Learning Guide 13: Linear Equations

Substituting Values into Equations

To substitute values into an equation:
Step 1: Replace the variable.

A letter that represents an unknown number

Find the value of y when $x = 5$.
$y = 2(x - 4) + 3$ Substitute.
$y = 2(5 - 4) + 3$ Brackets first.
$y = 2(1) + 3$ Multiply.
$y = 2 + 3$ Add.
$y = 5$

1. Find the value of y when $x = 4$.

a) $y = 2(x - 1) + 6$
    $y = 2(4 - 1) + 6$

    $y = 2(\underline{\hspace{2cm}}) + 6$

    $y = \underline{\hspace{2cm}} + 6$

    $y = \underline{\hspace{2cm}}$

b) $y = (3 + x - 7) \times 4$

    $y = (3 + \underline{\hspace{2cm}} - 7) \times 4$

    $y = (\underline{\hspace{2cm}} - 7) \times 4$

    $y = \underline{\hspace{2cm}} \times 4$

    $y = \underline{\hspace{2cm}}$

Modelling and Solving One-Step Equations

To solve a word problem, change the words into symbols, letters, and numbers to make an equation.

A number increased by 5 is 8.
$\downarrow$ $\downarrow$ $\downarrow\downarrow$
n +   5 = 8

4 times a number is 24.
$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow\downarrow$
4   × x = 24

Rewrite as $4x = 24$

To solve an equation, use the **opposite operation** to get x alone on 1 side of the equation.

opposite operation
- an operation that undoes another operation
- − and + are opposite operations, × and ÷ are opposite operations

1

Examples:

What you do to 1 side, do to the other.

$n + 5 = 8$
$n + 5 - \underline{5} = 8 - \underline{5}$
$n + \cancel{5} - \cancel{5} = 8 - 5$
$n = 3$

$4x = 24$
$$\frac{\cancel{4}x}{\cancel{4}} = \frac{24}{4}$$
$x = 6$

2. Write each sentence as an equation.

a) Seven more than a number is twelve.

$n + \boxed{} = \underline{\hspace{3cm}}$

b) Three less than a number is eleven.

$x - \boxed{} = \underline{\hspace{3cm}}$

c) Four times a number is twenty-eight.

_____

d) When a number is divided by nine, the result is nine.

_____

Solving Two-Step Equations

To solve a 2-step equation, find the value of *x*.

$2x + 3 = 7$
$2x + 3 - 3 = 7 - 3$ Subtract to undo addition.
$2x + \cancel{3} - \cancel{3} = 7 - 3$

$2x = 4$ Divide to undo multiplication.
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{4}{2}$$
$x = 2$

Get *x* alone on 1 side of the equation.

3. Solve the equations.

a) $5j + 9 = 29$

$5j + 9 - \underline{\hspace{2cm}} = 29 - \underline{\hspace{2cm}}$

$5j = \underline{\hspace{1.5cm}}$

$j = \underline{\hspace{1.5cm}}$

b) $2t - 2 = 14$

Warm Up

1. Write an equation for each diagram.

	represents x
	represents $+1$
	represents -1

a)

b)

_____

_____

c)

d)

_____

_____
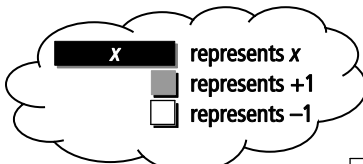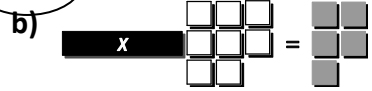
2. Show each equation using algebra tiles.

a) $3x + 5 = -1$

b) $2x + 1 = 5$

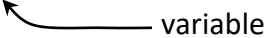**Modelling and Solving One-Step Equations:** $ax = b$, $\frac{x}{a} = b$

Linear equation
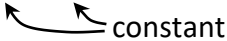• an equation whose points on a graph lie along a straight line

Numerical coefficient
• a number that multiplies the variable
• example: 4k + 3 ⟵ numerical coefficient

Variable
• a letter used to represent an unknown number
• example: 2y + 3 ⟵ variable

Constant
• a number that is not connected to a variable
• example: 2x + 5 = −3 ⟵ constant

Example: Solve an Equation

Solve each equation.

▢ represents a positive variable.
▢ represents a negative variable.
◼ represents +1.
▢ represents −1.

a) $3x = -12$

Solution

Method 1: Solve by Inspection
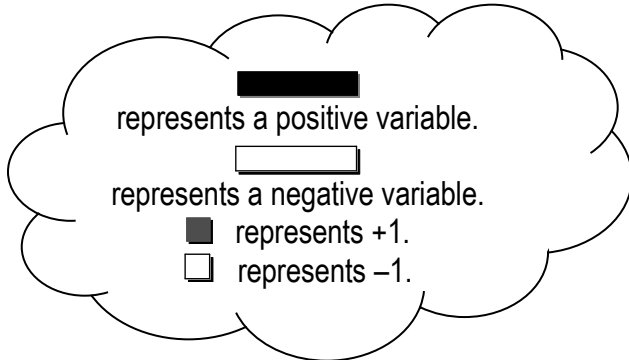
$3x = -12$

What number times 3 equals −12?

3 × ▢ = −12

3 × −4 = −12

So, $x =$ _____

Method 2: Solve Using Models and Diagrams

Use algebra tiles.

x		▢▢▢▢
x	=	▢▢▢▢
x		▢▢▢▢

3 black tiles = 3x
12 white tiles = −12
3 black tiles = 12 white tiles

How many white tiles = 1 black tile? _____

So, $x =$ _____ .

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Example: Divide to Apply the Opposite Operation

Solve by applying the opposite operation. Check your answer.

a) $-5b = -45$

$$\dfrac{-5b}{\boxed{}} = \dfrac{-45}{\boxed{}}$$

$b = $ _____

Check:

Left Side	Right Side
$-5b$	-45
$= -5 \times$ _____	
$= $ _____	

b) $6f = -12$

Check:

Left Side	Right Side

Example: Multiply to Apply the Opposite Operation

Solve by applying the opposite operation. Check your answer.

$$\frac{d}{-5} = 3$$

$$\frac{d}{-5} \times \underline{\hspace{1.5cm}} = 3 \times \underline{\hspace{1.5cm}}$$

$$d = \underline{\hspace{1.5cm}}$$

Check:

Left Side	Right Side
$\dfrac{d}{-5}$	3
$= \dfrac{\boxed{}}{-5}$	
$= \underline{\hspace{1.5cm}}$	

1. Five times a number is –15.

   Write an equation and solve for the variable in this sentence. _____

2. Raj is solving the equation $\frac{n}{9} = -4$.
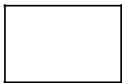
   $$\frac{n}{9} = -4$$
   $$\frac{n}{9} \times (-9) = -4 \times (-9)$$
   $$n = 36$$

   Raj's solution is wrong. Explain where he made his mistake.

   _____

3. Write the equation modelled by each diagram.

   a)

   ■ represents +1
   □ represents –1

   $t =$ _____

   b)

   _____
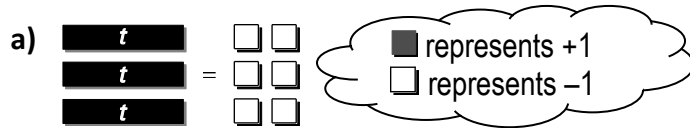
4. Solve by inspection.

   a) $-2j = 12$

   $-2 \times$ [ ] $= 12$

   $j =$ _____

   b) $5n = -25$

   $5 \times$ [ ] $= -25$

   $n =$ _____

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5. Solve each equation using the opposite operation. Check your answers.
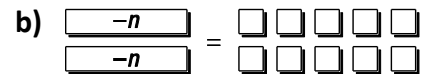
a) $4s = -12$

$$\frac{4s}{\boxed{}} = \frac{-12}{\boxed{}}$$

$s = $ _____

Check:

Left Side	Right Side

b) $-36 = -3j$

Check:

Left Side	Right Side

6. Nakasuk's snowmobile can travel for 13 km on 1 L of gas.

a) How far can he travel on 2 L of gas?

_____

_____

b) How far can he travel on 3 L of gas?

_____

_____

c) How far can he travel on 10 L of gas?

_____

_____

d) Nakasuk visits his aunt who lives 312 km away.
How many litres of gas will he need?
Use the equation $13x = 312$ to help you.

_____

_____
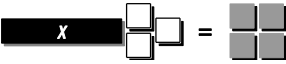
Warm Up

1. Write the equation for each diagram.

    a)

    b)

    _____

                                                                      _____

2. Evaluate.

    a) $b + 6$, when $b = 5$ **b)** $s - 3$, when $s = 12$

    $b + 6$

    $= $ _____ $ + 6$

    $= $ _____

3. Find the missing number by inspection.

    a) $5 \times$ _____ $= 15$ **b)** $4 \times$ _____ $= 24$
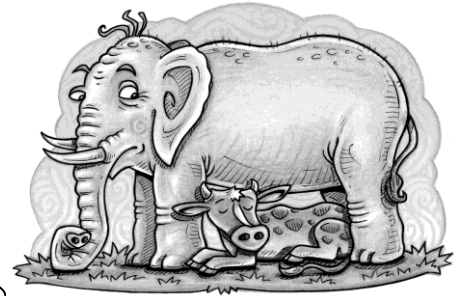
    c) _____ $\div 2 = 8$ **d)** $\dfrac{\square}{3} = 7$

Modelling and Solving Two-Step Equations: *ax + b = c*

Examples: Model With Algebra Tiles
1.   A cow sleeps 7 h a day.
This is 1 h less than twice the amount an elephant sleeps in a day.
How long does an elephant sleep?

> unknown value = number of hours an elephant sleeps

a) Write an equation for this situation.

Solution

Let *e* represent the number of hours an elephant sleeps.
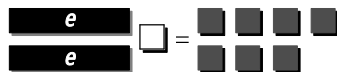"*Twice* what an elephant sleeps" = 2*e*
"1 h *less than* twice what an elephant sleeps" = 2*e* − ————————————

A cow sleeps 7 h, so the equation is 2*e* − 1 = ————————————

b) Solve the equation using algebra tiles.

Solution

The numerical coefficient is ——————————, so draw —————————— black tiles.

> the number that multiplies the variable

To isolate the black tiles on 1 side of the equal sign, add —————————— grey tile to both sides.
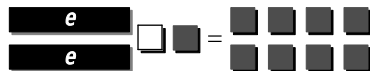
> The negative 1-tile and positive 1-tile on the left side equal 0.

—————————— black tile is equal to —————————— grey tiles.

e = 4

So, an elephant sleeps —————————— h a day.

2. Solve each equation using algebra tiles.

a) $2x + 4 = -6$

Fill in the missing tiles so your model represents the equation.

Isolate the black tiles.

Split up the tiles so you can see that 1 black tile is equal to ⸻ white tiles.

$x =$ ⸻

b) $3r - 2 = 13$

$r =$ ⸻

Example : Apply the Opposite Operations

Solve $4w + 3 = 19$ by applying the opposite operation.

+ and −
× and ÷

Solution
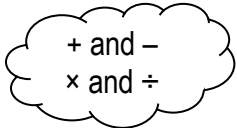
To isolate a variable, follow the reverse order of operations.

$4w + 3 = 19$

$4w + \cancel{3} - \cancel{3} = 19 - 3$ Subtract 3 from both sides of the equation.

$4w =$ ⸻ Divide both sides by 4.

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{\boxed{}}{4}$$

$w =$ ⸻

1. a) Draw algebra tiles to model $3x - 5 = 16$.

Isolate means to get the variable alone.

b) To isolate *x*, add _____ tiles to both sides of the equation.
Give 1 reason why you need to add this number of tiles. Hint: Use zero pairs.

_____

_____

c) To solve for *x,* divide both sides of the equation by _____.
Give 1 reason why you need to divide by this number.

_____

_____

2. Solve each equation modelled by the algebra tiles.

a)

$x = $ _____

b)
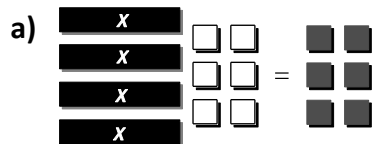
$t = $ _____

3. Complete the table.

Equation	First Operation to Solve	Second Operation to Solve
$4r - 2 = 14$	Add _____ to each side.	Divide both sides by _____ .
$-22 = -10 + 2n$		
$53 = -9k - 1$		
$3 - 3x = -9$		

4. Solve each equation and check your answer.

a) $\qquad 6r + 6 = 18$

$6r + 6 -$ _____ $= 18 -$ _____

$6r =$ _____

$$\frac{6r}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

$r =$ _____

Check:

Left Side	Right Side
$6r + 6$	18
$= 6(\underline{\quad\quad}) + 6$	
$= \underline{\quad\quad} + 6$	
$= \underline{\quad\quad}$	

b) $4m - 2 = 14$

Check:

Left Side	Right Side

5. You buy lunch at Sandwich Express.
A sandwich costs $4. Each extra topping costs $2.
You have $10. Use the equation $2e + 4 = 10$ to find
how many extra toppings you can get if you spend
all of your money.

MENU ▌▌▌▌

Your choice of extras,
only $2 each:
salad, fries, milk,
juice, jumbo cookie,
frozen yogurt.
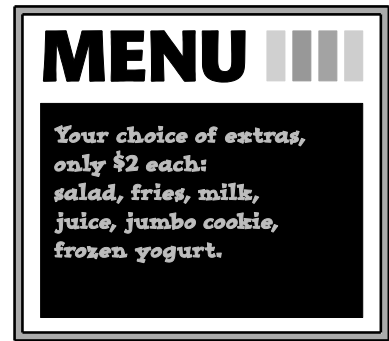
Sentence: _____

6. Jennifer is saving money to buy a new bike.
She doubled the money in her bank account, and then she took out $50.
She has $300 left in her account.

a) Write an equation to find the amount
in her account at the beginning.

$\boxed{}\, m - \underline{\hspace{2cm}} = 300$

m = money in
Jennifer's account

b) Solve the equation.

Jennifer had _____
in her bank account.

Warm Up

1. Use the opposite operation to solve.

    a) $p - 5 = 20$

    $p - \cancel{5} + \cancel{5} = 20 + \underline{\hspace{2cm}}$

    $p = \underline{\hspace{2cm}}$

    b) $x + 9 = 14$

    c) $3x = 15$

    $\dfrac{3x}{\boxed{}} = \dfrac{15}{\boxed{}}$

    $x = \underline{\hspace{2cm}}$

    d) $7 = \dfrac{d}{5}$

3. Draw a model for each equation. Do not solve.

    a) $x - 7 = 5$

    b) $5g + 3 = -1$

Modelling and Solving Two-Step Equations: $\dfrac{x}{a} + b = c$

Example 1: Model Equations

The elevation of Qamani'tuaq, Nunavut, is 1 m less than $\dfrac{1}{2}$ the elevation of Prince Rupert, B.C.

If the elevation of Qamani'tuaq is 18 m, what is the elevation of Prince Rupert?

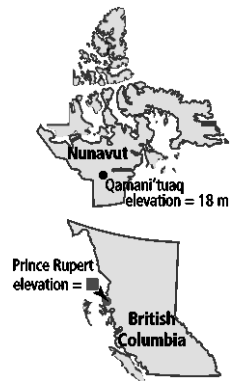a) Write an equation to find the elevation of Prince Rupert.

Solution

Let *p* represent the elevation of Prince Rupert.

" $\dfrac{1}{2}$ the elevation of Prince Rupert" = $\dfrac{1}{2}p$

"The elevation of Qamani'tuaq is 1 m *less than* $\dfrac{1}{2}$ " = $\dfrac{1}{2}p - 1$

The elevation of Qamani'tuaq is 18 m, so $\dfrac{1}{2}p - 1 =$ _____.

Elevation is the height above sea level.

Example: Apply the Reverse Order of Operations

Kristian Huselius played for the Calgary Flames during the 2006–2007 NHL season.

He had 41 more than $\dfrac{1}{2}$ the number of shots on goal as Jarome Iginla.

Huselius had 173 shots on goal.
How many shots on goal did Iginla have?

a) Write an equation to find the number of shots on goal
   Jarome Iginla had.

Solution

Let *j* represent the number of shots on goal Jarome Iginla had.

"Huselius had 41 more than $\dfrac{1}{2}$ the number of shots on goal" = $\dfrac{j}{2} +$ _____.

Since Huselius had 173 shots, $\dfrac{j}{2} + 41 =$ _____.

b) Solve the equation to find Jarome Iginla's number of shots on goal.

Solution

$$\frac{j}{2} + 41 = 173$$

$$\frac{j}{2} + 41 - \underline{\hspace{3cm}} = 173 - \underline{\hspace{2cm}}$$ Subtract ⎯⎯⎯ from both sides.

$$\frac{j}{2} = \underline{\hspace{2cm}}$$

$$\cancel{2} \times \frac{j}{\cancel{2}} = 2 \times \underline{\hspace{2cm}}$$ Multiply both sides by ⎯⎯⎯.

$$j = \underline{\hspace{2cm}}$$

Jarome Iginla had ⎯⎯⎯⎯⎯⎯ shots on goal during the 2006–2007 season.

Check:

Left Side	Right Side
$\frac{j}{2} + 41$	173 ✓
$= \dfrac{\boxed{}}{2} + 41$	
$= \underline{\hspace{1.5cm}} + 41$	
$= \underline{\hspace{1.5cm}}$ ✓	

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1. Draw a model for $\frac{g}{2} - 5 = 3$. Then, solve and check your answer.

   Model: *Solution:*

   Check:

Left Side	Right Side
$\frac{g}{2} - 5$	3

   $g =$ _____

2. Solve each equation using the reverse order of operations. Check your answers.

   a) $\qquad\qquad 2 + \dfrac{m}{3} = 18$

   $2 -$ _____ $+ \dfrac{m}{3} = 18 -$ _____

   $\dfrac{m}{3} =$ _____

   $\boxed{\diagup} \times \dfrac{m}{\cancel{3}} =$ _____ $\times$ _____

   $m =$ _____

   b) $\dfrac{c}{-8} - 8 = -12$

   Check:

Left Side	Right Side

   Check:

Left Side	Right Side

3. People 18 years old or younger need a certain number of hours of sleep each day.

The equation $s = 12 - \dfrac{a}{4}$ tells you how many hours of sleep they need.

s = amount of sleep needed, in hours
a = age of the person, in years

a) If Brian needs 10 h of sleep, how old is he?

$$s = 12 - \dfrac{a}{4}$$

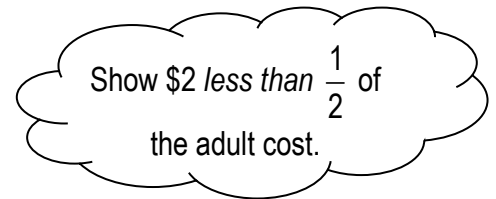**b)** Natasha is 13 years old. She gets 8 h of sleep a night. Is this enough sleep?

$$s = 12 - \dfrac{a}{4}$$

4. The cost of a concert ticket for a student is $2 less than $\dfrac{1}{2}$ of the cost for an adult.

a) Write an expression for the cost of a concert ticket for a student.
a = the cost for an adult

Cost of student ticket = $\boxed{}$ a _____

Show $2 *less than* $\dfrac{1}{2}$ of the adult cost.

b) If the cost of a student concert ticket is $5, how much does the adult ticket cost?

Equation: _____

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Warm Up

1. Fill in the blanks.

    a) The opposite of adding 4 is _____

    b) The opposite of dividing by 6 is _____

    c) The opposite of multiplying by –4 is _____

    d) The opposite of subtracting 7 is _____

2. Solve each equation.

Modelling and Solving Two-Step Equations: $a(x + b) = c$

 Example: Solve Equations

Kia is making a square quilt with a 4-cm border around it.
She wants the quilt to have a perimeter of 600 cm.
Find the lengths of the sides of Kia's quilt before she adds the border.
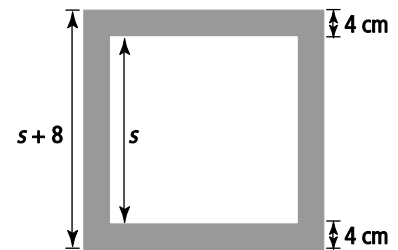
a) Write an equation.

 Solution

Let s represent the length of the side before the border is added.

A 4-cm border is added to each end of the side length: $s +$ _____

There are 4 sides to the quilt: perimeter = $4(s + 8)$ P of a square = $4s$

The perimeter is 600 cm, so the equation is $4(s + 8) =$ _____.

b) Solve the equation to find the side length of the quilt.

 Solution   **Distributive property**

- $a(b + c) = a \times b + a \times c$
- when you multiply each term inside the brackets by the term outside the brackets
- example: $2(x + 3) = (2 \times x) + (2 \times 3)$
                    $= 2x + 6$

Method 1: Divide First

$$4(s + 8) = 600$$

$$\frac{\cancel{4}(s+8)}{\cancel{4}} = \frac{600}{\boxed{}}$$

$$s + 8 = 150$$

$$s + 8 - \underline{\hspace{2cm}} = 150 - \underline{\hspace{2cm}}$$

$$s = \underline{\hspace{1.5cm}}$$

The side length of the quilt before the

border is added is \underline{\hspace{2cm}} cm.

Method 2: Use the Distributive Property

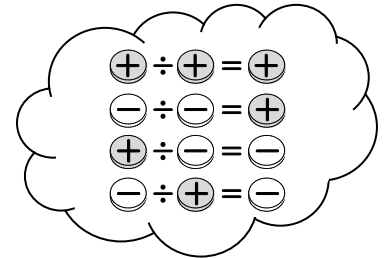$$4(s + 8) = 600$$

$$4s + 32 = 600$$

$$4s + 32 - \underline{\hspace{1.5cm}} = 600 - \underline{\hspace{1.5cm}}$$

$$\frac{\cancel{4}s}{\cancel{4}} = \frac{\boxed{}}{4}$$

$$s = \underline{\hspace{1.5cm}}$$

The quilt dimensions before adding
the border are 142 cm × 142 cm.

c) Solve $-4(x - 5) = 24$.

Method 1: Divide First

$$-4(x - 5) = 24$$

$$\frac{-\cancel{4}(x-5)}{-\cancel{4}} = \frac{24}{\boxed{}}$$

$$x - 5 = (-\underline{\hspace{2cm}})$$

$$x - 5 + 5 = (-\underline{\hspace{2cm}}) + 5$$

$$x = \underline{\hspace{2cm}}$$

Divide by -4.

$(+) \div (+) = (+)$
$(-) \div (-) = (+)$
$(+) \div (-) = (-)$
$(-) \div (+) = (-)$

Add \underline{\hspace{1.5cm}} to both sides.

Method 2: Use the Distributive Property First

$$-4(x - 5) = 24$$

Multiply x and -5 by -4.

$$(-4x) + (-4)(-5) = 24$$

$$-4x + \underline{\hspace{3cm}} = 24$$

$$-4x + 20 - 20 = 24 - 20$$

Subtract \underline{\hspace{1.5cm}} from both sides.

$$-4x = \underline{\hspace{2.5cm}}$$

$$\frac{-4x}{-4} = \frac{4}{\boxed{}}$$

Divide both sides by -4.

$$x = \underline{\hspace{2.5cm}}$$
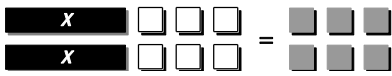
Check:

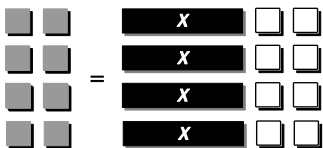| Left Side	Right Side
$-4(x - 5)$ $= -4(-1 - 5)$ $= -4(-6)$ $= 24$ ✓	24 ✓

Practise

1. Solve the equation modelled by each diagram. Check your answers.

a)

Check:

Left Side	Right Side

b)

Check:

Left Side	Right Side

2. Solve each equation by dividing first. Check your answers.

a) $6(r + 6) = -18$

$$\frac{6(r+6)}{\boxed{}} = \frac{-18}{\boxed{}}$$

$r + 6 = $ _____

$r + 6 - $ _____ $= 3 - $ _____

$r = $ _____

b) $4(m - 3) = 12$

$m = $ _____

3. Solve each equation using the distributive property.

a) $21 = 3(k + 3)$

$21 = 3(k + 3)$ Multiply $k + 3$ by 3.

$21 =$ _____ $+$ _____

$21 -$ _____ $=$ _____ $+$ _____ $-$ _____   Subtract 9 from both sides.

_____ $=$ _____

$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}k}{\boxed{}}$$ Divide both sides by 3.

$k =$ _____

b) $40 = -4(n - 3)$

$40 = -4(n - 3)$ Multiply n and -3 by -4.

$40 = ($ _____ $) + 12$

$40 -$ _____ $= (-$ _____ $) + 12 -$ _____   Subtract 12 from both sides.

_____ $= ($ _____ $)$

$$\frac{28}{\boxed{}} = \frac{-4n}{\boxed{}}$$ Divide both sides by -4.

$n =$ _____

c) $8(x - 3) = 32$ **d)** $3(1 + g) = 27$

4. An old fence around Gisel's tree is shaped like an equilateral triangle.
Gisel wants to build a new fence.
She wants to make each side 7 cm longer.
She wants the perimeter to be 183 cm.

the sides are the same length
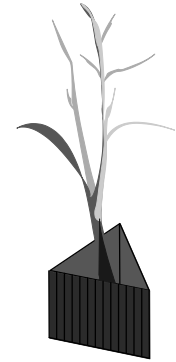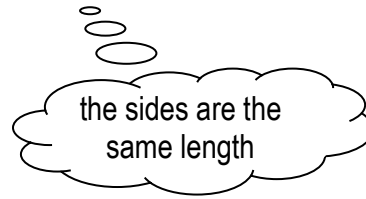
a) Write an equation for this problem.
f = length of fence before adding 7 cm

The length, f, with 7 cm added = _____

perimeter

Since all 3 sides are equal, the equation is $3(f + 7)$ = _____

b) Solve the equation to find the length of each side of the old fence.

The old fence measures _____ along each of its sides.

5. The formula $E = -125(t - 122)$ shows the amount of energy a hiker needs each day on a hike.
E is the amount of food energy, in kilojoules (kJ), and t is the outside temperature, in degrees Celsius.

a) If the outside temperature is −20 °C, how much food energy will the hiker need each day?

Does −20 replace E or t?
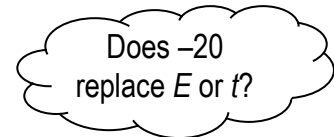
Sentence: _____

b) If a hiker uses 16 000 kJ of food energy, what is the outside temperature?

Does 16 000 replace E or t?