## LG 10 Adapted - Graphing

We have used number lines many times in the past to show how numbers can be added, subtracted, multiplied, and divided. This shows number lines can be used to show data very well.

We will be looking at a graph called the cartesian graph, named after René Descartes. A Cartesian graph is basically 2 number lines. The number line that goes up and down is called the y -axis. The number line that goes side to side is called the x -axis.


To use the graph, we need coordinates so that we know where to put data points (the dots on the graph that represent data).

## Coordinates can be given:

1) In an ordered pair. An ordered pair is always 2 numbers in brackets, separated by a comma.

Examples:

| $(-3,-4)$ | $(2,4)$ | $(-1,-2)$ | $(4,-2)$ | $(5,5)$ |
| :--- | :--- | :--- | :--- | :--- |

## Note:

- The ordered pair always lists the $\mathbf{x}$-value first. This is how far along the x -axis the point goes.
- The ordered pair always lists the $y$-value second. This is how far along the $y$-axis the point goes.
- $(\mathbf{0}, \mathbf{0})$ always represents the middle of the graph, and is called the origin.

Question: $(-3,-4)$ is already on the graph. Place the other coordinates on the graph.
2) As a table. We can list the $x$-values and the $y$-values in a table, where each row of the table represents one coordinate (one dot). It is standard to list the $x$-values in the left column, and $y$ values in the right column.

Example:

|  |  |
| :--- | :--- |
| -2 | 3 |
| 1 | 4 |
| 0 | 0 |
| 1 | 2 |
| 2 | -1 |
| -4 | -3 |
| -5 | -1 |



We plot each row by placing a dot in at the place where the $x$ and $y$ values match the ones in each of the table's rows.

Question: The coordinate $(-2,3)$ has already been placed on the graph. Place the rest of the coordinates on the graph.

Note: All graphs that have real-world meaning must have labels for the $x$ and $y$ axis that tell us what they represent. For example:

The $x$-axis could be labeled: Time in minutes (min)
The $\mathbf{y}$-axis could be labeled: distance away from school in kilometers (km)

## Linear graphs:

A linear graph is a graph that shows all the dots in a line.
A Graph is linear if the $y$ values always change by the same amounts when $x$ is changed by a consistent amount.

## Example:

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -1 | -4 |
| 0 | -2 |
| 1 | 0 |
| 2 | 2 |
| 3 | 4 |

Question: How much does the y change when $x$ goes up by 1? $\qquad$
Plot this graph and show the line it makes.


A graph is not linear if the $y$-values changes by a different amount each time $x$ is changed by a consistent amount.

Example:

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

Question: What happens to $y$ in this list when $x$ goes up by 1? $\qquad$
Plot this graph. Does it make a line with no turns?


## The slope-intercept form:

All straight lines can be represented by a formula called the slope-intercept form. The equation looks like this:

$$
y=m x+b
$$

Normally, there will be a number where $\mathbf{m}$ is and a number where $\mathbf{b}$ is.
Example:

$$
y=2 x+4
$$

The " $b$ " represents the $y$-intercept. This is where a line passes the $y$-axis (up and down number line). This can be a positive or negative number.

In the example, the line will pass the $y$-axis at $y=4$.
The " $m$ " represents the slope of the line (how steep the line is). The bigger the " $m$ " number, the steeper the slope.

The slope (steepness) can be represented by:
$m=\frac{\text { rise }}{\operatorname{run}}$

In the example, the slope is $\mathbf{2 . 2}$ can be written as:
$\perp$
Therefore, between each coordinate, the graph will have risen $\qquad$ , and run to the right by
$\qquad$ -.

Note:

1. If there is no number in front of $x$, assume there is an implied 1 and that the slope is 1 .
2. If there is no "b" number, assume the $y$-intercept is 0 .

## Practice questions:

Determine the $\mathbf{m}$ and $\mathbf{b}$ values of the following equations:


## Finding $m$ and $b$ from a graph:

To find the $\mathbf{y}$-intercept (b) from a graph:
Just find the $y$-value where the line passes the $y$-axis (up and down number line).
To find the slope (m) from a graph:

1) Find two coordinates that are on the grid
2) Find how much it rises between the two points
3) Find how much it runs between the two points
4) The slope is the rise over run

## Example 1:



## Example 2:



The line crosses the $y$-axis at -1 .
Therefore, the $\mathbf{b}$-value is -1

Let's use the points $(1,1)$ and $(2,3)$ because they are on the line.

Slope is rise over run:

- The line rises by 2 .
- The line runs by 1 .

Therefore, the slope is:
$\frac{2}{1}$, which is just 2
Therefore, the equation of the line will be:


The line crosses the $y$-axis at 2 .

Therefore, the $\mathbf{b}$-value is 2 .
Let's use the points $(0,2)$ and $(3,0)$ because they are on the line.

The line rises by -2 .

The line runs by 1.
Therefore, the slope is:
$m=-\frac{2}{3}$
Therefore, the equation of the line will be:


## Practice Questions:

Find the $\mathbf{m}$ and the $\mathbf{b}$ values of each graph and fill in the equation:
a)

b)

$\mathrm{m}=$ $\qquad$ $b=$
$\mathrm{m}=$ $\qquad$ $b=$ $\qquad$
Equation $: y=\square x+\square \quad$ Equation $: y=\square$
c)

$\mathrm{m}=$ $\qquad$
$b=$ $\qquad$
Equation :

Name:

$\mathrm{m}=\ldots \quad \mathrm{b}=\ldots$
Equation : $y=\square x+\square$
e)

$\mathrm{m}=$ $\qquad$
f)

$m=$

$b=$ $\qquad$
Equation :

## Graphing using an equation:

Many times, you will be asked to draw the graph of a given equation. To do this there are two main ways:

Method 1: Pick some sample $x$-values and build a table of $x$ vs $y$ values.
We will try with the equation:
$y=3 x-1$
The $x$-values chosen are a sample of numbers that will give us whole numbers for the $y$-column. In the table that follows, I randomly chose -2, -1, 0, 1, 2.
$y=3(-2)-1$
$y=\square-1$
$y=\square$

| $x$ | $y$ |
| :--- | :--- |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |



Method 2: The other way to graph lines without a table is to start at the $y$-intercept and rise/run using the slope given.

Example:
$y=\frac{1}{2} x-1$


We should start with the $y$-intercept ( $b$ value). We should place a dot at the point ( $0,-1$ ).
Then, we can use the slope to find where the next point will be.
The slope is $\mathbf{1}$ over 2, and the slope means rise over run. So, we need to move up by 1 and move to the right 2 .

Practice: Using the diagram, continue this pattern to create the line.

Name:
TA:

Questions: Create the graphs of the following linear equations.
${ }^{\text {1) }}=2 x+1$

2)
$y=-3 x+7$

3)
$y=\frac{3}{4} x+1$


Name:
TA:
4)
$y=5 x-2$

5) $y=2 x$

6)
$y=-\frac{3}{2} x+8$

7)

$$
y=-\frac{4}{5} x+7
$$



## Horizontal and Vertical Lines:

If a line goes straight left to right, it is a horizontal line.

Remember that slope is rise over run. A horizontal line has a rise of zero, because it is not going up or down. This means that the slope is zero.

Example:
$y=0 x+2$
$y=2$

In this case, the slope is zero, which, as seen on in the equation, $y=2$, the slope is often just not written down at all.

Because $y=2$, it does not matter what $x$ (the run) is, $y=2$ !
$\frac{0}{5}=0 \quad \frac{0}{2}=0 \quad \frac{0}{17}=0 \quad \frac{0}{20}=\square$

| $x$ | $y$ |
| :---: | :---: |
| -2 | 2 |
| -1 | 2 |
| 0 | 2 |
| 1 | 2 |
| 2 | 2 |



If a line goes straight up and down, it is a vertical line.
The vertical line has a run of zero. This means that for the slope, we are dividing the rise by a run of zero... which is impossible! So, we say the slope is undefined (no numerical answer). Try to put it into your calculator, it will not work!


To write the equation of a vertical line, we write it in terms of $x$.
Example:
$x=-3$
This means no matter what y is, x is equal to - 3

| $x$ | $y$ |
| :--- | :--- |
| -3 | -2 |
| -3 | -1 |
| -3 | 0 |
| -3 | 1 |
| -3 | 2 |



Draw the following vertical or horizontal lines AND state their slope:

1) $y=-2$

## Slope:

2) $x=5$

## Slope:


3) $y=-2$

Slope:

4) $x=-1$

## Slope:



Finding data on a graph (interpolation and extrapolation):
Once we have a graph, it can be used to find/approximate useful numbers that we want to know.

For example, to the right is a graph that shows the amount of money made per ice cream sold. The formula used was:

$$
y=4 x-2
$$

Where the $y$-axis represents the amount of money, and the $x$-axis represents the number of ice cream cones sold.

Where the slope of 4 means each ice cream is being sold for 4 dollars, and the -2 represents \$2 dollar's worth of costs to make some ice cream.

To find out how much we would have if we sold 2 ice cream cones (graph 1):

- Start on the $x$-axis (because it represents the ice cream cones)
- Find 2 and draw a line up until you hit the diagonal line.
- Then, draw a line over to the $y$-axis.
- We see the number that is on the $y$-axis is 6 dollars, so we would make 6 dollars for 2 ice creams sold.


To find out how many ice cream cones must be sold to have 12 dollars:

- Start on the $y$-axis (because it represents the money we will have)
- Find 12 and draw a line to the right until you hit the diagonal line.
- Then, draw a line down to the $x$-axis.
- We see we land right in between 3 and 4, so the answer is 3.5 ice creams, or 3 and a half ice creams. Since people are unlikely to buy half an ice cream, we would probably have to round up and sell 4 ice creams.


## Practice questions:

Use the graphs to find the numbers being asked for.

1) What is the $y$-value when $x$ is 4 ?

$y=$
2) What is the $x$-value when $y$ is 3 ?

$\mathrm{X}=$


X=
4) What is the $y$-value when $x$ is 2 ?

$y=$ $\qquad$
5) Approximate is the $x$-value when $y$ is 2? 6) What is the $y$-value when $x$ is 0.5 ?


$\mathrm{X}=$
$y=$ $\qquad$

