Learning Guide 15: Probability

Watch the following instructional video. In your handout:

i) Copy down the given notes and examples

ii) Complete the assigned questions

https://youtu.be/7K1K9Qc36wY

Probability

- the chance that an event will happen
- probability = $\dfrac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

Impossible event

- 0 or 0% probability (will not happen)

Certain event

- 1 or 100% probability (will definitely happen)

Examples:

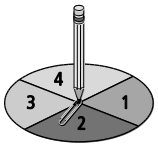
1. Find the probability that a coin will land heads up.

$$P(\text{H}) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} = \frac{1}{2} \leftarrow \frac{\text{number of heads}}{\text{total number of faces}}$$

What you want to happen.

The probability that the coin will land heads up is $\frac{1}{2}$, 0.5, or 50%.

2. You spin the spinner once. What is the probability of spinning a 2? Write the answer as a fraction, a decimal, and a percent.



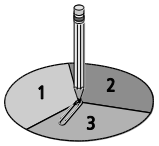
$P(2)$ means the probability of spinning a 2.

$$P(2) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{\square}{4}$$

The probability of spinning a 2 is $\frac{\square}{\square}$, 0._____, or _____%.

Using Tables and Tree Diagrams

Flip a coin AND spin the spinner.

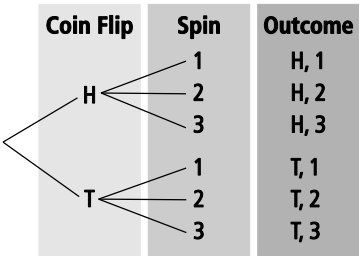


You can use a table and a tree diagram to organize the outcomes.

Table

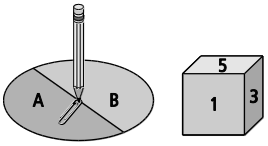
Coin	Spinner		
	1	**2**	**3**
Heads (H)	H, 1	H, 2	H, 3
Tails (T)	T, 1	T, 2	T, 3

Tree Diagram



There are 6 possible outcomes: (H, 1), (H, 2), (H, 3), (T, 1), (T, 2), (T, 3).

3. **a)** Use a table to show the possible outcomes when you spin this spinner and roll a 6-sided die.



Spinner	Die					
	1	**2**	**3**	**4**	**5**	**6**
A	A, 1					
B						

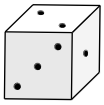
b) List the outcomes.

(_____, _____), _____

_____

Warm Up

1. You roll a 6-sided die. Find the probabilities. Write your answer as a decimal.



a)

$$P(6) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{\square}$$

$$= \_\_\_\_\_$$

b)

$$P(\text{even number}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{\square}$$

$$= \_\_\_\_\_$$

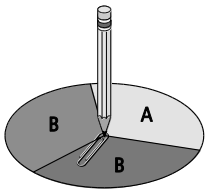
Determining Probabilities Using Tree Diagrams and Tables

Example 1: Determine Probabilities from a Tree Diagram

Independent events

- 2 or more events that do not affect each other
- example: spinning a spinner does not affect what happens if you also roll a die

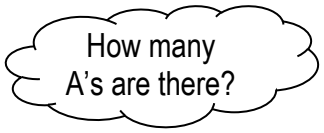
a) What is the probability of spinning A on the first spin? Write your answer as a fraction, a decimal, and a percent.



Solution

There are 3 sections on the spinner: A, B, and B.

How many favourable outcomes are there? _____



$$P(\text{A}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{3}$$

C 1 ÷ 3 = 0.333333333

$$= 0.\overline{3}$$

The probability of spinning A is $\frac{\square}{3}$, 0._____, or $33.\overline{3}\%$.

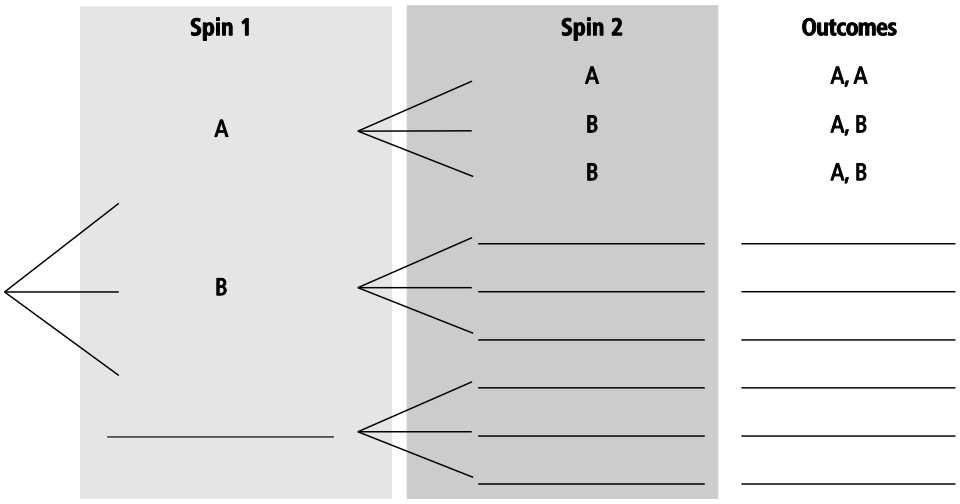
b) Draw a tree diagram to show the sample space when you spin the spinner twice.

FOLDABLES™ Study Tool

Sample Space

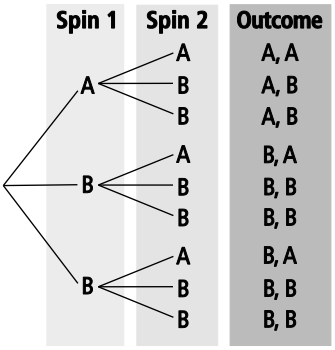
- all the possible outcomes of the event

Solution



c) What is the probability of spinning A and then spinning B, *P*(A then B)? Write the answer as a fraction, a decimal, and a percent.

Solution



The favourable outcome is (A, B).
Look at the tree diagram.
How many times does (A, B) happen? _____

$$P(\text{A then B}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{9}$$

C 2 ÷ 9 = 0.222222222

$$= \text{_____}$$

Use bar notation to show a repeating decimal.

The probability of spinning A and then B is $\frac{\square}{\square}$, 0._____, or _____%.

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ii) Complete the assigned questions

https://youtu.be/jIe-YhOGZc0

d) Use the tree diagram to find the probability of getting the same letter on both spins, $P(\text{A, A})$ or $P(\text{B, B})$. Write your answer as a fraction, a decimal, and a percent.

Solution

$$P(\text{A, A}) \text{ or } P(\text{B, B}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{9}$$

$$= \_\_\_\_\_$$

How many outcomes are (A, A) or (B, B)? Add the numbers.

The probability of spinning the same letter is $\frac{\square}{\square}$, 0._____, or _____ %.

Example 2: Determine Probabilities From a Table

You roll two 6-sided dice; 1 is white and 1 is grey.

a) Make a table to show the sample space.

Solution

Complete the table to show all the possible combinations.

		Grey Die					
		1	**2**	**3**	**4**	**5**	**6**
White Die	**1**	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	_____, _____	2, 3	_____, _____	2, 5	_____, _____
	3	3, 1	_____, _____	3, 3	_____, _____	3, 5	_____, _____
	4	4, 1	_____, _____	4, 3	_____, _____	4, 5	_____, _____
	5	5, 1	_____, _____	5, 3	_____, _____	5, 5	_____, _____
	6	6, 1	_____, _____	6, 3	_____, _____	6, 5	_____, _____

b) What is the probability of rolling a sum greater than 10?
Write your answer as a fraction, a decimal, and a percent.

Sum means add.

Solution

Add the numbers in each cell.

A *cell* is 1 box from the table.

$$P(\text{sum} > 10) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

How many cells add to more than 10?

> means *greater than*

$$= \frac{\square}{36}$$

3 **36** 0.083333333

$$= 0.08\overline{3}$$

The probability of a sum greater than 10 is $\frac{3}{36}$, 0._____, or $8.\overline{3}\%$.

c) What is the probability that the number on the white die is 1 larger than the number on the grey die?
Write your answer as a fraction, a decimal, and a percent.

Solution

In the table, shade all the outcomes where the white die number is 1 larger than the grey die number.

		Grey Die					
		1	**2**	**3**	**4**	**5**	**6**
White Die	**1**	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$$P(\text{white die 1 number larger than grey die}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{36}$$

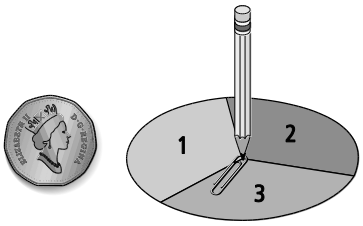
C **5** **÷** **36** **=** 0.13888888

$$= \_\_\_\_\_$$

The probability of the white die number being 1 number larger than the grey die number is $\frac{\square}{\square}$, 0._____, or _____%.

Practise

1. Jake flips a coin and spins the spinner.



a) Complete the tree diagram to show the sample space.

Coin	Spinner	Outcomes
	_____	_____ , _____
H	2	H , 2
	_____	_____ , _____
	_____	_____ , _____
T	2	T , 2
	_____	_____ , _____

b) List the sample space. (_____, _____), (_____, _____),

_____

c) What is the $P(H, 2)$? Write your answer as a fraction, a decimal, and a percent.

$$P(H, 2) = \frac{\square}{6} \leftarrow \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

= _____ ← decimal

= _____ ← percent

d) What is $P(T, \text{odd number})$?
Write your answer as a fraction, a decimal, and a percent.

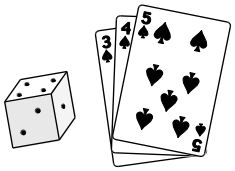
$$P(T, \text{odd number}) = \frac{\square}{\square} \leftarrow \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

Odd numbers end in 1, 3, 5, …

= _____ ← decimal

= _____ ← percent

2. Mason chooses 1 card and rolls a 6-sided die.



a) Complete the table to show the sample space.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Cards	**3**	3, 1					
	4						
	5						

b) What is the probability that both numbers are the same?
Write your answer as a fraction, a decimal, and a percent.

P(both same number) = $\frac{\square}{\square}$ ← $\frac{\text{favourable outcomes}}{\text{possible outcomes}}$

= _____ ← decimal

= _____ ← percent

c) What is the probability that the sum of the die and the card is equal to 6?
Write your answer as a fraction, a decimal, and a percent.

P(sum is 6) = $\frac{\square}{\square}$ ← $\frac{\text{favourable outcomes}}{\text{possible outcomes}}$

= _____ ← decimal

= _____ ← percent

d) What is the probability that the number on the die is larger than the number on the card?
Write your answer as a fraction, a decimal, and a percent.

P(larger number on die) = $\frac{\square}{\square}$ ← $\frac{\text{favourable outcomes}}{\text{possible outcomes}}$

= _____ ← decimal

= _____ ← percent

3. Ruby is fishing. She has an equal chance of catching a whitefish, a trout, an arctic char, or losing a fish off the hook. What might she catch if she makes two attampts?

a) Complete the table to show all the possible combinations.

		Second Catch			
		_____	_____	_____	_____
First Catch	**Whitefish (W)**				
	Trout (T)				
	_____				
	_____				

There are _____ possible combinations.

b) What is the probability she will catch 2 arctic char: *P*(char, char)?

c) What is *P*(whitefish, char) in either order?

d) What is the probability she will catch nothing at all?

Warm Up

1. List the possible outcomes of each event.

a) rolling a 6-sided die _____, _____, _____, _____, _____, _____

b) flipping a coin _____, _____

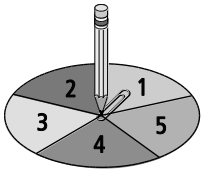
c) spinning this spinner _____, _____, _____

2. How many possible outcomes are there?

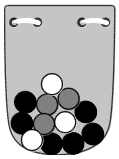
a) rolling a 4-sided die _____

b) choosing a card _____

c) spinning this spinner _____



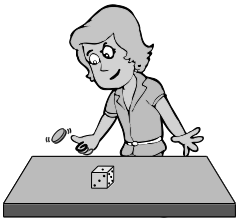
d) choosing a marble _____



Outcomes of Independent Events

Example 1: Determine the Total Number of Outcomes From Two Events

Clara flips a coin and rolls a 6-sided die. How many possible outcomes are there?



Solution

Method 1: Create a Table

Complete the table.

		Number on Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin Flip	**H (head)**	H, 1	H, 2				
	T (tail)	T, 1	T, 2				

There are _____ possible outcomes.

Method 2: Use Multiplication

Total number of possible outcomes = possible outcomes on die × possible outcomes on coin

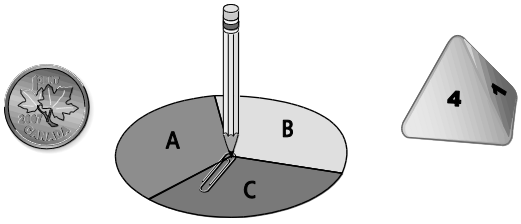
= _____ × 2

= _____

There are _____ possible outcomes.

Example 2: Determine the Total Number of Outcomes From Three or More Events

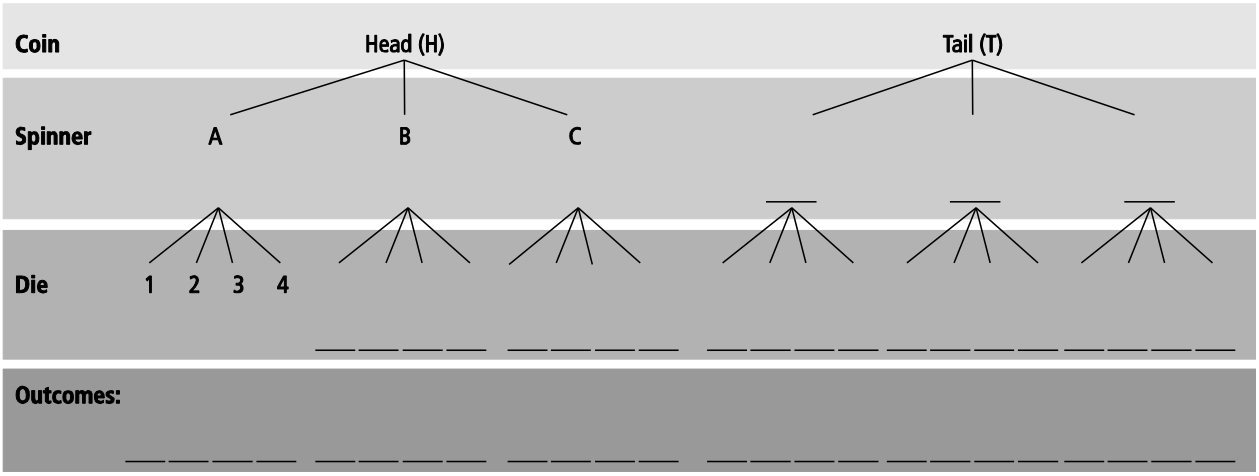
Zaira flips a coin, spins the spinner, and rolls a 4-sided die.



a) How many possible outcomes are there?

Solution

Method 1: Use a Tree Diagram



The tree diagram shows _____ possible outcomes.

Method 2: Use Multiplication

Number of possible outcomes for

- a coin: _____
- the spinner: _____
- a 4-sided die: _____

Total possible outcomes
= possible outcomes for coin × possible outcomes for spinner × possible outcomes for die

= _____ × _____ × _____

= _____

There are _____ possible outcomes.

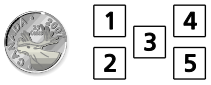
b) Why would it be difficult to show the sample space for this experiment in a table?

Solution

You can use a table for 2 events.

You can show 1 event in the columns and 1 event in the _____.
For 3 events, you would need more than 1 table to show all of the outcomes.

Practise



1. Fiona has 5 tiles numbered from 1 to 5 and a coin.
She chooses 1 tile and flips the coin. What are all the possible outcomes?

a) Use 3 different methods to show how to find the number of possible outcomes.

Tree Diagram:

Table:

		Tiles				
Coins						

Multiplication:

b) Which method do you like best? Give 1 reason for your answer.

_____

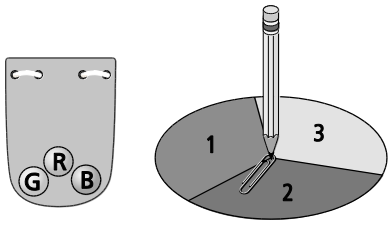
_____

2. Explain why you cannot use 1 table to find the possible outcomes when you have 3 or more events. Use an example to help you explain your answer.

_____

_____

3. A bag holds 3 marbles—1 red, 1 green, and 1 blue. A spinner has 3 equal sections labelled 1, 2, and 3. You choose a marble and spin the spinner.



a) Complete the table to show the sample space.

		Spinner		
		1	**2**	**3**
Marbles	**Red (R)**			
	Green (G)			
	Blue (B)			

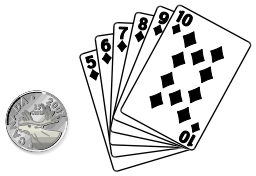
b) How many possible outcomes does the table show? _____

c) Write a multiplication statement to show the outcomes.

_____ × _____ = _____

There are _____ possible outcomes.

4. Caine flips a coin and chooses a card.



a) Complete the table to show all the possible outcomes.

		Cards					
		5	_____	_____	_____	_____	_____
Coin	**Heads (H)**						
	Tails (_____)						

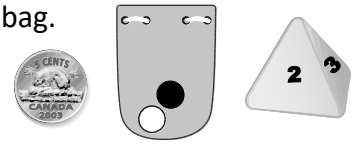
There are _____ possible outcomes.

b) Use multiplication to check the number of possible outcomes.

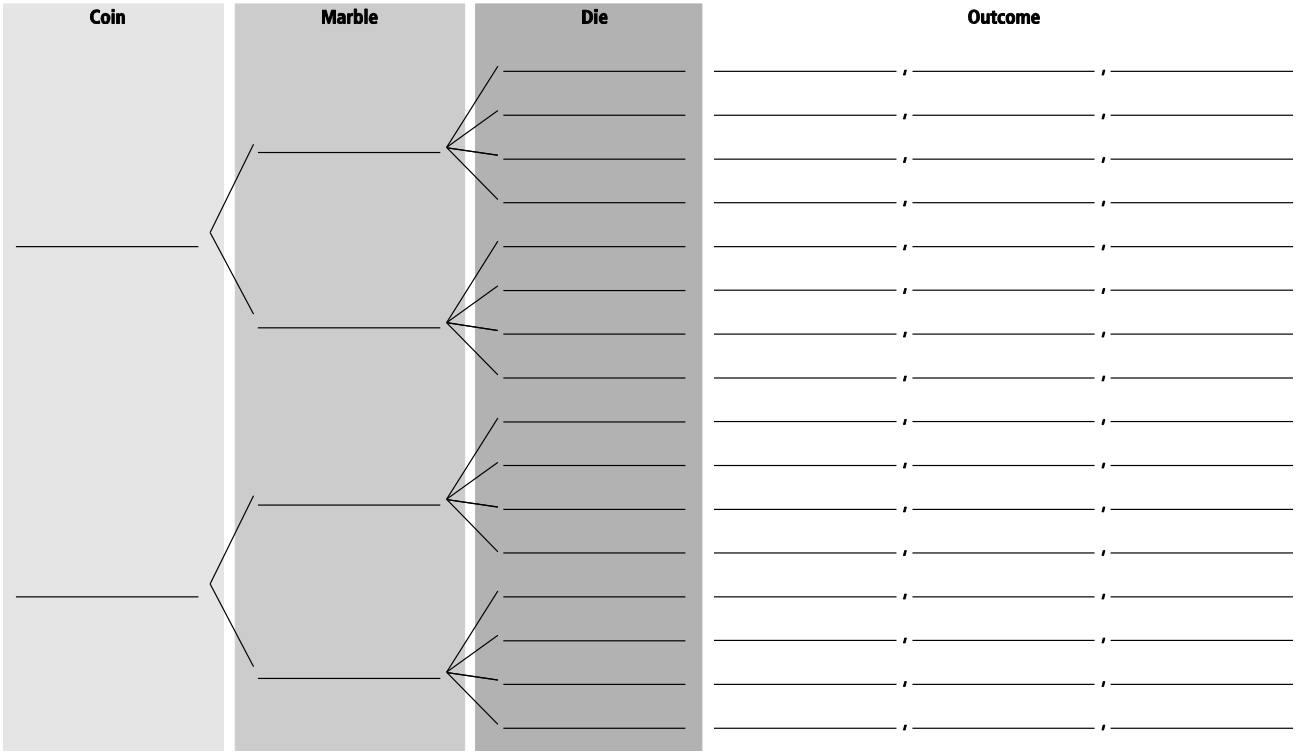
_____ × _____ = _____

There are _____ possible outcomes.

5. Jacqueline flips a coin, rolls a 4-sided die, and chooses a marble from a bag. Show the number of possible outcomes 2 different ways.



Tree Diagram:



Multiplication: _____ × _____ × _____ = _____

There are _____ possible outcomes.

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https://youtu.be/2o6BV-KsZHY

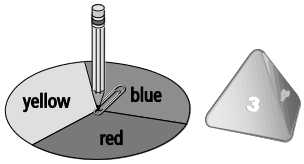
Warm Up

1. Vince flips a coin and rolls a 6-sided die.
Show the sample space using a tree diagram.

Coin	Die	Outcomes
Heads	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____
Tails	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____
	_____	_____ , _____

Determining Probabilities Using Fractions

Example 1: Calculating Probabilities Using a Table and Multiplication



Mackenzie spins a spinner and rolls a 4-sided die.

a) Complete the table to show the sample space.
How many possible outcomes are there?

Solution

Spinner	Die			
	1	**2**	**3**	**4**
Blue (B)	B, 1	B, 2	B, 3	B, 4
Red (R)				
Yellow(Y)				

Number of possible outcomes: _____.

b) What is *P*(B, 2)?
Write your answer as a fraction.

Solution

Shade the cells that have a B and a 2.

$$P(\text{B, 2}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{12}$$

The probability of getting blue and 2 is _____.

c) Use multiplication to find *P*(blue, 2).

Solution

Find each probability separately.

$$P(\text{B}) = \frac{\text{number of blue sections on spinner}}{\text{total number of sections on spinner}}$$

$$= \frac{\square}{\square}$$

$$P(2) = \frac{\text{number of 2s on die}}{\text{number of sides on die}}$$

$$= \frac{\square}{\square}$$

Multiply the probabilities of the single events.

$$P(\text{blue, 2}) = P(\text{B}) \times P(2)$$

$$= \frac{\square}{3} \times \frac{\square}{4}$$

$$= \frac{\square}{12}$$

d) Using the information in the table, what is *P*(red *or* blue, 4)?
Write your answer as a fraction.

Spinner	Die			
	1	**2**	**3**	**4**
Blue (B)	B, 1	B, 2	B, 3	B, 4
Red (R)	R, 1	R, 2	R, 3	R, 4
Yellow(Y)	Y, 1	Y, 2	Y, 3	Y, 4

Solution

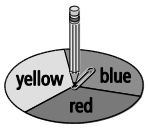
In the table, circle (R, 4) and (B, 4).

$$P(\text{R } or \text{ B, } 4) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{12}$$

e) Use multiplication to check your answer in part d).

Solution



$$P(\text{R or B}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{3}$$

$$P(4) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{4}$$

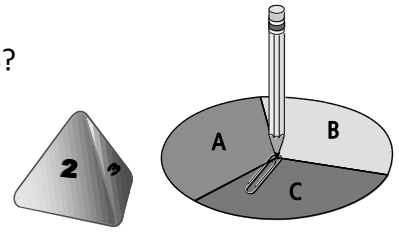
Multiply the probabilities of the 2 events.

$$P(\text{R } or \text{ B}) \times P(4) = \frac{\square}{3} \times \frac{\square}{4}$$

$$= \frac{\square}{12}$$

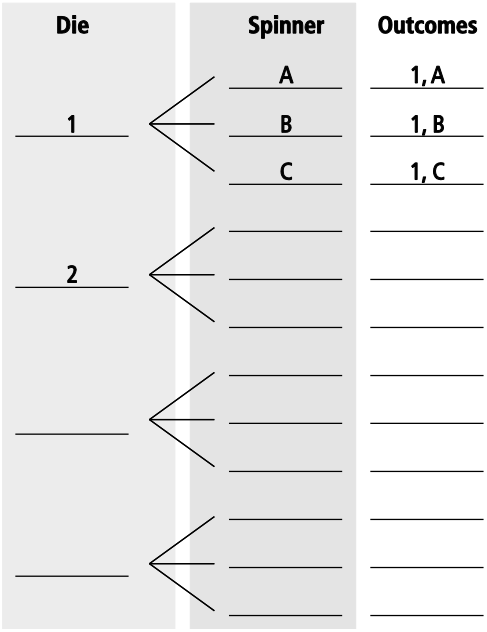
Example 2: Calculating Probabilities Using a Tree Diagram and Multiplication

Oliver rolls a 4-sided die and spins the spinner.
What is the probability of him rolling an even number and spinning a B?
Show your answer using a tree diagram and multiplication.



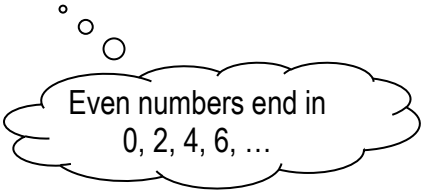
Solution

Tree Diagram:



There are _____ possible outcomes.

There are _____ favourable outcomes (look for even numbers with Bs).



$$P(\text{even number and B}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{\square}{\square}$$

Multiplication:

Die: $P(\text{even number}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$

$$= \frac{\square}{4}$$

Spinner: $P(\text{B}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$

$$= \frac{\square}{3}$$

$$P(\text{even number, B}) = P(\text{even number}) \times P(\text{B})$$

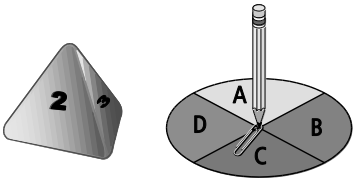
$$= \frac{\square}{4} \times \frac{\square}{3}$$

$$= \frac{\square}{\square}$$

Both methods give the same result.

Practise

1. Kostyn spins the spinner and rolls a 4-sided die.



a) Complete the table to show the sample space.

Die	Spinner			
	A	B	C	D
1				
2				
3				
4				

b) What is $P(2, A)$?

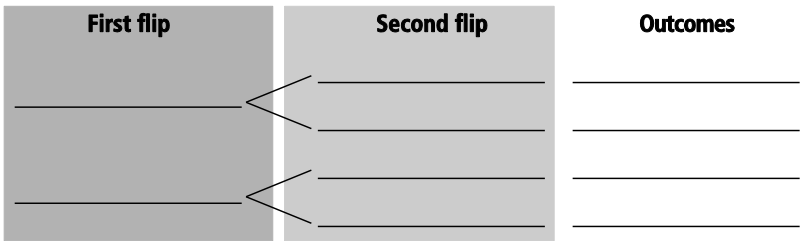
c) Use multiplication to find $P(2, A)$.

$P(2, A) = P(2) \times P(\_\_\_\_\_)$

$=$

2. Sydney flips a coin twice.

a) Complete the tree diagram to show the sample space.



b) What is $P(H, H)$?

$=$

c) Check your answer using multiplication.

$P(H, H) = P(\_\_\_\_\_) \times P(\_\_\_\_\_)$

3. Grade 8 students are planting 4 types of flowers: daisy (D), marigold (M), rose (R), and tulip (T).
The students can plant them in 4 places: school (S), flowerpot (F), park (P), and hospital (H).
Tamira does an experiment to see where the different flowers will be planted.
The sample space is (M, F), (R, H), (D, S), (M, F), (T, H), (T, P), (D, H), (R, P), (M, P), (R, F).

a) What is the experimental probability of getting *P*(marigold, flowerpot)?

b) Use multiplication to find the theoretical probability of *P*(marigold, flowerpot).

P(marigold) = $\frac{\square}{\square}$ *P*(flowerpot) = $\frac{\square}{\square}$

P(marigold, flowerpot) = *P*(marigold) × *P*(flowerpot)

4. Finn is ordering pizza for his soccer team.
There are 3 specials: Peppy Pepperoni, Happy Hawaiian, and Cheery Cheese.
He can choose from 2 types of crust: regular or thin.

a) How many different combinations of pizza are there? Show your thinking.

b) What is the probability he will choose Happy Hawaiian with thin crust?
Use 2 different ways to show your answer.

c) Finn finds out 1 of the players is allergic to pepperoni.
How many combinations of pizza are there without pepperoni?