

Learning Guide 12: Linear Relations

Watch the following instructional video. In your handout:

i) Copy down the given notes and examples

ii) Complete the assigned questions

<https://youtu.be/wyN-ROxgLhE>

Describe Patterns Using an Expression

Expression

- a number *or* letter *or* combination of numbers and letters connected by $+$, $-$, \times , or \div

Examples: 5 , r , $8t$, $x + 9$, $2y - 5$

Variable

- a letter that represents an unknown number or amount

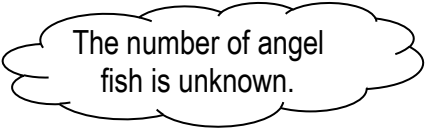
Example:

1. A fish tank holds 3 Goldfish and some Angel fish.

How many fish are there in total?

Number of angel fish: \circ \circ \circ

Number of goldfish: 3



The number of angel fish is unknown.

Total number of fish in the tank: $f + 3$

2. Write an expression. Tell what your variable describes.

- a) Simon has many shirts.
He gives 2 shirts away.
How many shirts does he still have?

Variable: s = the number of _____

Expression: $s -$ _____

- b) Shay has 5 packages of pencils. Each package has the same number of pencils.
How many pencils are in all 5 packages?

Variable: _____

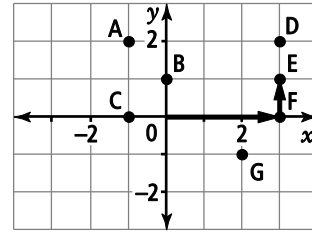
Expression: _____

Using a Coordinate Grid (x -axis and y-axis)

You can describe points on a coordinate grid using **ordered pairs**: (x, y) .

- The **x-coordinate** tells you how many units to move left or right starting at the origin $(0, 0)$.
- The **y-coordinate** tells you how many units to move up or down starting at the x -axis.

To plot the point E $(3, 1)$, start at $(0, 0)$.
Move 3 units right and 1 unit up.



3. Use the grid above. Write the coordinates of each point in the table of values.

Point	A	B	C	D	E	F	G
x			-1		3		
y			0		1		

Warm Up

1. For each chart, describe a pattern to go from the input to the output.

Ask yourself: "Do I add or subtract?"

Input	Output
2	5
4	7
3	6
8	11

Pattern:

Input	Output
6	5
3	2
7	6
10	9

Pattern:

Input	Output
1	7
3	9
5	11
6	12

Pattern:

2. For each chart, describe a pattern to go from the input to the output.

Ask yourself: "Do I multiply or divide?"

Input	Output
4	8
6	12
3	6
5	10

Pattern:

Input	Output
6	2
12	4
3	1
15	5

Pattern:

Input	Output
3	30
1	10
4	40
8	80

Pattern:

3. Write the coordinates of each point on the grid.

A (3, 4)

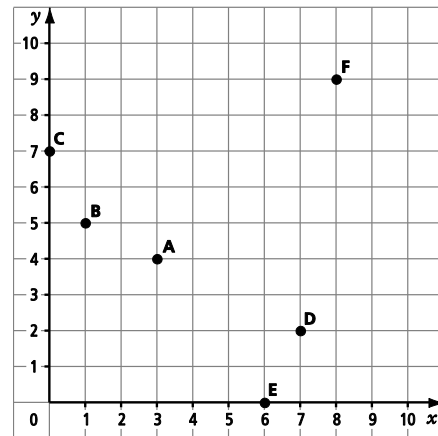
B (_____, _____)

C (_____, _____)

D (_____, _____)

E (_____, _____)

F (_____, _____)



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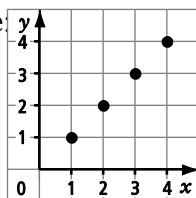
<https://youtu.be/e6jS3ZWtjpM>

Analyzing Graphs of Linear Relations

Linear relation

- a pattern made by a set of points that lie in a straight line

• Example



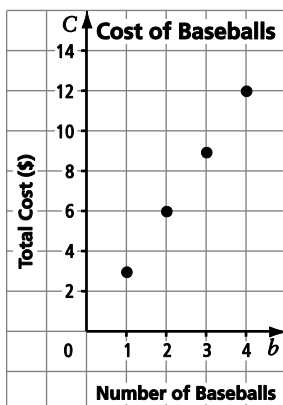
linear = straight line

Table of Values

- shows 2 sets of related numbers

Example 1: Make a Table of Values From a Graph

The graph shows that the total cost depends on the number of baseballs you buy.
Total cost and number of baseballs are related to each other.



The variable b shows the number of baseballs.
The variable C shows the total cost in dollars.

a) Describe the patterns you see on the graph.

Solution

- The graph shows data on the cost of baseballs.

One ball costs \$3, 2 balls cost _____, 3 balls cost _____,

- The total cost increases by \$_____ each time you buy a baseball.

To move from 1 point to the next, go 1 unit horizontally (\leftrightarrow) and _____ units vertically (\updownarrow).

- The points lie in a _____ .
The graph shows a linear relation.

b) Make a table of values from the graph.

Solution

Read each point as an ordered pair to make a table of values. Complete each table.

In a horizontal table of values, the top row shows the x -coordinates.
The bottom row shows the y -coordinates.

Number of Baseballs (b)	1	2	3	4
Total Cost (C)	3	6		

In a vertical table of values, the first column shows the x -coordinates.
The second column shows the y -coordinates.

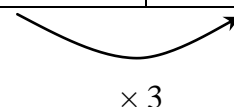
Number of Baseballs (b)	Total Cost (C)
1	3
2	6
3	
4	

c) Write an expression that represents the cost of buying b baseballs.

Solution

Let b describe the number of baseballs.
The expression $3b$ describes the cost of buying b baseballs.

Number of Baseballs (b)	Total Cost (C)
1	3
2	6
3	9
4	12



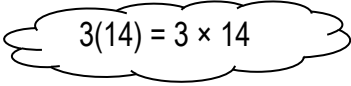
d) If the relationship in the graph continues, how much will it cost to buy 14 baseballs?

Solution

Use the expression $3b$ to find the cost of 14 baseballs.

Cost of 14 baseballs = $3(\rule{1.5cm}{0.4pt})$ ← Substitute 14 for b .

= $\rule{1.5cm}{0.4pt}$

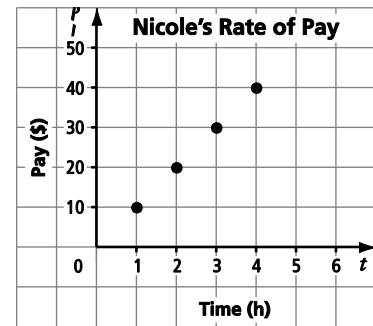

$$3(14) = 3 \times 14$$

The cost of 14 baseballs is \$ $\rule{1.5cm}{0.4pt}$.

Example 2: Analyse Data on a Graph of a Linear Relation

Nicole has a part-time job.

The graph shows her pay related to the number of hours she works.



- a) Describe the patterns you see in the graph.

Solution

- The graph shows data on the pay Nicole receives for each hour of work.

The pay for 1 h is _____, the pay for 2 h is _____,
the pay for 3 h is _____,

- The points lie in a straight _____, so the graph is a linear relation.
- To move from 1 point to the next, move _____ unit horizontally (\leftrightarrow) and 10 units vertically (\updownarrow).

The Pay axis
counts by 10s.

- b) Make a table of values.

Solution

Time (h)	1	2	3	4
Pay (\$)	10			40

- c) How much does Nicole make per hour?

Solution

Look at the table of values.

In 1 h of work, Nicole makes \$_____.

For each hour she works, Nicole makes \$10 more, so she makes \$10/hr.

- d) Is it possible to have points between the points shown on the graph?
Explain why or why not.

Solution

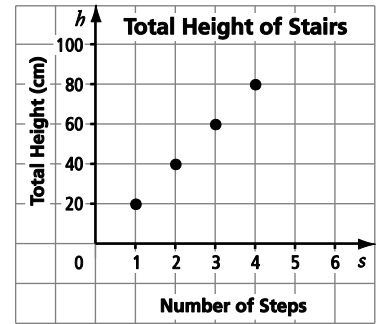
Yes, it is possible to have points between the points on the graph.
Nicole could work full hours and half hours.

If she worked 2.5 h, she would be paid $2.5 \times \$10 = \$$ _____.

On the graph, this point would be (2.5, 25).

Practise

1. The graph shows how much higher you get each time you go up a step of a staircase.



- a) Describe the 2 patterns you see in the graph.

- The pattern lies in a _____

_____.

- To move from 1 point to the next:

- b) Use the graph to complete the table of values.

Number of Steps	1	2	3	4
Total Height (cm)				

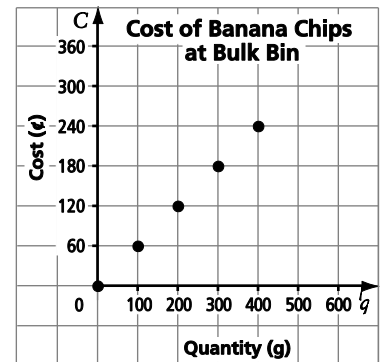
- c) Describe the pattern in the table of values.

The total height starts at _____ cm and increases by _____ cm.

- d) Write an expression for the total height after climbing s stairs: _____

- e) If the relationship in the graph continues, what is the total height on step 10?

2. Tessa and Vince go shopping at Bulk Bin.
The graph shows the cost of banana chips.



- a) Does the graph show a linear relation? Explain why or why not.

- b) Describe 2 patterns shown on this graph.

- The pattern of the points: _____

- To move from 1 point to the next: _____

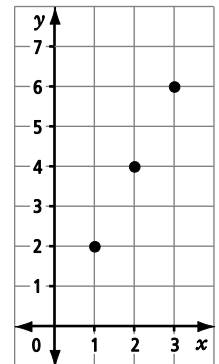
- c) Complete the table of values for this graph.

Quantity (g)	0	100	200	300	400
Cost (\$)					

d) Can the graph show the cost of 250 g of banana chips? Explain your answer.

3. a) Complete the table of values for the ordered pairs on the graph.

x	1		
y			



b) Describe the 2 patterns you see in the graph.

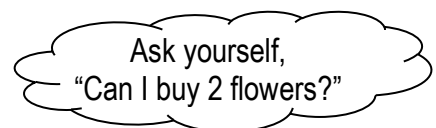
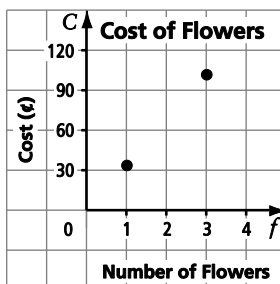
- The pattern of the points:

- To move from 1 point to the next:

c) Extend your table of values so the x -column goes to 9.

d) If this pattern continues, what is the value of y when $x = 9$? _____

4. The graph shows part of a linear relation that describes the cost of cake flower decorations.



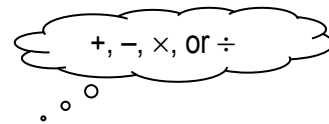
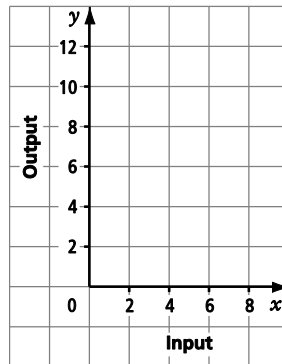
Is it reasonable to have points between the ones on the graph? Explain your answer.

Warm Up

1. Graph each set of points.

a)

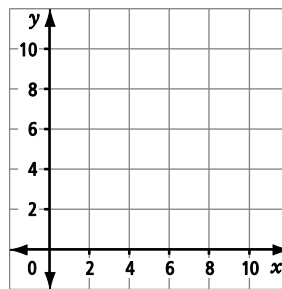
Input	Output
2	5
4	7
3	6
8	11



To go from the input to the output, the pattern is _____.

b)

x	y
6	5
3	2
7	6
10	9



To go from x to y , the pattern is _____.

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<https://youtu.be/tqAn7rmX51Y>

Patterns in a Table of Values

Example 1: Identify the Relationship in a Table of Values

Relationship

- a pattern formed by 2 sets of numbers

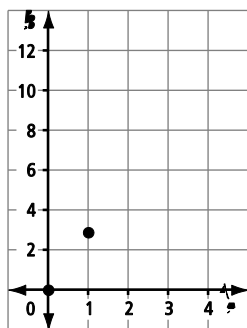
The pattern in this table of values describes a linear relation.

a) Graph the ordered pairs.

Solution

The ordered pairs are (0, 0), (1, 3), (_____, _____),

(_____, _____), and (_____, _____).



Plot the last 3 ordered pairs.

A	B
0	0
1	3
2	6
3	9
4	12

Consecutive numbers go in order from smallest to largest (e.g: 6, 7, 8, 9 ...).

b) What is the difference between consecutive A-values?

Solution

$$3 - 2 = \underline{\hspace{2cm}} \quad 2 - 1 = \underline{\hspace{2cm}} \quad 1 - 0 = \underline{\hspace{2cm}}$$

Consecutive A-values have a difference of _____.

c) What is the difference between consecutive B-values?

Solution: $9 - 6 = \underline{\hspace{2cm}}$ $6 - 3 = \underline{\hspace{2cm}}$ $3 - 0 = \underline{\hspace{2cm}}$

Consecutive B-values have a difference of _____.

d) Look at the graph. Describe how to move from (0, 0) to the next point.

Solution

Starting at $(0, 0)$, move _____ unit horizontally (\leftrightarrow) and _____ units vertically (\updownarrow).

e) Write an expression for B in terms of A .

Solution

Look at the table of values: To get from A to B , multiply by _____.

Look at the graph: When A increases by _____, B increases by 3.
There are 3 ways to write B in terms of A :

Words	Ordered Pair (x, y)	Expression
B is 3 times A	$(A, 3A)$	$3 \times A$ or $3A$

Example 2: Use a Table to Determine a Linear Relation

Table 1

x	2	4	6	8
y	3	7	11	15

Table 2

m	1	2	3	4
n	1	4	7	8

a) Complete the chart to show the pattern in the values for each variable.

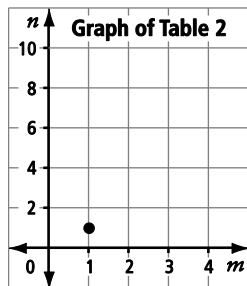
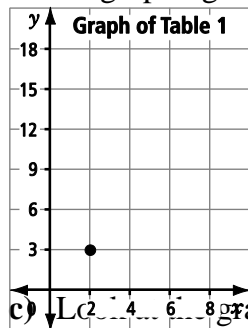
Solution

	Table 1	Table 2
Difference Between Consecutive First Variables	2	_____
Difference Between Consecutive Second Variables	$7 - 3 = \underline{\hspace{2cm}}$ $11 - 7 = \underline{\hspace{2cm}}$ $15 - 11 = \underline{\hspace{2cm}}$ The y -values have a difference of _____.	$4 - 1 = \underline{\hspace{2cm}}$ $7 - 4 = \underline{\hspace{2cm}}$ $8 - 7 = \underline{\hspace{2cm}}$ The n -values have differences of 3, 3, and _____.

b) Graph each table of values. Which relation is linear?

Solution

Finish graphing the points.



The graph of Table _____ shows a linear relation.

c) _____ graphs.

Table 1 is a linear relation and Table 2 is not. Explain how you know.

Solution

Table 1: The difference in the y -coordinates is _____. The relation is linear.
(*the same or not the same*)

Table 2: The difference in the n -coordinates is _____. The relation is *not* linear.
(*the same or not the same*)

Example 3: Use a Table of Values in Solving a Problem

Sam is paid \$7 for every hour of babysitting.

- a) The table of values shows how much she is paid for 1 h, 2 h, and 3 h of babysitting. Complete the table.

Solution

Number of Hours, n	Sam's Pay, P
1	7
2	14
3	21
4	
5	

b) Is this a linear relation? Explain how you know.

Solution

Look at the table of values.

Consecutive n -values have a difference of _____, or they increase by _____.

Consecutive P -values have a difference of _____, or they increase by _____.

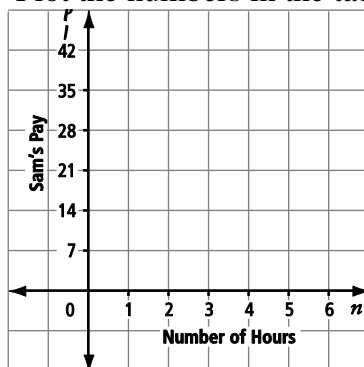
This relation is **linear** because:

- the number of hours (n) changes by the same amount
- Sam's pay (P) changes by the same amount

c) Graph this relation.

Solution

Plot the numbers in the table of values on the grid



d) Write an expression to show Sam's pay.

Solution

Look at the table of values. To get from n to P , multiply by _____.

Look at the graph. When n increases by _____, P increases by _____.

There are 3 ways to write Sam's pay in terms of the number of hours she babysits:

Words	Ordered Pair	Expression
P is _____ times n	$(n, \boxed{} n)$	$\boxed{} n$

- e) How much will Sam be paid for 9 h of babysitting?

Solution

$$\text{Sam's pay} = 7 \times n$$

$$= 7 \times \underline{\hspace{2cm}} \quad \text{Substitute 9 for } n.$$

$$= \underline{\hspace{2cm}}$$

Sam will be paid \$.

Practise

1.

x	y
1	5
2	8
3	11
4	14

- a) The difference between consecutive x -values is .

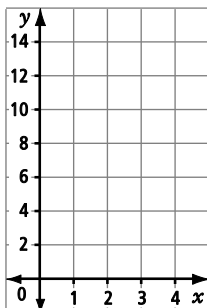
$$(4 - 3 = \underline{\hspace{1cm}}, 3 - 2 = \underline{\hspace{1cm}}, \text{ and } 2 - 1 = \underline{\hspace{1cm}})$$

- b) The difference between consecutive y -values is .

$$(14 - 11 = \underline{\hspace{1cm}}, 11 - 8 = \underline{\hspace{1cm}}, \text{ and } \underline{\hspace{1cm}})$$

- c) Does this table of values describe a linear relation? Circle YES or NO.
Give 1 reason for your answer.

- d) Graph the table of values.



- e) Look at the graph. Describe the movement from (1, 5) to the next point.

Starting at (1, 5), move

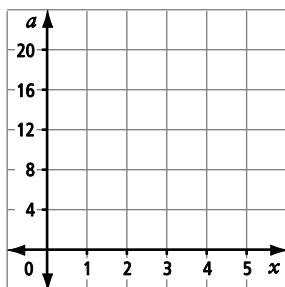
 unit horizontally and

 units vertically.

2. The table of values describes a linear relation.

x	0	1	2	3	4	5
a	0	4	8	12	16	20

- a) The difference between consecutive x -values is _____.
- b) The difference between consecutive a -values is _____.
- c) Graph the ordered pairs.
- d) Look at the graph. Describe in words how to move from $(0, 0)$ to the next point.



- e) Write a in terms of x .

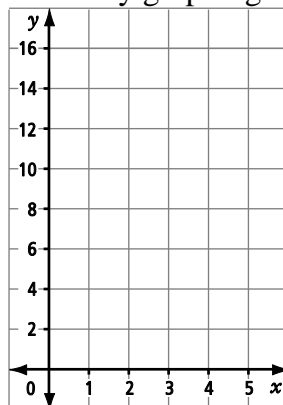
Words	Ordered Pair	Expression
a is _____ times x	$(x, \boxed{}x)$	$\boxed{}x$

3.

x	y
2	7
3	10
4	13
5	16

- a) What is the difference between consecutive x -values? _____
- b) What is the difference between consecutive y -values? _____
Is the difference the same for consecutive values? Circle YES or NO.
- c) Is the relationship in the table of values a linear relation? Circle YES or NO.
Give 1 reason for your answer.

- d) Check by graphing.



4. Mara reads 90 words per minute.

- a) Complete the table of values.

Number of Minutes, m	1	2	3	4	5	6
Number of Words, w						

- b) Explain how you can find out if this is a linear relation.

- c) If the number of minutes is m , then the expression for the number of words is

_____ $\times m$.

- d) How many words can Mara read in 15 min?

5. A community centre has a new banquet hall.
The centre charges \$5 per person to rent the hall.

- a) Complete the table of values.

Number of People, p	1	20	40	60	80	100
Rental Cost, C (\$)	5					

- b) If the number of people is p , then the expression for the rental cost is _____.

- c) How much will it cost for 150 people?

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<https://youtu.be/znDX8LZNOc4>

Warm Up

1. Complete the patterns.

a) 3, 8, 13, _____, _____, _____ b) 9, 7, 5, _____, _____

2. Whole numbers start at 0 and increase by 1 each time.

a) List the first 10 even whole numbers.

b) List the first 10 odd whole numbers.

3. Integers are ... -3, -2, -1, 0, 1, 2, 3, ...
Zero is not positive or negative.

a) List the first 5 positive integers.

b) List the first 5 negative integers.

4. Use substitution to evaluate.

a) $3x - 4$, when $x = 5$

$3(\text{_____}) - 4$ Substitute.
 $= \text{_____} - 4$ Multiply.
 $= \text{_____}$

b) $8 + 2x$, when $x = 3$

$8 + 2(\text{_____})$ Substitute.
 $= 8 + \text{_____}$ Multiply.
 $= \text{_____}$

5. Let n describe Nancy's age.
The expression $n - 5$ represents Jane's age.
If Nancy is 12 years old, how old is Jane?

Jane is _____ years old.

Linear Relationships

Example 1: Graph From a Linear Formula

Formula

- an equation that shows how 1 variable is related to another
- example: $P = 2l + 2w$ shows how the perimeter of a rectangle is related to its length and width

A car travels at 60 km/h.

Use the formula $d = 60t$ to express this relationship, where d is the distance travelled, in kilometres, and t is the time, in hours.

a) Make a table of values.

Solution

Time, t	0	1	2	3
Distance, d				

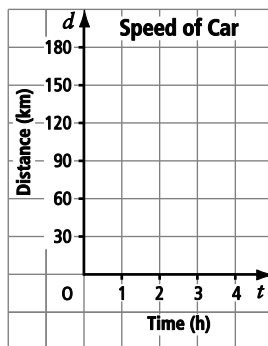
Time cannot be negative in this question. So, t cannot be a negative integer.

To find d , substitute each value of t from the table into the formula $d = 60t$.

$d = 60t$	$d = 60t$	$d = 60t$	$d = 60t$
$d = 60(0)$	$d = 60(1)$	$d = 60(\text{_____})$	$d = 60(\text{_____})$
$d = \text{_____}$	$d = \text{_____}$	$d = \text{_____}$	$d = \text{_____}$

b) Graph the ordered pairs in your table of values.

Solution



c) Is it possible to have points between the ones on this graph?

105 km is halfway between 90 km and 120 km.

Solution

It is possible to have points between the ones on the graph.

The distance at 2.5 h is _____ km. The time at 105 km is _____ h.

d) How far will the car travel in 3.5 h?

Solution

$$d = 60t$$

$$d = 60 \times (\text{_____}) \quad \text{Substitute 3.5 for } t.$$

$$d = \text{_____}$$

The car will travel 210 km in 3.5 h.

Example 2: Graph From a Linear Equation Using Integers

Equation

- 2 expressions that have the same value and are joined with an equal sign
- examples: $2x + 3 = 10$ and $y = x - 5$

$y = -3x + 4$ is a linear equation.

a) Use the linear equation to make a table of values.

Solution

You do not know what x describes, so, use integer values for x .

To find y , substitute each of the values for x into the equation $y = -3x + 4$.

x	y
-2	10
-1	
0	
1	
2	
3	

For $x = -2$:
 $y = -3x + 4$
 $y = -3(-2) + 4$
 $y = 6 + 4$
 $y = 10$

For $x = -1$:
 $y = -3x + 4$
 $y = -3(-1) + 4$
 $y = \text{_____} + 4$
 $y = \text{_____}$

For $x = 0$:
 $y = -3x + 4$
 $y = -3(0) + 4$
 $y = \text{_____} + 4$
 $y = \text{_____}$

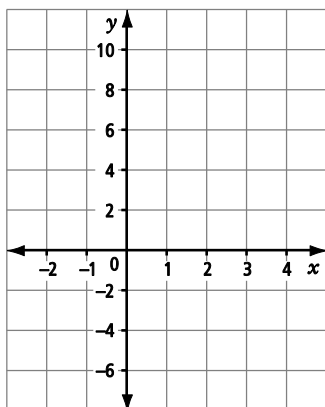
For $x = 1$:
 $y = -3x + 4$
 $y = -3(1) + 4$
 $y = \text{_____} + 4$
 $y = \text{_____}$

For $x = 2$:
 $y = -3x + 4$
 $y = -3(2) + 4$
 $y = \text{_____} + 4$
 $y = \text{_____}$

For $x = 3$:
 $y = -3x + 4$
 $y = -3(3) + 4$
 $y = \text{_____} + 4$
 $y = \text{_____}$

b) Graph the ordered pairs.

Solution



c) Find the y-value in the ordered pair (11, y).

Solution

The point (11, y) tells you that the x-value is 11.
Substitute 11 into the formula $y = -3x + 4$.

$$y = -3x + 4$$

$$y = -3(\text{_____}) + 4 \quad \text{Substitute.}$$

$$y = \text{_____} + 4 \quad \text{Multiply.}$$

$$y = \text{_____}$$

The value of the y-coordinate is _____.

The ordered pair is (11, _____).

Practise

1. Find the value of each equation.

a) $y = 5x - 3$ when $x = 6$

$$y = 5(\text{_____}) - 3$$

$$y = \text{_____} - 3$$

$$y = \text{_____}$$

b) $y = 3x + 2$ when $x = -4$

$$\leftarrow \text{Substitute} \rightarrow \quad y = 3(\text{_____}) + 2$$

$$\leftarrow \text{Multiply} \rightarrow \quad y = \text{_____} + 2$$

$$y = \text{_____}$$

c) $y = x - 8$ when $x = 5$

d) $y = -5x$ when $x = -2$

2. Use the formula $C = 6t$ to describe a long-distance telephone plan, where C is the cost in cents and t is the time in minutes.

- a) Make a table of values.

Use at least 4 whole number values for t .

t	C
1	

Let $t = 1$

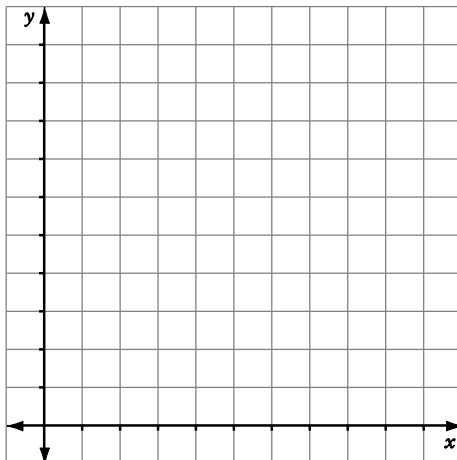
$$C = 6t$$

$$C = 6(1)$$

$$C = \underline{\hspace{2cm}}$$

1	2	3	<i>Only 6¢ per minute anytime for calls across Canada!</i>
4	5	6	
7	8	9	
*	0	#	
			CallCanada

- b) Graph the ordered pairs from your table of values.



- c) If you round part minutes up to the next whole minute, is it possible to have points between the ones on your graph? Explain.

3. Complete each table of values.

- a) $y = 3x + 2$

x	y
-2	
0	
2	
4	

Let $x = -2$
 $y = 3(-2) + 2$

$y = \underline{\hspace{2cm}} + 2$

$y = \underline{\hspace{2cm}}$

Let $x = 0$

Let $x = 2$

Let $x = 4$

b) $y = -4x$

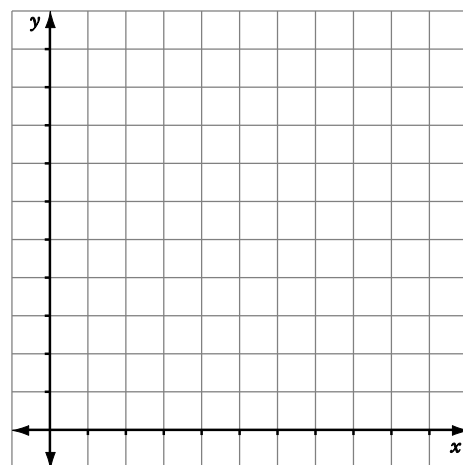
x	y
-2	
0	
2	
4	

4. An animal shelter pays you \$5 for each dog you walk.
Use the formula $M = 5d$ to relate the money you make to the number of dogs you walk.
 M is the money you make and d is the number of dogs you walk.

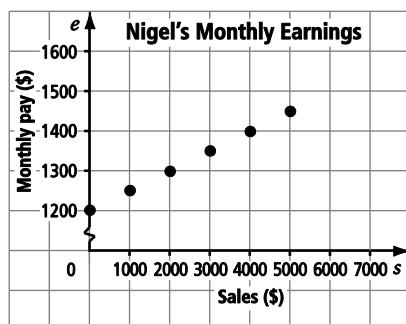
a) Make a table of values.

d				
M				

b) Graph the ordered pairs.



5. The graph shows Nigel's monthly pay.



A break in the y-axis means the length of the axis has been shortened. It can be shown as

- a) If Nigel does not make any sales, what is his monthly pay? _____
- b) Nigel has sales of \$4000 in 1 month. How much does he make? _____
- c) Nigel earns \$1300 in 1 month. What are his sales? _____

6. You can buy work gloves from a web site.
 Use the formula $C = 5g + 2$ to find the price.
 C is the cost in dollars and g is the number of pairs of gloves.

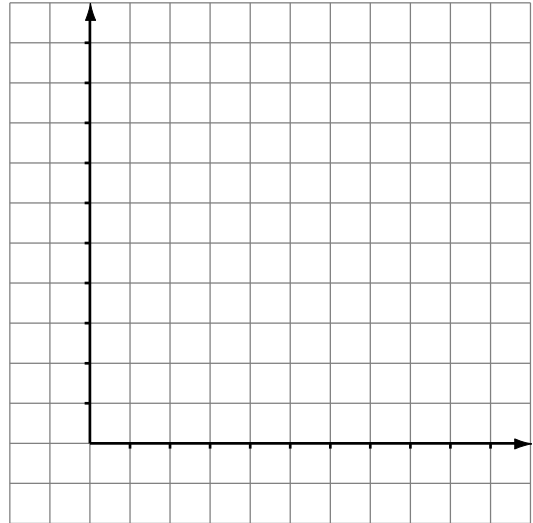
a) Complete the table of values using whole numbers.

g	C

b) Graph the ordered pairs.

To draw a graph:

- ☐ Label each axis using g and C .
- ☐ Describe each axis.
- ☐ Mark the intervals on each axis.
- ☐ Give the graph a title.
- ☐ Plot the points.



c) Is this a linear relation? Circle YES or NO.
 Give 1 reason for your answer.
