ cans}} = \dfrac{\$15}{\boxed{}}$

4. Delia was paid $35 for 5 h of babysitting.

How much will she earn for 3 h?

5. Set up a proportion for each situation.

a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.

$$\frac{17 \text{ g}}{\boxed{} \text{ beans}} = \frac{\boxed{} \text{ g}}{\boxed{} \text{ beans}}$$

b) There are 15 boys for every 13 girls in each classroom in a school.
If there are 75 boys in the school, then there are 65 girls.

$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

6. A gardener charges $25 to mow a lawn that measures 600 m².
How much should he charge for a lawn that measures 1200 m²?

$$\frac{\text{cost}}{\text{area}} = \frac{\text{cost}}{\text{area}}$$

Sentence: _____

7. It costs $7.50 for 3 rides at an amusement park.

   a) What is the unit rate per ride?   **b)** At this rate, what would it cost for 18 rides?

   _____   _____

   _____   _____

8. At a different amusement park, it costs $10 for 4 rides. What is the cost for 12 rides?

Sentence: _____