# Learning Guide 4: Exploring the Pythagorean Relationship

**Watch the following instructional video. In your handout:**

**i) Copy down the given notes and examples**

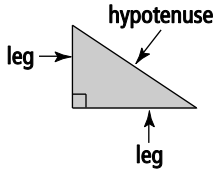
**ii) Complete the assigned questions**

https://youtu.be/jgA3LOWWRCs

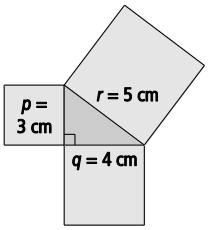
## Example 1: Describe Relationships in Right Triangles

**Right triangle**

- a triangle with a right angle (90°)
- the right angle is marked with a small square
- the 2 shorter sides are called the legs
- the longest side is called the hypotenuse



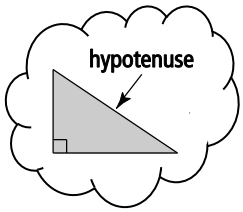

**a)** What is the area of each square?




***Solution***

| $p = 3$ cm | $q = 4$ cm | $r = 5$ cm |
|---|---|---|
| $A = s^2$ | $A = s^2$ | $A = s^2$ |
| $A = 3^2$ | $A =$ _____ | $A =$ _____ |
| $A =$ _____ × _____ | $A =$ _____ × _____ | $A =$ _____ × _____ |
| $A =$ _____ cm$^2$ | $A =$ _____ cm$^2$ | $A =$ _____ cm$^2$ |

**b)** Which side is the hypotenuse of the triangle?

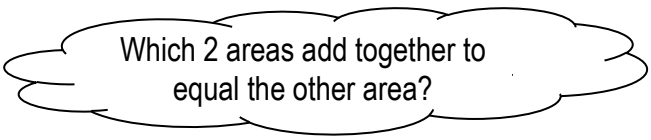


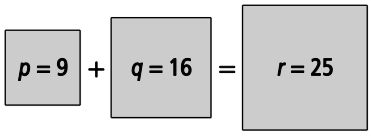

***Solution***

Side _____ is the hypotenuse.

**c)** Write an addition statement to show how the areas are related.

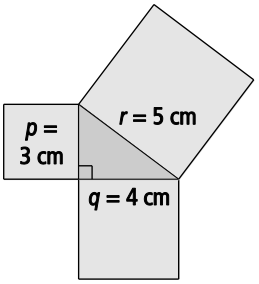
***Solution***




_____ + _____ = 25

**d)** Describe the relationship between the side lengths of the triangle.
Use words and symbols.



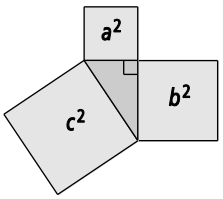

***Solution***

*Words*: The sum of the areas of the squares attached to legs $p$

and _____ equals the area of the square attached to hypotenuse $r$.

*Symbols:* $p^2 +$ _____$^2 = r^2$

**Pythagorean relationship**
• the relationship between the lengths of the sides of a right triangle
• $a^2 + b^2 = c^2$, where $c$ is the hypotenuse




The sides of a right triangle are 9 cm, 12 cm, and 15 cm.

**a)** Sketch the triangle.
Draw a square on each side of the triangle.

**b)** What is the area of each square?

| $p = 9$ | $q = 12$ | $r = 15$ |
|---|---|---|
| $A = s^2$ | | |
| $= 9^2$ | | |
| $=$ _____ $\times$ _____ | | |
| $=$ _____ cm$^2$ | | |

**c)** Write an addition statement using the areas of the 3 squares.

□ + □ = □

_____ + _____ = _____

## Example 2: Identify a Right Triangle

A triangle has side lengths of 5 cm, 7 cm, and 9 cm.

**a)** What are the areas of the 3 squares that can be drawn on the sides of the triangle?

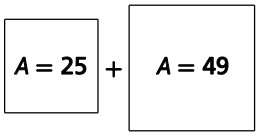
***Solution***

| | | |
|---|---|---|
| $A = 5 \times 5$ | $A = 7 \times 7$ | $A =$ _____ × _____ |
| $A =$ _____ $cm^2$ | $A =$ _____ $cm^2$ | $A =$ _____ $cm^2$ |

**b)** Is the triangle a right triangle? Explain.

***Solution***

*Step 1*: Add the areas of the 2 smaller squares.




25 + 49 = _____

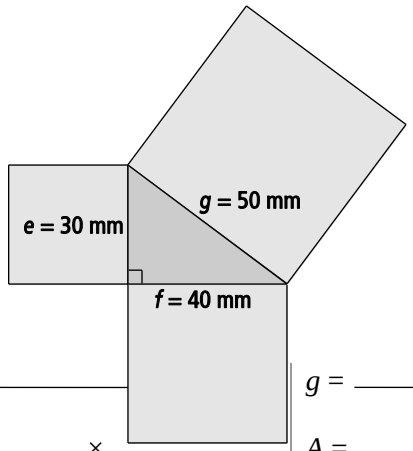
*Step 2*: Does the sum of the areas of the smaller squares equal the area of the large square?

$74\ cm^2 \neq 81\ cm^2$

The triangle _____ a right triangle.
*(is* or *is not)*

## Practise

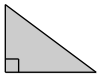
**1.** What are the areas of the 3 squares?




| | | |
|---|---|---|
| $e =$ _____ | $f =$ _____ | $g =$ _____ |
| $A =$ _____ × _____ | $A =$ _____ × _____ | $A =$ _____ × _____ |
| $A =$ _____ $mm^2$ | $A =$ _____ | $A =$ _____ |

**2.** A right triangle has side lengths of 9 mm, 12 mm, and 15 mm.

**a)** Draw a square on each side of the triangle.



**b)** What is the area of each square?

| Square 1 | Square 2 | Square 3 |
|---|---|---|
| | | |

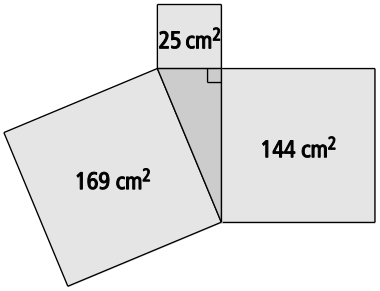
**c)** Is this a right triangle? Show your work.

Area of 2 small squares = area of large square

_____ + _____ = _____

The triangle _____ a right triangle.
*(is* or *is not)*

**3.** **a)** Write an addition statement using the areas of the 3 squares.




Area of 2 small squares = area of large square

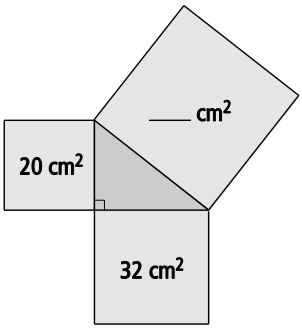
_____ + _____ = _____

**b)** What is the side length of each square?

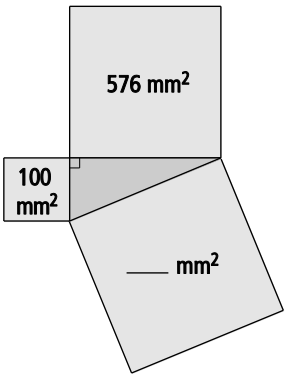
| Square 1 | Square 2 | Square 3 |
|---|---|---|
| $A$ = 25 cm$^2$ | $A$ = _____ | $A$ = _____ |
| Side length = _____ cm | Side length = _____ cm | Side length = _____ cm |

**4.** Use the Pythagorean relationship to find the unknown area of each square.

**a)**




**b)**




Area of 2 small squares

= _____ + _____

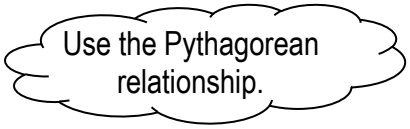
= _____

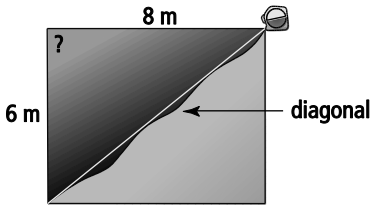
Area of 2 squares = area of large square

The area of the large square

is _____.

**5.** Construction workers are digging a hole for a swimming pool.
They want to check that the angle they have dug is 90°.
The diagonal of the rectangle is also the hypotenuse of a right angle triangle.
The diagonal measures 9.5 m.
Is the angle 90°? Explain.




Sentence: _____

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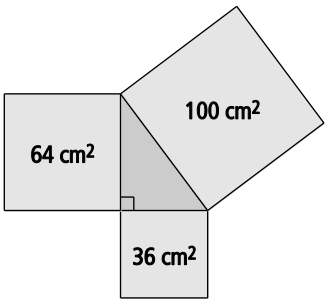
https://youtu.be/D47QpJ8TnLI

## Warm Up

**1.** **a)** List the areas of the 3 squares, from smallest to largest.




_____, _____, _____

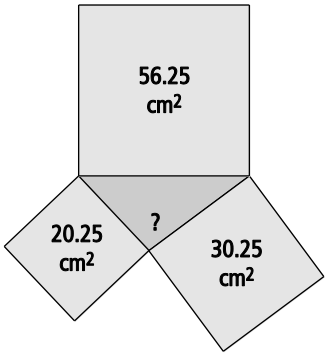
**b)** Write an addition statement with the areas of the 3 squares.

**c)** Calculate the side length of each of the squares.

Find the square root of each area.

The side lengths of the squares are _____, _____, and _____.

**2.** Is the triangle a right triangle? Show your work.



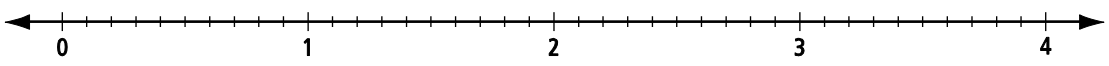

This triangle _____ a right triangle.
*(is* or *is not)*

**3.** Round to the nearest tenth (one decimal place).

**a)** 2.34 ≈ _____ **b)** 10.56 ≈ _____

**c)** 5.98 ≈ _____ **d)** 30.01 ≈ _____

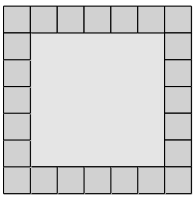
**4.** Label 1.3, 2.5, and 3.8 on the number line.



## Estimating Square Roots

### Example: Estimate the Square Root of a Number

Felicity wants to know if a wading pool will fit in her yard.
She estimates the length of the sides to see if it will fit.
The pool is square and has an area of 7 $m^2$.



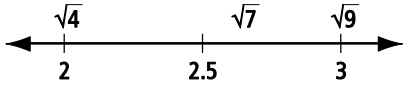
**a)** What is a reasonable estimate for the side length of the pool?
Use perfect squares to estimate. Round your answer to 1 decimal place.

***Solution***

The side length of the pool is $\sqrt{7}$ (the symbol √ means square root. $\sqrt{7}$ will give you the side length of a square that has an area of 7…the number that you square to get 7).

What number do you square to get 7?

$2^2 = 4$, $3^2 =$ _____
The perfect squares closest to 7 are 4 and 9:




The square root of 7 is closer to the square root of 9.

$\sqrt{9}$ = _____, so $\sqrt{7}$ will be a little less than 3. A reasonable estimate for $\sqrt{7}$ is 2.7 m.

**b)** Use a calculator to find the side length of the pool. Round your answer to 1 decimal place.

***Solution***

C 7 √ = _____

Round your answer to 1 decimal place. 2.645751311 m ≈ _____ m
Is this answer close to the estimate? Circle YES or NO.

Use perfect squares to estimate the square root.
Round your answer to 1 decimal place.
Check your answer with a calculator.

**a)** $\sqrt{18}$

Perfect squares on either side of 18:

_____ and _____

The closer square root is _____.

A reasonable estimate is _____.

Check with a calculator: _____

**b)** $\sqrt{35}$

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https://youtu.be/YMD-rP25etA

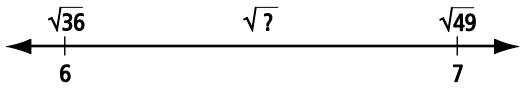
## Example 2: Identify a Number With a Square Root Between 2 Numbers

**a)** Name a whole number that has a square root between 6 and 7.

***Solution***

*Step 1*: Find the square of 6: $6^2$ = _____. Find the square of 7: $7^2$ = _____

*Step 2*: Plot these numbers on a number line:



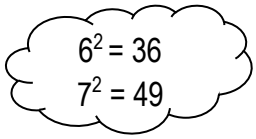

*Step 3*: Estimate a number between $\sqrt{36}$ and $\sqrt{49}$.

$\sqrt{40}$ is between $\sqrt{36}$ and $\sqrt{49}$. The value is between 6 and 7.

*Step 4*: Check: C 40 √ = _____

6.32455532 is between 6 and 7, so it is a possible answer.

**b)** How many whole numbers have a square root between 6 and 7?




***Solution***

List the whole numbers larger than 36 and smaller than 49: 37, _____, _____,

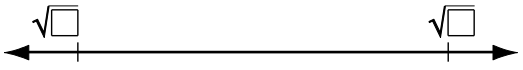
_____, _____, _____, _____, _____, _____, _____, _____, 48

There are _____ whole numbers that have square roots between 6 and 7.

**a)** Find a whole number with a square root between 8 and 9.

*Step 1*: $8^2$ = _____ $9^2$ = _____

*Step 2*: Plot these numbers on a number line.




*Step 3*: Estimate a number between the 2 square roots: _____

*Step 4*: Check:

**b)** How many whole numbers have a square root between 8 and 9?

_____

## Practise

**1.** Juan is explaining how to estimate $\sqrt{28}$ to 1 decimal place without using a calculator.
Finish Juan's explanation.

*Step 1*: Find the perfect squares on either side of 28. They are 25 and _____.

*Step 2*: The perfect square that is closer to 28 is _____.

*Step 3*: _____

**2.** Explain how to find a whole number that has a square root between 3 and 4.

*Step 1*: Find the squares of 3 and 4. $3^2$ = _____, $4^2$ = _____

*Step 2*: Draw a number line.

*Step 3*: _____

**3.** Estimate the square root of each number.
Round your answer to 1 decimal place.
Check your answer with a calculator.

**a)** 32
Perfect squares on either side of 32:

$5^2$ = _____ $6^2$ = _____

$\sqrt{32}$ is between _____ and _____.

The closer square root is _____.

An estimate is _____.

Check with a calculator: _____

**b)** 55
Perfect squares on either side of 55:

$7^2$ = _____ $\square^2$ = _____

$\sqrt{55}$ is between _____ and _____.

The closer square root is _____.

An estimate is _____.

Check with a calculator: _____

**4.** Estimate each value.
Round your answer to 1 decimal place.
Check your answer with a calculator.

**a)** $\sqrt{14}$

Perfect squares on either side of 14:

_____, _____

The closer square root is _____.

An estimate is _____.

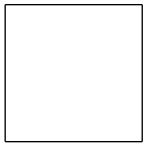
Check with a calculator: _____

**b)** $\sqrt{86}$

**5.** What are all the whole numbers with a square root between 2 and 3?

$2^2$ = _____
$3^2$ = _____

**6.** Kai uses 1 whole can of paint on a square wall.
One can covers 27 $m^2$.
Estimate the side length of the wall. Round your answer to 1 decimal place.

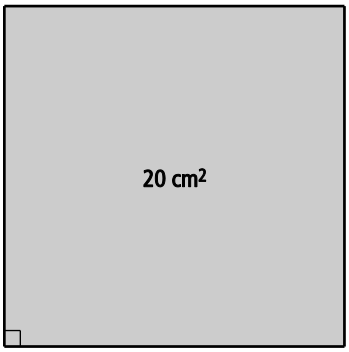



_____ m

_____ m

Sentence: _____

**7.** The square has an area of 20 $cm^2$.




**a)** Estimate the side length to 1 decimal place.

Perfect squares on either side of 20:

$4^2$ = _____ $\square^2$ = _____

$\sqrt{20}$ is between _____ and _____.

The closer square root is _____.

An estimate is _____.

Check with a calculator: _____

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https://youtu.be/7QzvW9aJuWc

### 3.4 Warm Up

**1.** Estimate $\sqrt{28}$ to 1 decimal place.

Perfect squares on either side of 28:

$5^2 =$ _____ $\square^2 =$ _____

$\sqrt{28}$ is between _____ and _____.

The closer square root is _____.

An estimate is _____.

**2.** Use a calculator to check your answer to #1. _____

**3.** List the numbers with a square root between 10 and 11.

$10^2 =$ _____ $11^2 =$ _____

The numbers are 101, _____, 120.

**4.** Find the square root.

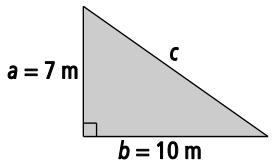
**a)** $\sqrt{36} =$ _____ **b)** $\sqrt{16} =$ _____

**c)** $\sqrt{100} =$ _____ **d)** $\sqrt{64} =$ _____

**e)** $\sqrt{49} =$ _____ **f)** $\sqrt{81} =$ _____

## Using the Pythagorean Relationship

### Example: Determine the Length of the Hypotenuse of a Right Triangle




Find the length of hypotenuse $c$.
Round your answer to the nearest tenth of a metre (1 decimal place).

***Solution***

The length of the hypotenuse is $c$.
The lengths of the legs are $a$ and $b$.

Use the Pythagorean relationship, $c^2 = a^2 + b^2$.

$a =$ _____ $b =$ _____

$c^2 = a^2 + b^2$

$= 7^2 + 10^2$

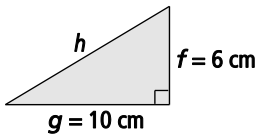
$=$ _____ + _____

$=$ _____

$c = \sqrt{\square}$

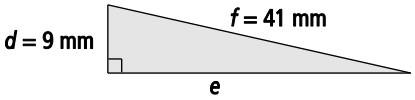
$c \approx$ _____

The length of the hypotenuse is about _____ m.



## Example : Determine the Length of a Leg of a Right Triangle

What is the length of leg $e$?




***Solution***

Use the Pythagorean relationship.
The letter for the length of the hypotenuse is _____.

The letters for the lengths of the legs are _____ and _____.
So, $f^2 = d^2 + e^2$
Substitute $d$, $e$, and $f$ into the equation.

$41^2 = 9^2 + e^2$

_____ = _____ $+ e^2$

$1681 - 81 = \cancel{81} + e^2 - \cancel{81}$ Subtract 81 from both sides.

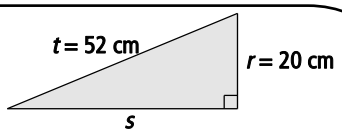
_____ $= e^2$

$\sqrt{\square} = e$

_____ $= e$

The length of the leg is _____ mm.

Find the length of leg $s$.




The letter for the length of the hypotenuse is _____.

The letters for the lengths of the legs are _____ and _____.

$t^2 = r^2 + s^2$

$\square^2 = \square^2 + s^2$

_____ = _____ $+ s^2$

_____ – _____ = _____ $+ s^2$ – _____

_____ $= s^2$

$\sqrt{\square} = s$

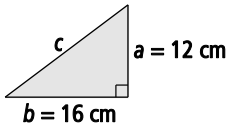
_____ $= s$

The length of the leg is _____ cm.

## Practise

**1.** Find the length of each hypotenuse.

**a)**




$c^2 = a^2 + b^2$

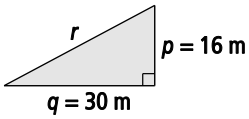
$c^2 =$ $\square^2$ + $\square^2$

$c^2 =$ _____ + _____

$c =$ $\sqrt{\square}$

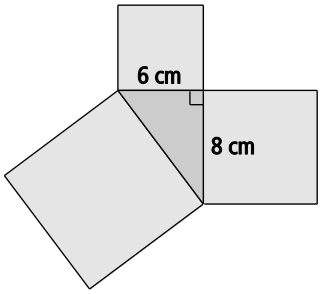
$c =$ _____ cm

**b)**




$r^2 =$ _____ + _____

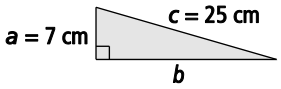
**2.** **a)** What is the area of each square attached to the legs of the right triangle?




**b)** What is the area of the square attached to the hypotenuse?

**c)** What is the length of the hypotenuse?

**3.** Find the missing length of the leg of the right triangle.




$c^2 = a^2 + b^2$

$\square^2 = \square^2 + b^2$

$\_\_\_\_\_ = \_\_\_\_\_ + b^2$

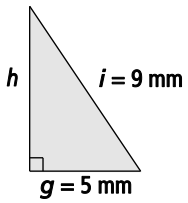
$\_\_\_\_\_ = b^2$

$\sqrt{\square} = b$

$\_\_\_\_\_ = b$

The length of the leg is _____ cm.

**4.** Find the missing length of the leg for the triangle.
Round your answer to the nearest tenth of a millimetre (1 decimal place).

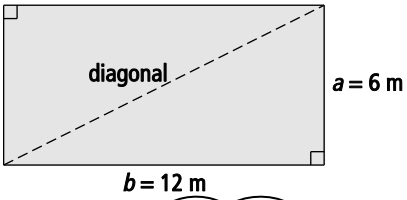



**5.** Tina wants to build a path across the diagonal of her yard.
How long will the path be?
Round your answer to the nearest tenth of a metre (1 decimal place).

Formula →

Substitute →

Solve →




What side of the triangle does the diagonal represent?

**Watch the following instructional video. In your handout:**

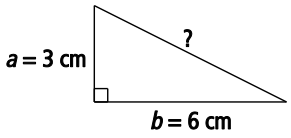
**i) Copy down the given notes and examples**

**ii) Complete the assigned questions**

https://youtu.be/B5jy2Edx0dI

## Warm Up

**1.** What is the length of the hypotenuse in the triangle? Round your answer to the nearest tenth of a centimetre (1 decimal place).

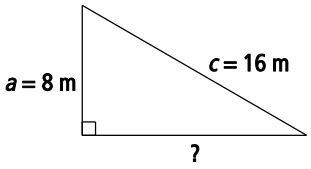



Formula →

Substitute →

Solve →

**2.** What is the length of the leg in the triangle? Round your answer to the nearest tenth of a metre (1 decimal place).



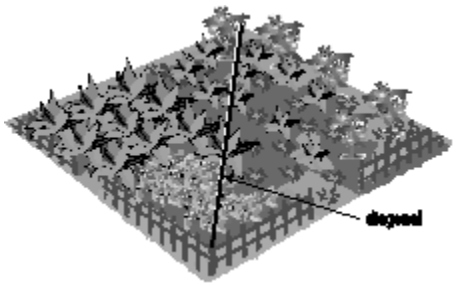

Formula →

Substitute →

Solve →

**3.** A square garden has an area of 36 $m^2$.

**a)** What is the length of each side?

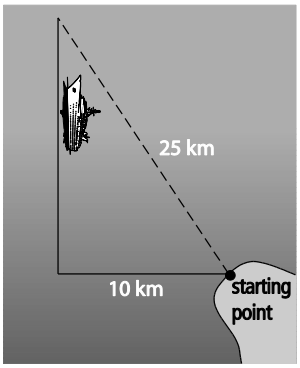


**b)** How long is a path running diagonally across the garden? Round your answer to the nearest hundredth of a metre (2 decimal places).

## Applying the Pythagorean Relationship

## Example: Determining Distances With Right Triangles

A ship travels west for 10 km.
Then, it turns and travels north.
If the ship is 25 km from its starting point, how far north did it travel?
Round your answer to the nearest tenth of a kilometre
(1 decimal place).




**a)** $c^2 = a^2 + b^2$

_____ = _____ + $b^2$

_____ = _____ + $b^2$

_____ = $b^2$

$\sqrt{\square}$ = $b$

_____ = $b$

Sentence: _____

**b)** The ship did not take the shortest route.
How much farther did the ship travel than the shortest route?

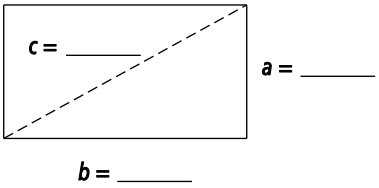
_____

### Example: Verify a Right Angle Triangle

*Verify* means check.

A construction company is digging a rectangular hole with a width of 17 m and a length of 20 m. A worker measures the diagonal length to be 26.25 m.

**a)** Label the rectangle with the dimensions.




**b)** Is the corner a right angle? Explain your answer.

$a^2 + b^2 = c^2$

Left Side:

$a^2 + b^2$

$= \square^2 + \square^2$

$=$ _____ + _____

$=$ _____

Right Side:

$c^2 = \square^2$

$c =$ _____

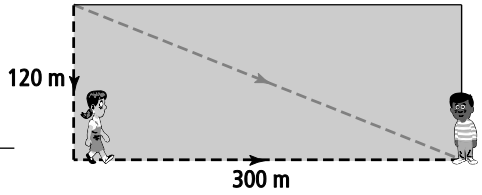
The left side is _____ to the right side.
*(equal* or *not equal)*

The corner _____ a right angle.
*(is* or *is not)*

## Practise

**1.** Walter and Maria live beside a rectangular field.

**a)** Maria walked around 2 sides of the field. How far did Maria walk?



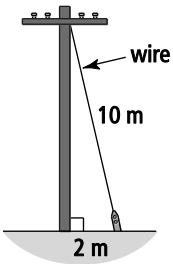

_____ + _____ = _____

Sentence: _____

**b)** Walter walked across the field in a diagonal line. How far did Walter walk? Round your answer to the nearest metre.

**c)** Who walked farther? By how much?

**2.** A wire is attached to the top of a pole and to the ground.
Find the height of the pole.
Round your answer to the nearest tenth of a metre (1 decimal place).




$c^2 = a^2 + b^2$

_____ = _____ + $b^2$

_____ = _____ + $b^2$

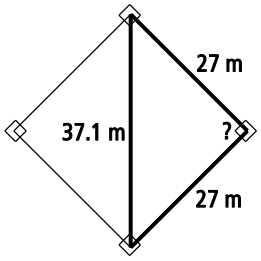
_____ = $b^2$

$\sqrt{\square}$ = $b$

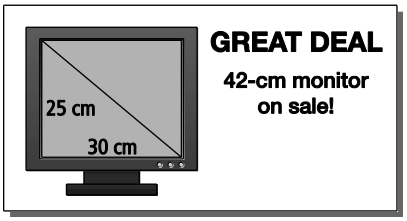
_____ = $b$

Sentence: _____

**3.** You are checking the design plans for a baseball diamond.
Is the triangle a right triangle? Explain.




**4.** The size of a computer monitor is based on the length of the diagonal of the screen.




**a)** How long does the ad say the diagonal is?

_____

**b)** Shahriar thinks that the diagonal is not as large as the ad says.
Is he correct? Calculate the length of the diagonal to find out.