Watch the following instructional video. In your handout:

i) Copy down the given notes and examples

ii) Complete the assigned questions

https://youtu.be/GTGZUcPg_os

Adding Integers

zero pair

- includes one $+$ and one $-$
- equals 0 because $(+1) + (-1) = 0$

Use integer chips or a number line to show $(+4) + (-1) = +3$.

Circle the zero pairs to show they equal 0.

$+4$

-1

2. Use the diagram to complete the addition statement.

a) $+$ $+$ $+$ $+$ $+$ $+$ $+$
    $-$ $-$ $-$ $-$

    $(+7) + (-4) =$ _____

b)

    −8 −7 −6 −5 −4 −3 −2 −1 0 +1

    Start at 0.

    $(-3) + ($ _____ $) =$ _____

3. Calculate.

    −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10

    a) $(+4) + (+5) =$ _____   b) $(-2) + (+8) =$ _____
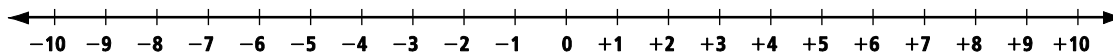
Subtracting Integers

Use integer chips or number lines to subtract integers.

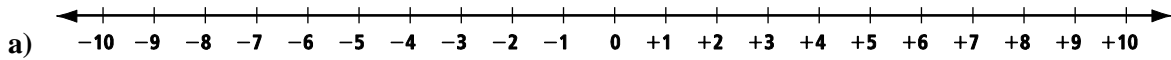**opposite integer**

- two integers with the same number but the opposite sign (+3 and –3)
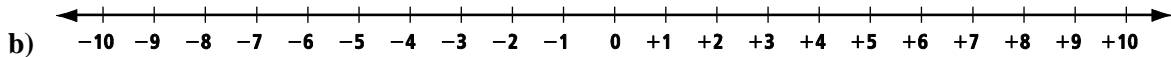
Subtract integers by adding the **opposite integer**.

$(+5) - (-4)$ $(-3) - (+2)$
$= (+5) + (+4)$ $= (-3) + (-2)$
$= +9$ $= -5$

4. Use a number line to subtract.

a)

$-10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad +4 \quad +5 \quad +6 \quad +7 \quad +8 \quad +9 \quad +10$

$(-6) - (-2) = (-6) + (\underline{\hspace{2cm}}) = \underline{\hspace{2.5cm}}$

b)

$-10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad +4 \quad +5 \quad +6 \quad +7 \quad +8 \quad +9 \quad +10$

$(-2) - (+6) = (-2) + (\underline{\hspace{2cm}}) = \underline{\hspace{2.5cm}}$

Order of Operations

Steps in the order of operations:

$8 \div 4 + (3 + 2) \times 6 - 7$ Do brackets first.

$= 8 \div 4 + 5 \times 6 - 7$ Multiply and divide in order from left to right.

$= 2 + 30 - 7$ Add and subtract in order from left to right.
$= 25$

5. Calculate. Show your work.

a) $8 + 6 \times 5 - 1$ Multiply.

    $= 8 + \underline{\hspace{1.5cm}} - 1$ Add.

    $= \underline{\hspace{1.5cm}} - 1$ Subtract.

    $= \underline{\hspace{1.5cm}}$

b) $3 \times (7 - 2) + 16 \div 4$ Brackets.

    $= 3 \times (\underline{\hspace{1.5cm}}) + 16 \div 4$ Multiply.

    $= \underline{\hspace{1.5cm}} + 16 \div 4$ Divide.

    $= \underline{\hspace{1.5cm}} + \underline{\hspace{1.5cm}}$ Add.

    $= \underline{\hspace{1.5cm}}$

Warm up

1. Draw a number line to solve each equation.

   a) (+4) + (–5)

   b) (+3) + (–2)

2. Subtract.

Example: (–1) – (+2)

(–1) – (+2)   Add the opposite.
= (–1) + (–2)
= –3

   a)   (–3) – (+4)

   = (–3) + (–4)   Add the opposite.

   = _____

   b)   2 – (–6)

   = 2 + (_____)

   = _____

   c) (–5) – (–2)

   d) 5 – (–6)

Practice:

Calculate the following (use any method):

1) -2 + 5

2) 8 + (-3)

3) -11 + (-2)

4) -9 + 11

5) 13 + (-11) + 4 + (-9)

6) -1 + 5 + 2 + (-5) + 3

Calculate the following by adding the opposite (the first one is done for you)

Original Problem	Problem Rewritten Adding the Opposite	Final Answer
7) -6 – (-1)	-6 + (+1)	-5
8) 5 – 13		
9) 8 – (-19)		
10) -7 – 9		
11) 3 – 5		

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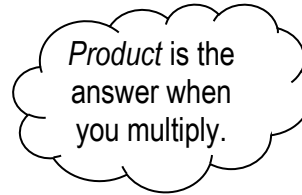
8.1 Exploring Integer Multiplication

Example: Multiply Using Integer Chips

Find each product using integer chips.
Write the multiplication statement for each equation.

> *Product* is the answer when you multiply.

a) $(+5) \times (+2)$

Solution

Draw 5 groups of 2 positive integer chips $(+2)$.

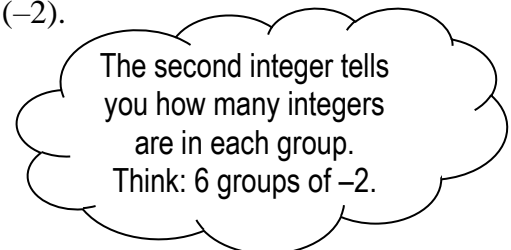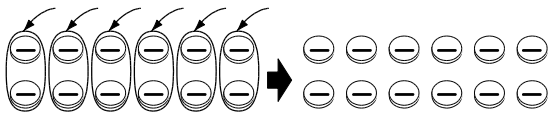> The first integer tells you the number of groups to draw.
> Think: 5 groups of +2.
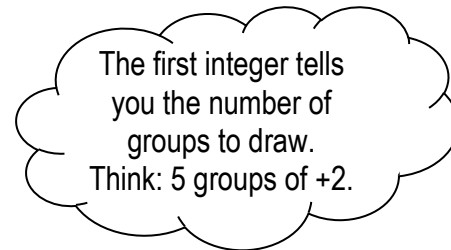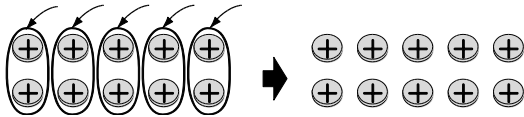
There are 10 positive integer chips.
The product is $+10$.

The multiplication statement is $(+5) \times (+2) = +$_____.

b) $(+6) \times (-2)$

Solution

Draw _____ groups of 2 negative integer chips (-2).

> The second integer tells you how many integers are in each group.
> Think: 6 groups of –2.

There are 12 negative integer chips.

The product is $-$_____.

The multiplication statement is $(+6) \times (-2) = -$_____.

Find each product. Draw integer chips to show your thinking.

a) $(+4) \times (+2)$
Draw 4 groups of 2 positive integer chips.

There are ———————— integer chips.

$(+4) \times (+$————————$) = $ ————————

b) $(+5) \times (-2)$

Draw ———————— groups of 2 ———————————— integer chips.

There are ———————— integer chips.

$($————————$) \times (-2) = -$————————

c) $(-4) \times (+2)$. This can also be written as $(+2) \times (-4)$

Draw 2 groups of 4 negative integer chips.

There are ———————— integer chips.

$(-4) \times (+2) = $ ————————

Practise

1. Write each repeated addition as a multiplication statement.

    a) $(+1) + (+1) + (+1) + (+1) + (+1)$

       $= (\underline{\hspace{2cm}}) \times (+1)$

    b) $(-6) + (-6)$

       $= (\underline{\hspace{2cm}}) \times (\underline{\hspace{2cm}})$

2. Write each multiplication statement as a repeated addition.

    a) $(+3) \times (+8)$

       $= (\underline{\hspace{1.5cm}}) + (\underline{\hspace{1.5cm}}) + (\underline{\hspace{1.5cm}})$

    b) $(+5) \times (-6)$

3. Write the multiplication statement for each diagram.

    a)

    $(+2) \times (\underline{\hspace{1.5cm}}) = \underline{\hspace{1.5cm}}$

    b)

    $(\underline{\hspace{1.5cm}}) \times (\underline{\hspace{1.5cm}}) = \underline{\hspace{1.5cm}}$
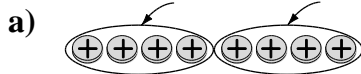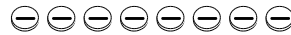
4. Complete each multiplication statement.

    a) $(+4) \times (+6)$

   Draw $\underline{\hspace{2.5cm}}$ groups of

   $\underline{\hspace{2.5cm}}$ positive integer chips.

   There are $\underline{\hspace{2.5cm}}$ integer chips.

   $(+4) \times (+6) = \underline{\hspace{2.5cm}}$

    b) $(+7) \times (-2)$

   Draw $\underline{\hspace{2cm}}$ groups of 2

   $\underline{\hspace{2.5cm}}$ integer chips.

   There are $\underline{\hspace{2cm}}$ integer chips.

   $(+7) \times (\underline{\hspace{1.5cm}}) = \underline{\hspace{2cm}}$

5. Complete each multiplication statement.
Draw integer chips to help you.

a) $(-1) \times (+5)$

b) $(-8) \times (+2)$

There are ＿＿＿＿＿＿＿

＿＿＿＿＿＿＿＿＿ integer chips.

(＿＿＿＿) × (＿＿＿＿) = ＿＿＿＿

There are ＿＿＿＿＿＿＿

＿＿＿＿＿＿ integer chips.

(＿＿＿＿) × (＿＿＿＿) = ＿＿＿＿

6. Write a multiplication statement to represent each problem.

Draw integer chips or a thermometer to help you.

a) The temperature increased 2 °C per hour for 6 h.
What was the total temperature change?

6 h = (+＿＿＿＿＿＿)
Temperature increase of 2 °C = +2

(＿＿＿＿＿＿) × (＿＿＿＿＿＿) = ＿＿＿＿＿＿

The temperature increased by ＿＿＿＿＿ °C.

b) Ayesha repaid some money she owed in 4 payments of $8 each.
How much money did Ayesha repay?

4 payments = (+＿＿＿＿＿＿)

$8 payments = (＿＿＿＿＿＿)

Is 8 positive or negative?
Repaid means she lost money.

(＿＿＿＿＿＿) × (＿＿＿＿＿＿) = ＿＿＿＿＿＿

Ayesha repaid $＿＿＿＿＿＿.

7. An oil rig is drilling a well at 2 m/min.
How deep is the well after the first 8 min?

2 m deep = (–_____)

8 min = (_____)

(_____) × (_____) = _____

The well is _____ m deep.

Deep means the answer has a negative sign.
Do not write the negative sign in your statement.

8. An aircraft descends at 3 m/s for 12 s.
How far does it descend?

Multiply 2 integers.

Sentence: _____

8.2 Multiplying Integers

Example: Multiply Integers

sign rule for multiplication

• the product of two integers with the same sign is positive

$$\oplus \times \oplus = \oplus \qquad \ominus \times \ominus = \oplus$$

• the product of two integers with different signs is negative

$$\oplus \times \ominus = \ominus \qquad \ominus \times \oplus = \ominus$$

a) Calculate $(+3) \times (+4)$.

Solution

Multiply the numbers: $3 \times 4 = $ _____
Apply the sign rule: The product of 2 integers with the same signs is positive.

$(+3) \times (+4) = +$ _____ $\oplus \times \oplus = \oplus$

b) Calculate $(+2) \times (-9)$.

Solution

Multiply the numbers: $2 \times$ _____ $=$ _____
Apply the sign rule: The product of 2 integers with different signs is negative.

$(+2) \times ($ _____ $) = -$ _____ $\oplus \times \ominus = \ominus$

c) Calculate $(-6) \times (-4)$.

Solution

Multiply the numbers: _____ $\times$ _____ $=$ _____

Apply the sign rule: The product of 2 integers with the same signs is _____ .

_____ $\times$ _____ $=$ _____ $\ominus \times \ominus = \oplus$

Example: Apply Integer Multiplication

Tina takes $35 out of her bank account each month to give to charity.
Estimate and calculate the amount she gives in a year.

Solution
Write a multiplication statement:
Taking out $35 = –35
12 months = +12
$(+12) \times (–35)$

Calculate:

$(+12) \times (–35) =$ _____

Tina gives _____ a year to charity.

Give means the answer has a negative sign.
Do not write the negative sign in your
statement.

Duane puts $65 a month into a savings account.
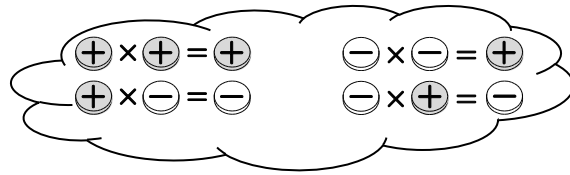How much money does he have in the account after 18 months?

Savings of $65 = (_____)   18 months = (_____)

(_____) × (_____) = _____

Duane has $_____ in his account.

Practise

$$(+) \times (+) = (+) \qquad (-) \times (-) = (+)$$
$$(+) \times (-) = (-) \qquad (-) \times (+) = (-)$$

1. Find the product.

   a) $(+10) \times (+4) =$ _____

   b) $(+6) \times (-5) =$ _____

   c) $(-7) \times (+5) =$ _____

   d) $(-8) \times (-4) =$ _____

2. A telephone company offers a $15 discount per month.
   How much is the annual discount?

   Discount means $15 off your bill.
   Annual means each year.

   $15 discount = (_____)

   12 months = (_____)

   Multiplication statement: (_____) × (_____) = _____

   The discount is $ _____.

3. Complete each statement.

   a) $(+6) \times ($_____$) = +18$

   b) $($_____$) \times (-2) = -10$

   c) $($_____$) \times (+3) = -12$

   d) $(-4) \times ($_____$) = +16$

4. A hot-air balloon is descending at 60 m/min.
   How far does it go down in 25 min?

   Descending means
   going down.

   Descending 60 m = (_____)

   25 min = (+_____)

   Multiplication statement: (_____) × (_____) = _____

   Sentence: _____

5. Astronauts train for space using deep dives on a plane.
The plane can descend at 120 m/s for 20 s.
How far does the plane descend?

Sentence: _____

8.3 Warm Up

1. Multiply.

   a) $(+2) \times (-7) =$ _____   **b)** $(-5) \times (-6) =$ _____

2. Divide.

   a) $12 \div 6 =$ _____   **b)** $18 \div 2 =$ _____

3. Fill in the missing integers.

   a) $(+10) \times (-4) =$ _____   **b)** $(-2) \times$ _____ $= (-10)$

4. Fill in the blanks.

  a) If 2 integers have the same sign, the product is _____.

     Examples: $(-7) \times (-4) =$ _____ and $(+7) \times (+4) =$ _____

  b) If 2 integers have different signs, the product is _____.

     Examples: $(-5) \times (+6) =$ _____ and $(+5) \times (-6) =$ _____

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https://youtu.be/PeJo9fx_TAI

8.3 Dividing Integers

Example 1: Divide Integers

sign rule for division

- the quotient of two integers with the same sign is positive

$$(+) \div (+) = (+) \qquad (-) \div (-) = (+)$$

- the quotient of two integers with different signs is negative

$$(+) \div (-) = (-) \qquad (-) \div (+) = (-)$$

a) Calculate $(+6) \div (+2)$.

Solution

Divide the numbers: $6 \div 2 = 3$
Apply the sign rule: The quotient of 2 integers with the same sign is positive.

$(+6) \div (+2) = +3$

b) Calculate $(-12) \div (-6)$.

Solution

Divide the numbers: $12 \div 6 =$ _____

Apply the sign rule: The quotient of 2 integers with the same sign is _____.

$(-12) \div (-6) = +$_____

c) Calculate $(-20) \div (+4)$.

Solution

Divide the numbers: $20 \div 4 =$ _____
Apply the sign rule: The quotient of 2 integers with different signs is negative.

$(-20) \div (+4) = -$_____

d) Calculate $(+42) \div (-14)$.

Solution

Divide the numbers: $42 \div 14 = $ _____
Apply the sign rule: The quotient of 2 integers with different signs is _____.

$(+42) \div (-14) = $ _____

Example 2: Apply Integer Division

Daria and 4 friends went out for lunch. The total cost was $85. They divided the cost equally.

How much did each person pay?

Solution

Write a division statement.

Total cost of $85 = (–85)

Daria plus 4 friends = (+_____)

(–85) ÷ (+5) = (–_____)

Each person has to pay $_____.

Pay means the answer has a negative sign.
Do not write the negative sign.

Check:

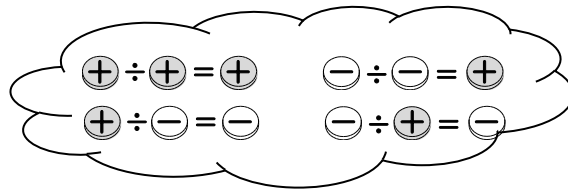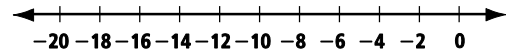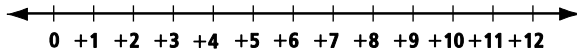Use multiplication to check the division.

Answer × divisor = cost of the bill

(–_____) × (+5) = _____

1. Find the quotient using a number line.

    a) $(+12) \div (+6) =$ _____

    b) $(-20) \div (-4) =$ _____

$$+ \div + = + \qquad - \div - = +$$
$$+ \div - = - \qquad - \div + = -$$

2. Calculate.

    a) $(+20) \div (+5) =$ _____

    b) $(+36) \div (-6) =$ _____

    c) $(-57) \div (+19) =$ _____

    d) $(-84) \div (-42) =$ _____

3. Complete each statement.

    a) $(+15) \div$ _____ $= (-3)$

    b) _____ $\div (+2) = (+10)$

    c) $(-8) \div (+4) =$ _____

    d) $(-30) \div$ _____ $= (-6)$

4. Raoul borrowed $15 per month from his mother for art supplies.
At the end of his art course, he owed his mother $60.
How long was the course?
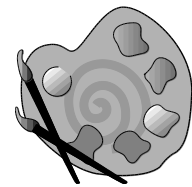
Amount borrowed per month = (_____)

Amount Raoul owes his mother = (_____)

Division statement: (_____) ÷ (_____) = _____

The course was _____ months long.

Check:

_____ × _____ = _____

5. The school spent $384 to buy 32 calculators.
What was the cost of 1 calculator?

$384 spent = (———————)

Number of calculators = (———————)

Division statement: ——————————————

Sentence: _____

https://youtu.be/oWB7cGuSFrc

8.4 Warm Up

1. Complete each division statement.

a) $(+12) \div$ _____ $= (+3)$ 　　　　**b)** (_____) $\div (+5) = (-4)$

2. Add

a) $(-8) + (5) =$ _____ 　　　　**b)** $(-3) + (-8) =$ _____

3. Subtract.

a) 　$(-4) - (+8)$ 　　　　　　　　　　　　**b)** 　$(+7) - (-5)$

　　　$= (-4) + (-8)$ 　← Add the opposite. → 　　$= (+7) + ($ _____)

　　　$=$ _____ 　　　　　　　　　　　　$=$ _____

4. Solve. 　⟨ Use the sign rules. ⟩

a) $(+3) \times (-7) =$ _____ 　　　　**b)** $(-2) \times (-11) =$ _____

c) $(+14) \div (-7) =$ _____ 　　　　**d)** $(-32) \div (-4) =$ _____

5. Calculate. 　⟨ Use the order of operations. ⟩

a) 　$10 + 24 \div 6$ 　　　　　　　　　　　　**b)** 　$7 - 3 \times (2 + 1)$

　　　$= 10 +$ _____ 　　　　　　　　　$= 7 - 3 \times$ _____

　　　$=$ _____ 　　　　　　　　　　　　$= 7 -$ _____

　　　　　　　　　　　　　　　　　　　　　　　　　$=$ _____

c) 　$3 + 3 + 4 \div 2$ 　　　　　　　　　　　**d)** $5 \times 4 \div 2$

8.4 Applying Integer Operations

Example 1: Use the Order of Operations

order of operations

- the order of steps for a calculation

  Step 1: Brackets.

  Step 2: Multiply and divide in order from left to right.

  Step 3: Add and subtract in order from left to right.

a) Calculate $(-15) \div (-3) - (+4) \times (-2)$.

Solution

$(-15) \div (-3) - (+4) \times (-2)$ Multiply and divide in order.

$= (+5) - (+4) \times (-2)$

$= (+5) - (\underline{\hspace{3cm}})$ Subtract.

$= (+5) + (+\underline{\hspace{3cm}})$ Add the opposite of -8.

$= +\underline{\hspace{2.5cm}}$

b) Calculate $(-6) - (-9) + (-14) \div (+2)$.

Solution

$(-6) - (-9) + (-14) \div (+2)$ Divide.

$= (-6) - (-9) + (\underline{\hspace{3cm}})$ Add and subtract in order.

$= (-6) + (+9) + (\underline{\hspace{3cm}})$ Add the opposite.

$= (+3) + (-7)$

$= \underline{\hspace{3cm}}$

c) Calculate $-8 + (-2) \times [4 + (-1)]$.

Solution

$-8 + (-2) \times [4 + (-1)]$ ⟍⟋ Brackets.

$= (-8) + (-2) \times (+\underline{\hspace{2cm}})$ Multiply.

$= (-8) + (-\underline{\hspace{2cm}})$ Add.

$= -\underline{\hspace{2cm}}$

Practise

1. Calculate using the order of operations.

a) $(+30) \div (-10) + (-20) \div (-1)$ ⟍⟋ Divide.

$= \underline{\hspace{2cm}} + (-20) \div (-1)$ Divide.

$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$ Add.

$= \underline{\hspace{2cm}}$

b) $(-2) \times [(+1) + (+2)] + (-7)$ ⟍⟋ Brackets.

$= (-2) \times \underline{\hspace{2cm}} + (-7)$ Multiply.

$= \underline{\hspace{2cm}} + (-7)$ Add.

$= \underline{\hspace{2cm}}$

2. Calculate.

a) $(4 - 7) \times 2 + 12$

$= \underline{\hspace{2cm}} \times 2 + 12$

$= \underline{\hspace{2cm}} + 12$

$= \underline{\hspace{2cm}}$

b) $-10 \div 5 + 3 \times (-4)$

$= \underline{\hspace{2cm}} + 3 \times (-4)$

$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

3. The temperature of a new freezer, before it is plugged in, is 22 °C. When it is plugged in, the temperature drops to –10 °C.

a) Find the temperature change.

Start temperature of 22 °C = (—————)

End temperature of –10 °C = (—————)

Temperature change = end temperature – start temperature

Sentence:

_____